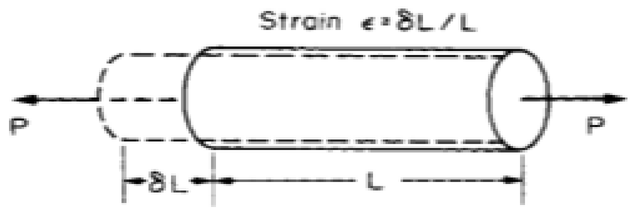
Chapter Tow

Simple strains

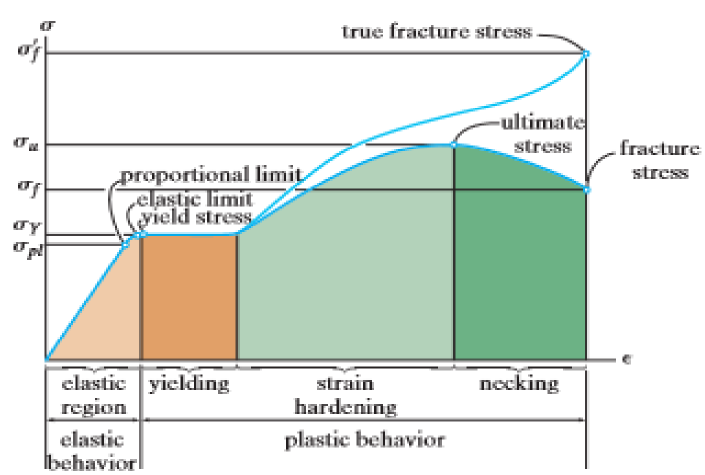


If a bar is subjected to a direct tension, the bar will change in length. If the bar has an original length "L" and change in length by an amount "δL" the strain produces is defined as follows

$$Strain = \frac{change\ in\ length}{original\ length} = \frac{\delta L}{L}$$

Strain represents a change in length divided by the original length, strain is dimensionless quantity. Strain assumed to be constant over the length under certain condition: -

1. The specimen must be constant cross section.
2. The material must be homogenous.
3. The load must be axial, that is produces uniform stress.



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Proportional Limit (Hook's Law)

From the origin to the point called proportional limit, the stress strain curve is a straight line. This linear relation was first noticed by Robert Hook in 1678 and is called Hook's Law that within the proportional limit, the stress is directly proportional to strain or

$\sigma \propto \epsilon$ or $\sigma = k\,\epsilon$

The constant of proportionality k is called the Modulus of Elasticity (E) or Young' Modulus and is equal to the slope of the stress-strain diagram from the origin to proportional limit. Then

$\sigma = E\,\epsilon$

Elastic limit

The elastic limit is the limit beyond which the material will no longer go back to its original shape when the load is removed, or it is the maximum stress that may be developed such that there is no permanent deformation when the load is entirely removed.

Elastic and Plastic Range

The region in stress -strain diagram from the origin to proportional limit is called the elastic range. The region from proportional limit to rupture is called plastic range.

Yield Point

Yield point is the point at which the material will have an appreciable elongation or yielding without any increase in load.

Ultimate Strength

The maximum coordinate in the stress -strain diagram is the ultimate strength or tensile strength.

Modulus of resilience

Modulus of resilience is the work done on a unit volume of material as the force is gradually increased from the origin to plastic limit in $N.m/m^3$. This may be calculated as the area under the stress-strain curve from the origin up to the elastic limit (the shaded area in the figure). The resilience of the material is its ability to absorb energy without creating permanent distortion.

Modulus of toughness

Modulus of toughness is the work done on the unit volume of material as the force is gradually increased from the origin to rupture in $N.m/m^3$. This may be calculated as the area under the entire stress-strain curve from the origin to rupture. The toughness of a material is its ability to absorb energy without causing it to break.

True Stress–Strain Diagram

Instead of always using the original cross-sectional area and specimen length to calculate the (engineering) stress and strain, we could have used the actual cross-sectional area and specimen length at the instant the load is measured. The values of stress and strain found from these measurements are called true stress and true strain, and a plot of their values is called the true stress–strain diagram

Necking

(a)

Failure of a ductile material

(b)

Working Stress, Allowable Stress and Factor of Safety.

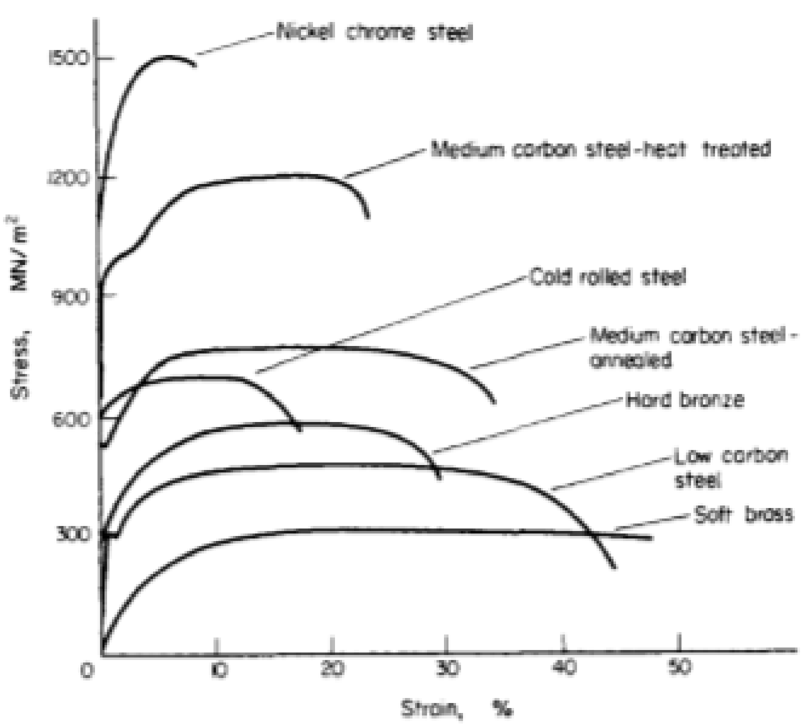
Working stress is defined as the actual stress of a material under a given loading. The maximum safe stress that the material can carry is termed as the allowable stress. The45678 allowable stress should be limited to value not exceeding the proportional limit. However, since proportional limit is difficult to determine accurately, the allowable stress is taken as either the yield point or ultimate strength divided by a factor of safety. The ratio of this strength (ultimate or yield strength) to allowable strength is called the factor of safety.

$$Factor\ of\ Safety = \frac{Maximum\ Stress}{Allowable\ Working\ Stress}$$

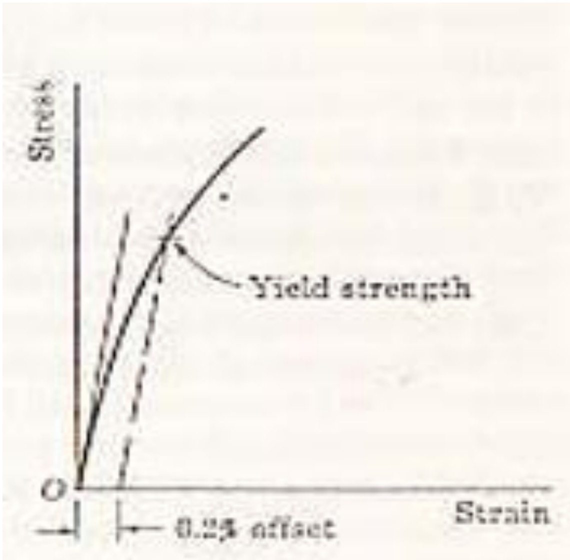
$\sigma w = \frac{\sigma yp}{Nyp}$ or $\sigma w = \frac{\sigma ult}{Nult}$

Typical values changes from **2.5**(for static loads) to **10** (for shock loads)

$\sigma w = \frac{1}{2}\ \sigma yp$



Comparative stress-strain diagram for different materials.



Yield strength determined by offset method.

Axial Deformation

In the liner portion of the stress -strain diagram, the stress is proportional to strain and is given by,

$$\sigma = E\epsilon$$

Since $\sigma = {}^{\delta}/_{L}$ and $\epsilon = {}^{\delta}/_{L}$,then

${}^{P}/_{A} = E\,{}^{\delta}/_{L}$

Solving for δ,

$\delta = \frac{PL}{AE}$ $\delta = \frac{\sigma L}{E}$

To use this formula, there are three restrictions:-

1- The load must be axial.
2- The bar must have constant cross-section and be homogenous.
3- The stress must not exceed the proportional limit.

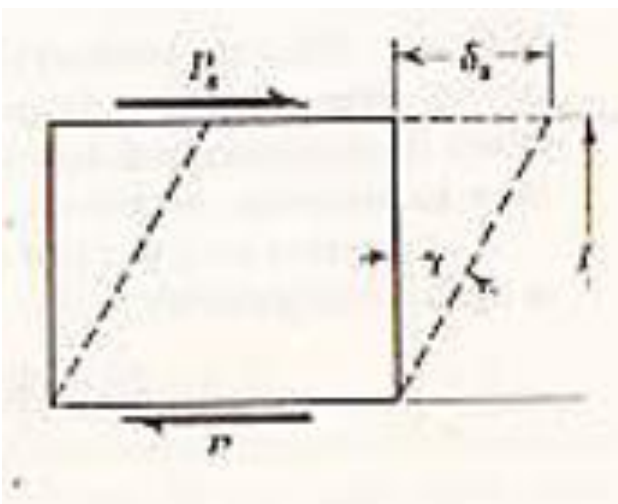
Shearing Strain

Shearing strain defined as the angular change between tow perpendicular faces of differential element.

The relation between shearing stress and shearing strain assuming Hook's Law to apply to shear

$$\frac{shear\ stress\ \tau}{shear\ strain\ \gamma} = constant = G$$

$$G: modulus\ of\ rigidity\ or\ shear\ modulus.$$



Shear deformation

$$tan\gamma = \frac{\delta s}{L}$$

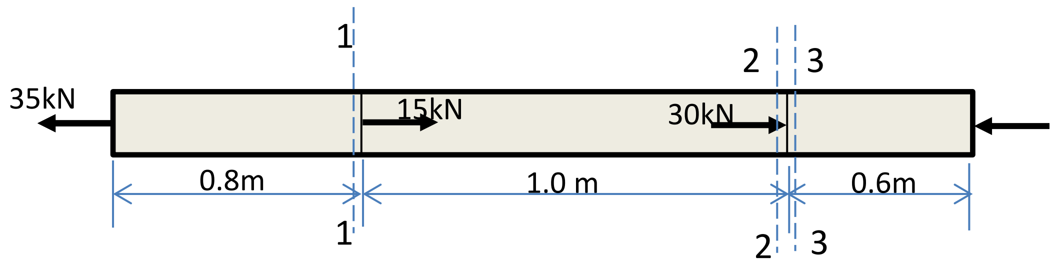
The angle γ is usually very small ,$tan\gamma = \gamma$

$$\gamma = \frac{\delta s}{L}$$

$\tau = G\gamma$, τ=V/A

$\delta s = \frac{VL}{AE}$, V is the shearing force acting over the shearing area As.

108 (singer): An aluminum bar having a cross-sectional area of 160 mm^2 carries the axial loads at positions shown in figure, if E=70Gpa, compute the total deformation of the bar. Assume the bar is suitably braced to prevent buckling.



Solution

$$\delta 1 = \frac{P1L1}{A1E1}$$

$$= \frac{\mathbf{35} * 10\text{^}380.8}{\mathbf{160} * 10^{-6} * 70 * 10^9} = \mathbf{0.0025\ m}\ \ (\boldsymbol{elongation})$$

$$\boldsymbol{\delta 2} = \frac{\boldsymbol{P2L2}}{\boldsymbol{AE}}$$

$$= \frac{\mathbf{20} * \mathbf{10^3} * \mathbf{1.0}}{\mathbf{160} * \mathbf{10^{-6}} * \mathbf{70} * \mathbf{10^9}} = \mathbf{0.00179\ m}\ (\boldsymbol{elongation})$$

$$\boldsymbol{\delta 3} = \frac{\boldsymbol{P3L3}}{\boldsymbol{AE}}$$

$$= \frac{\mathbf{10} * \mathbf{0.6} * \mathbf{10^3}}{\mathbf{160} * \mathbf{10^{-6}} * \mathbf{70} * \mathbf{10^9}} = \mathbf{0.00054\ m}\ \ (\boldsymbol{cntraction})$$

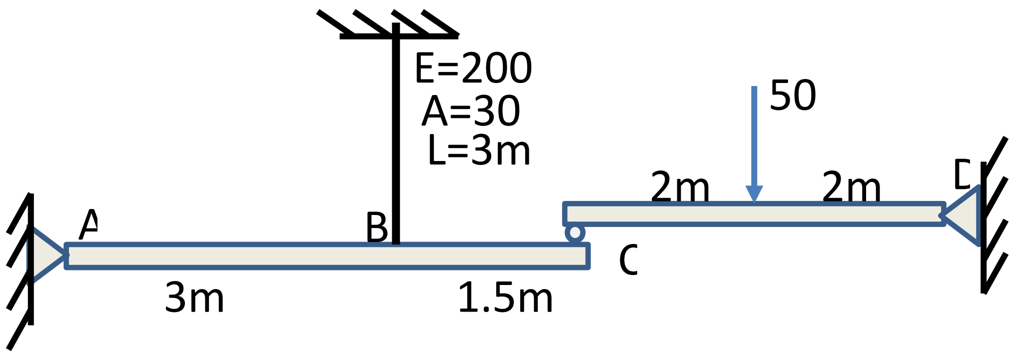
$$\boldsymbol{\delta} = \boldsymbol{\delta 1} + \boldsymbol{\delta 2} + \boldsymbol{\delta 3}$$

$$\boldsymbol{\delta} = \mathbf{0.0025} + \mathbf{0.00179} - \mathbf{0.00054}$$

$$\boldsymbol{\delta} = \mathbf{0.00375\ m}$$

$$= \mathbf{3.75\ mm}$$

211(singer): The rigid bars shown in figure are supported by roller at C and pinned at A and D. A steel rod at B helps support the load of 50 KN. Compute the vertical displacement of the roller at C.



Solution:

$$\sum MD = 0$$

$$50 \times 20 = Rcy \times 4$$

$$Rcy = 25\ kN$$

$$\sum MA = 0$$

$$25 \times 4.5 = 3T$$

$$T = 37.5\ kN$$

$$\delta B = \frac{TL}{AE}$$

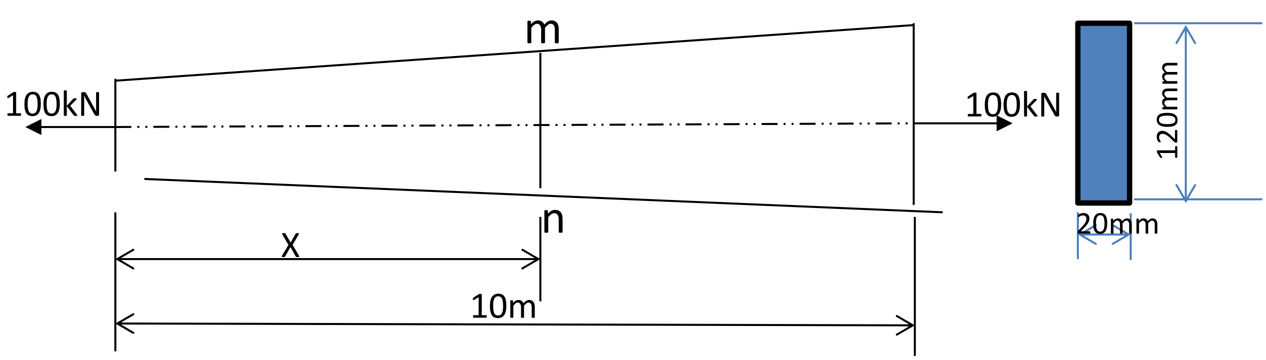
$$= \frac{37.5 \times 10^3 \times 3}{300 \times 10^{-6} \times 200 \times 10^9} = 0.00188\ \ mm$$

$$\frac{\delta c}{4.5} = \frac{\delta B}{3}$$

$$\delta c = 0.00281\ mm$$

$$= 2.81\ m$$

Example (201) Compute the total elongation caused by axial load of 100 KN applied to a bar 20mm thick,tapering from a width of (120mm)to (40mm) in length of (10m)as shown in figure.Assume E=200*10^9 N/m^2 .



Solution:

$$\frac{y-20}{x}=\frac{60-20}{10}\Rightarrow y=(4*x+20)$$

$$A=20(2y)=(160x+800)mm^2$$

@ sec m-n ,in a differential length dx ,the elongation may be found from

$$\delta=\frac{PL}{AE}=(\frac{100*10^3dx}{160X+800)*10^{-6}(200*10^9)}$$

$$=\frac{0.5dx}{160x+800}$$

From which the total elongation will be

$$\delta=0.50\int_0^{10}\frac{dx}{160x+800}=\frac{0.5}{160}\left[\ln(160x+800)\right]_0^{10}$$

$$=(3.13*10^{-3}\ln\frac{2400}{800}=3.44*10^{-3}=3.44mm$$

Example: For the shown member what is the maximum force " P " applied s that the maximum deformation is 0.25 mm , plot deformation diagram. Es=210 KN/mm^2 ,Eal=70 KN/mm^2 .

Solution :

$$\delta = \delta s + \delta al$$

$$\delta s = \frac{Ps\ Ls}{As\ Es} = \frac{P \times 300}{(50)^2 \times 210}$$

$$\delta al = \frac{P \times 380}{(100)^2 \times 70}$$

$$\frac{P \times 300}{(50)^2 \times 210} + \frac{P \times 380}{(100)^2 \times 70} = 0.25$$

$$P = 225.2\ \ kN$$

204 (singer): A uniform bar of length " L" ,cross-sectional area A, and unit mass ρ is suspended vertically from one end .Show that it's total elongation is $\delta = \rho g L^2/2E$. If the total mass of the bar is M, show also that $\delta = MgL/2AE$.

Solution:

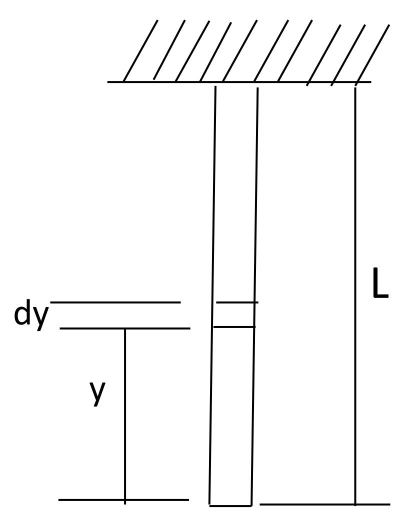
$$\delta = d\delta$$

$$P = Wy\ g = (\rho AY)g$$

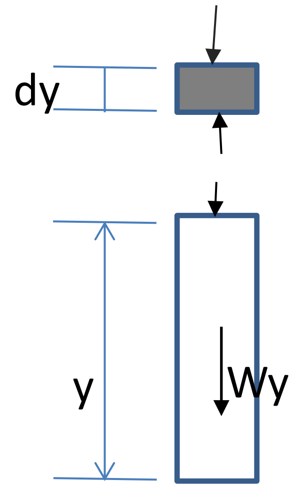
$$L = dy$$

$$d\delta = \frac{\rho AY}{AE} dy$$

$$\delta = \int_0^L \frac{\rho AYg}{AE}\ dy$$



$$= \frac{\rho g}{E} \int_0^L Y dy \quad ; \quad \delta = \frac{\rho g}{E} \frac{L^2}{2}$$



Given that total mass M:

$$\rho = \frac{M}{V} \; ; \; \rho = M/AL$$

$$\delta = \frac{M}{AL} \cdot \frac{g}{E} \cdot \frac{L^2}{2}$$

$$\delta = \frac{MgL}{2E}$$

OR another solution :-

Mass per unit length =M/L

Mass at $\quad y = \frac{M}{L} . y$

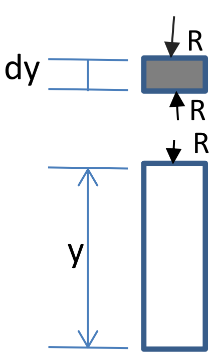
$$Wy = \frac{M}{L} . y$$

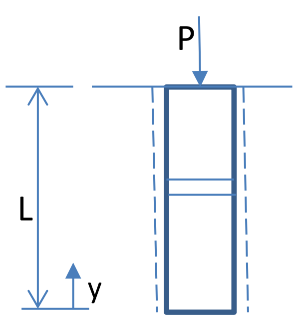
$$d\delta = \frac{\frac{M.g.y}{L} dy}{AE}$$

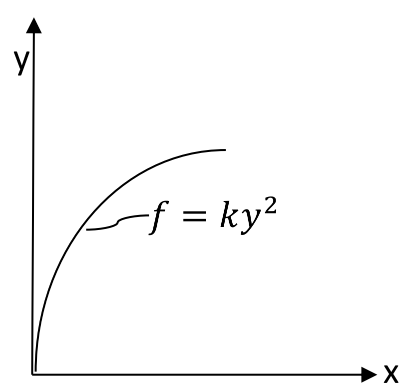
$$\delta = \frac{Mg}{LAE} \int_0^L y \, dy$$

$$= \frac{Mg}{LAE} \frac{y^2}{2} \Big]_0^L \quad ; \quad \delta = \frac{MgL}{2AE}$$

<u>Example :</u> A driven pile is supported to resist the force P. The force is transmitted to the soil by side friction (variable) as shown in figure. Find the total shortening in the pile.







Solution

$$P = \int f\ dy$$

$$P = \int_0^L ky\text{^}2dy$$

$$= \frac{ky\text{^}3}{3}\begin{matrix} L \\ 0 \end{matrix}\Bigg] = \frac{kL\text{^}3}{3}$$

$$k = \frac{3P}{L\text{^}3}$$

$$f = \frac{3P}{L\text{^}3}y^2$$

$$R = \int_0^y \frac{3P}{L\text{^}3}y^2 dy$$

$$= \frac{3P}{L\text{^}3}\frac{y^3}{3}\ \Big]_0^y$$

$$R = \frac{Py3}{L^3}$$

$$\delta = \frac{PL}{AE}$$

$$d\delta = \frac{R}{AE}dy$$

$$\delta = \int_0^L \frac{Rdy}{AE}$$

$$= \int_0^L \frac{Py^3}{L^3} \frac{dy}{AE}$$

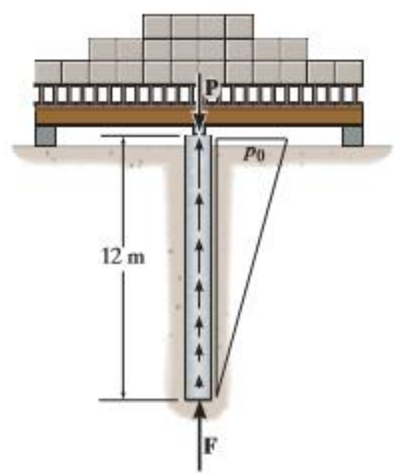
$$\delta = \frac{P}{AEL^3} \frac{y^4}{4} \Big]_0^L$$

$$\delta = -\frac{PL}{4AE}$$

$$A = \frac{\pi}{4} D^2 \qquad ; \quad \delta = \frac{PL}{\pi D^2 E}$$

<u>**4-30 (Hibbler)**</u>: The weight of kentlege exerts an axial force of P=1500 KN on the 300-mm diameter high strength concrete bore pile. If the distribution of the resisting skin friction developed from the interaction between the soil and the surface of the pile is approximated as shown ' and the resisting bearing force F is required to be zero' determine the maximum intensity P_0 KN/m for equilibrium . Also , find the corresponding elastic shortening of the pile. Neglect the weight of the pile. Ec=29 Gpa.

<u>**Solution**</u>:

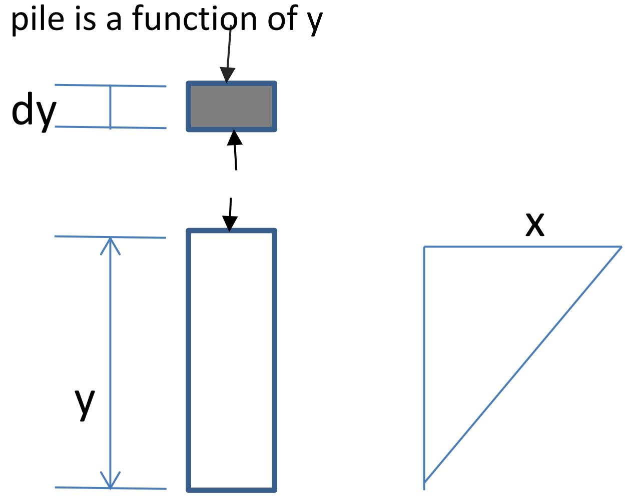


$$\sum Fy = 0 \uparrow +$$

$$F + \frac{Po * 12}{2} = 1500$$

$$Po = 250$$

*The normal force developed in the pile is a function of y



$$\delta = \frac{PL}{AE}$$

$$d\delta = \frac{P(y)dy}{AE}$$

$$\delta = \int_0^{12} \frac{P(y)dy}{AE} \quad ;$$

$$P(y) = \frac{x.y}{2} = \frac{Po.y^2}{24} = 10.42y^2$$

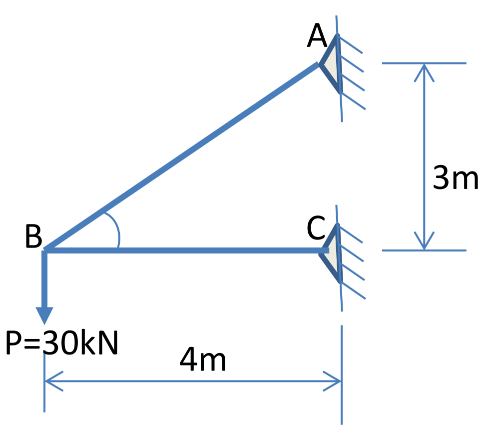
$$\delta = \int_0^{12} \frac{10.42.y^2}{AE} dy$$

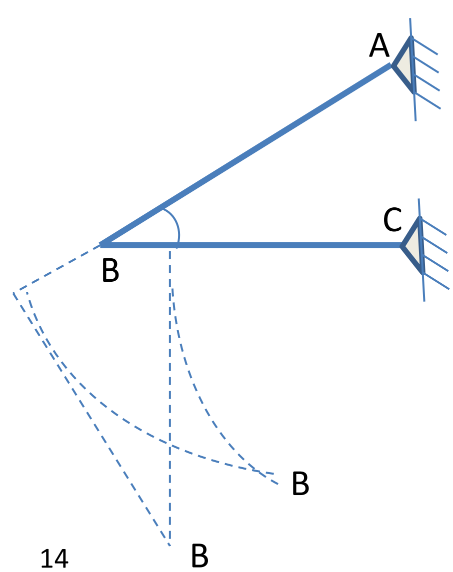
$$= = \frac{10.42}{AE}\left[\frac{y^3}{3}\right]_0^{12} \quad ; \; A = \frac{\pi}{4}(0.3)^2$$

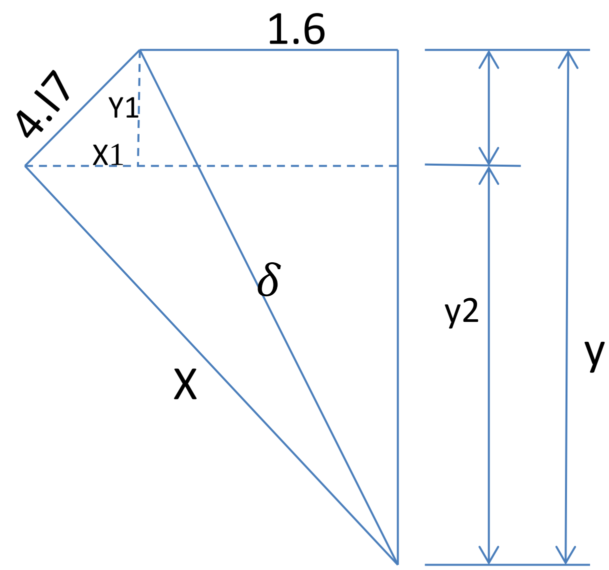
$$\delta = 2.93 * 10^{-3} m$$

$$= 2.93mm$$

<u>202(singer) :</u> Tow steel bars AB and BC support a load P=30KN as shown in figure .Area of ABis 300mm^2 ,area of bc is 500mm^2 .If E=200 GPa ,compute the horizontal and vertical movement of B.







From statics :

P_{AB} =50 kN (tension)

P_{BC}= 40 k N (compression)

$$\delta = \frac{PL}{AE}$$

$$\delta AB = \frac{50*10^3*5000}{300*200*10^9} = 4.17mm(lengthening)$$

$$\delta BC = \frac{40*10^3*4000}{500*200*10^9} = 1.6mm(shortening)$$

$$\frac{x1}{4.17} = \frac{4}{5} \Rightarrow x = 3.336$$

$$\frac{x1+1.6}{y2} = \frac{3}{4}$$

$$y2 = 6.58mm$$

$$\frac{y1}{4.17} = \frac{3}{5} \Rightarrow y1 = 2.502mm$$

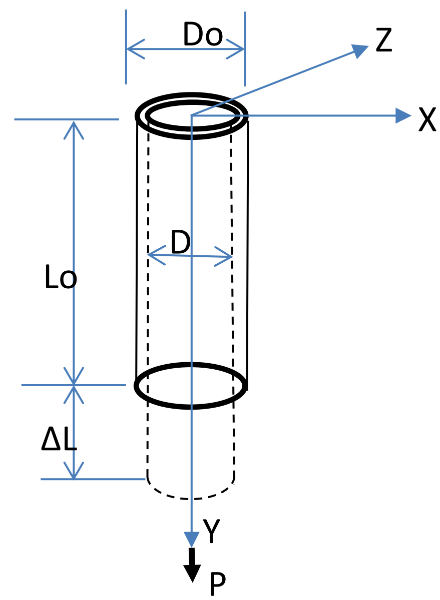
$$y = 6.58 + 2.502 = 9.082mm$$

$$BB' = \sqrt{(9.082)^2 + (1.6)^2} = 9.22mm$$

Poisson's Ratio :Biaxial and Triaxial Deformations

Hook's Law

Stress α Strain



$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

$$E = \frac{Stress}{Strain} = \frac{\sigma}{\epsilon}$$

Uniaxial Stress

$$\epsilon x = \frac{Do - D}{Do} = \frac{\Delta D}{Do}$$

$$\epsilon y = \frac{\Delta L}{L}$$

$$\upsilon = \frac{\epsilon x}{\epsilon y}$$

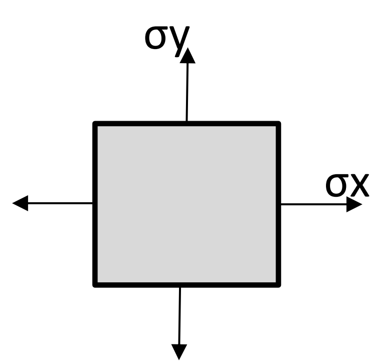
$$\upsilon = \frac{Lateral\ Strain}{Longitudinal\ Strain}$$

$\upsilon : Poison's\ Ratio$

Poison's Ratio : is the ratio between the lateral strain to the longitudinal strain.

Application of Poison's Ratio to a two -dimensional stress system

Tow dimensional stress system is one in which all the stresses lie within one plane such as the x-y plane.



In the x- direction resulting from σx , $\epsilon x = \sigma x/E$

In the y-direction resulting from σy , $\epsilon y = \sigma y/E$

In the x-direction resulting from σy , $\epsilon x = -\upsilon(\frac{\sigma x}{E})$

In the y-direction resulting from the σx , $\epsilon y = -\upsilon(\frac{\sigma x}{E})$

The total strain in the x-direction will be:

$$\epsilon x = \frac{\sigma x}{E} - \upsilon\frac{\sigma y}{E} = \frac{1}{E}(\sigma x - \upsilon\sigma y)$$

The total strain in the y-direction will be:

<u>Triaxial tensile stresses:</u>

$\epsilon x = \frac{1}{E}[\sigma x - \upsilon(\sigma y + \sigma \text{z})]$

$\epsilon y = \frac{1}{E}[\sigma \text{y-} \upsilon(\sigma \text{z+}\sigma x)]$

$\epsilon \text{z=}\frac{1}{E}[\sigma z - \upsilon(\sigma x + \sigma y)]$

The relation between E, G and υ

$$G = \frac{E}{2(1+\upsilon)}$$

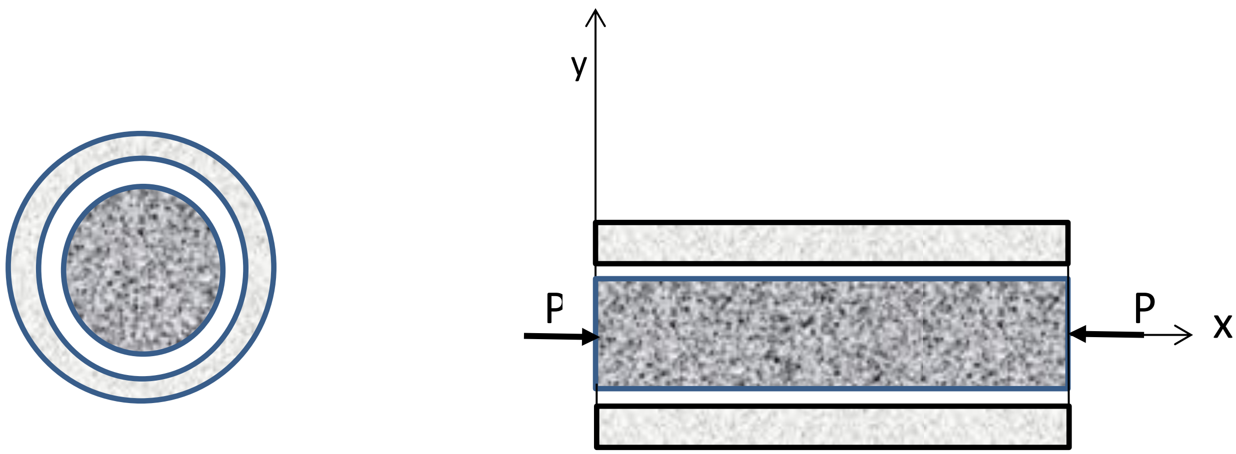
*The common values of Poisson's ratio are:

0.25-0.3 for steel.

0.33 for most other metals.

0.2 for concrete.

<u>221(singer)</u>: A solid aluminum shaft if 80-mm diameter fits concentrically in a hollow steel tube. Compute the minimum internal diameter of the steel tube so that no contact pressure exist when the aluminum shaft carries an axial compressive load of 400 KN .Assume x



<u>Solution:</u>

The axial compressive stress in the aluminum is:

$$\sigma = \frac{P}{A} \qquad , \sigma x = -\frac{400 * 10^3}{\frac{\pi}{4}(0.08)^2} = -79.6 \;\; MN/m^2$$

For uniaxial stress , the transverse strain is:

$$\epsilon y = -\upsilon \epsilon x = -\frac{\upsilon \sigma x}{E} \qquad : \epsilon y = -\frac{1}{3}\left(\frac{-79.6*10^6}{70*10^9}\right)$$

$$\epsilon y = 379 * 10^{-6} \; \frac{m}{m}$$

Therefore the required diameter clearance is

$$\delta = \epsilon L \qquad , \delta y = (379 * 10^{-6})(80) = 0.0303 \, mm$$

- $\epsilon y = \frac{Do-D}{D}$

The required internal diameter of the tube is found by adding this clearance to the original diameter of the aluminum shaft, thus giving,

$$D = 80 + 0.0303 = 80.0303 \ mm \qquad Ans.$$

223(Singer): A rectangular aluminum block is 100 mm long in the X-direction 75 mm wide in the Y-direction, and 50 mm thick in the Z-direction .It is subjected to triaxial loading consisting of uniform distributed tensile forces of (200KN) in the X-direction and uniformly distributed compressive forces of (160 KN) in the Y-direction and (220KN) in the Z-direction .If υ=1/3 and E=70GPa,determine a single distributed loading in the X-direction that would produce the same Z-deformation as the original loading.

Solution:

$$\sigma x = \frac{200 * 10^3}{0.075 * 0.05} = 53.333 * 10^6 \ N/m^2$$

$$\sigma y = -\frac{160 * 10^3}{0.05 * 0.1} = -32 * 10^6 \ N/m^2$$

$$\sigma z = -\frac{220 * 10^3}{0.075 * 0.1} = -29.333 * 10^6 \ N/m^2$$

$$\epsilon z = \frac{1}{E}[\sigma z - v(\sigma x + \sigma y)]$$

$$= \frac{1}{70*10^9}\left[-29.333 * 10^6 - \frac{1}{3}(-53.333 * 10^6 - 32 * 10^6)\right]$$

$$= -0.52 * 10^{-3} \ m/m$$

*Single distributed load in the x-direction that would produce the same z deformation as the original loading means that the loads in the y&z directions will be zero.

$$\epsilon z = \frac{1}{E}[\sigma z - \upsilon(\sigma x + \sigma y)]$$

$$-0.52 * 10^{-3} = \frac{1}{70 * 10^9}\left[0 - \frac{1}{3}(\sigma x + 0)\right]$$

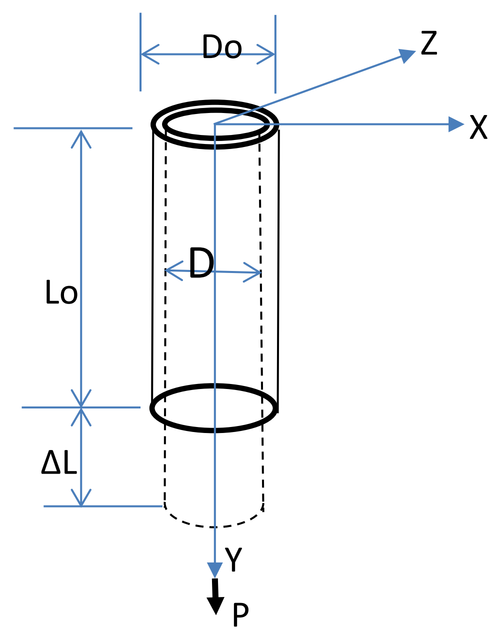
$$\sigma x = 109.2 * 10^6 \ \frac{N}{m^2}$$

$$P = \sigma * A$$

$$= 109.2 * 10^6 * 0.05 * 0.075$$

$$= 409.5 \quad kN$$

<u>Example:</u> In a tensile test on an aluminum bar of 57mm diameter and a length of 305 mm the total elongation is (0.24 mm) a tensile force of 124 kN ,and the contraction in diameter is (0.015mm).Find E ,υ and G for aluminum.



Solution:

$$\epsilon x = \frac{1}{E}[\sigma x - \ \upsilon(\sigma y + \sigma z)]$$

$$\boldsymbol{\sigma x = 0} \quad , \boldsymbol{\sigma y = 0}$$

$$\epsilon x = -\upsilon \frac{\sigma y}{E}$$

$$\epsilon y = \frac{\sigma y}{E}$$

$$\epsilon z = -\frac{\upsilon \sigma y}{E}$$

$$\sigma y = \frac{P}{A} = \frac{142 * 10^3}{\frac{\pi}{4}(57)^2 * 10^6} = 55.648 * 10^6 \; N/m\text{^}2$$

$$\sigma y = 55.648 \quad MPa$$

Lateral strain ϵy

$$\epsilon x = \frac{Do - D}{Do} = \frac{\Delta D}{Do} = -\frac{0.015}{57} = -0.00026 \;\; mm/mm$$

Longitudinal strain

$$\epsilon y = \frac{\Delta L}{L} = \frac{0.24}{305} = 0.000787 \;\; mm/mm$$

$$\epsilon y = \frac{\sigma y}{E}$$

$$E = \frac{55.648}{0.00787} = 70709 \; MPa$$

$$\upsilon = \frac{lateral \; strain}{longitudinal \; strain}$$

$$= \frac{-(0.000263)}{0.000787} = -0.334$$

$$G = \frac{E}{2(1+\upsilon)} = \frac{70709}{2(1-0.334)} = 53000 \quad MPa$$

Problem 225(Singer) A welded steel cylindrical drum made of a 10-mm plate has an internal diameter of 1.20 m. Compute the change in diameter that would be caused by an internal pressure of 1.5 MPa. Assume that Poisson's ratio is 0.30 and E = 200 GPa.

Solution 225

σ_y = longitudinal stress

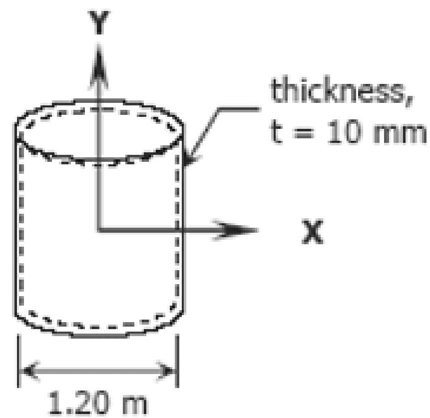
$$\sigma_y = \frac{pD}{4t} = \frac{1.5(1200)}{4(10)}$$

$\sigma_y = 45$ MPa

σ_x = tangential stress

$$\sigma_x = \frac{pD}{2t} = \frac{1.5(1200)}{2(10)}$$

$\sigma_x = 90$ MPa



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\varepsilon_x = \frac{90}{200000} - 0.3\left(\frac{45}{200000}\right)$$

$\varepsilon_x = 3.825 \times 10^{-4}$

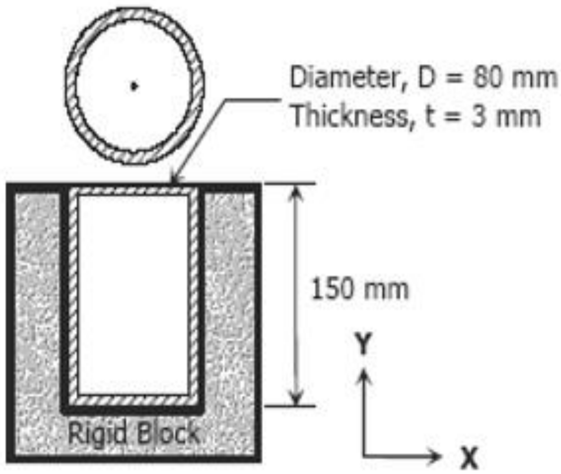
$$\varepsilon_x = \frac{\Delta D}{D}$$

$\Delta D = \varepsilon_x D = (3.825 \times 10^{-4})(1200)$

$\Delta D = \mathbf{0.459\ mm}$

Problem 227

A 150-mm-long bronze tube, closed at its ends, is 80 mm in diameter and has a wall thickness of 3 mm. It fits without clearance in an 80-mm hole in a rigid block. The tube is then subjected to an internal pressure of 4.00 MPa. Assuming ν = 1/3 and E = 83 GPa, determine the tangential stress in the tube.



Longitudinal stress:

$$\sigma_y = \frac{pD}{4t} = \frac{4(80)}{4(3)}$$

$$\sigma_y = \frac{80}{3} \text{ MPa}$$

The strain in the x-direction is:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

$$\sigma_x = \nu\sigma_y = \text{tangential stress}$$

$$\sigma_x = \frac{1}{3}\left(\frac{80}{3}\right)$$

$$\sigma_x = 8.89 \text{ MPa}$$

Statically Indeterminate Members

When the reactive force or the internal resulting forces over a cross section exceed the number of independent equations of equilibrium, the structure is statically indeterminate.

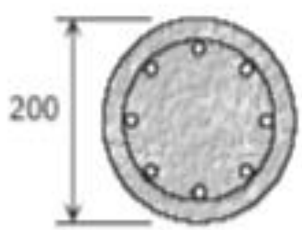
So how to solve such problems?

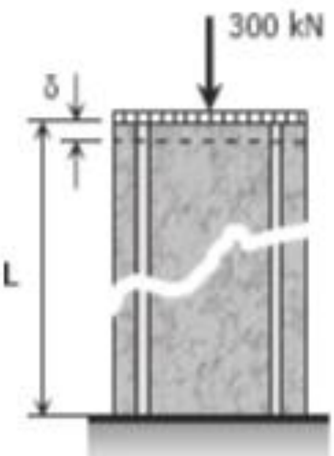
To a free body diagram of the structure or a part of it , apply the equation s of static equilibrium.

If there are more unknowns, obtain additional equations from the geometric relations between the elastic deformations produces by the loads.

Problem 334 A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel, respectively. Use Eco = 14 GPa and Est = 200 GPa.

Solution





$$\delta_{co} = \delta_{st} = \delta$$

$$\left(\frac{PL}{AE}\right)_{co} = \left(\frac{PL}{AE}\right)_{st}$$

$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{co} L}{14000} = \frac{\sigma_{st} L}{200000}$$

$$100\sigma_{co} = 7\sigma_{s}$$

When $\sigma_{st} = 120$ MPa

$100\sigma_{co} = 7(120)$

$\sigma_{co} = 8.4$ MPa > 6 MPa (not ok!)

When $\sigma_{co} = 6$ MPa

$100(6) = 7\sigma_{st}$

$\sigma_{st} = 85.71$ MPa < 120 MPa (ok!)

Use $\sigma_{co} = 6$ MPa and $\sigma_{st} = 85.71$ MPa

$$\Sigma F_V = 0$$

$$P_{st} + P_{co} = 300$$

$$\sigma_{st} A_{st} + \sigma_{co} A_{co} = 300$$

$$85.71A_{st} + 6[\tfrac{1}{4}\pi(200)^2 - A_{st}] = 300(1000)$$

$$79.71A_{st} + 60\,000\pi = 300\,000$$

$$A_{st} = 1398.9 \text{ mm}^2$$

The three A-36 steel bars shown in Fig. 4–14*a* are pin connected to a *rigid* member. If the applied load on the member is 15 kN, determine the force developed in each bar. Bars AB and EF each have a cross-sectional area of 50 mm^2, and bar CD has a cross-sectional area of 30 mm^2.

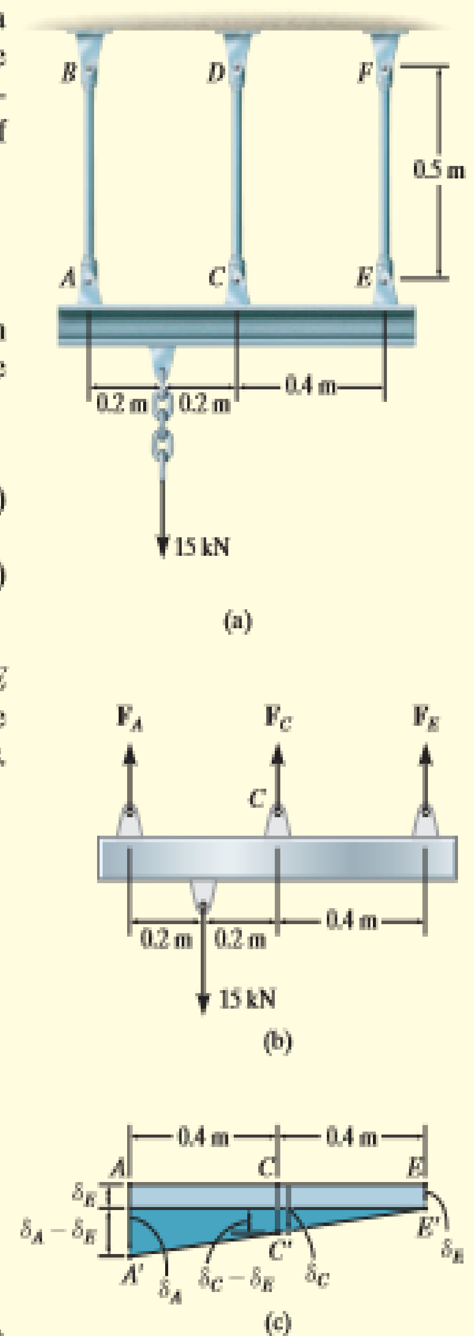


Fig. 4–14

SOLUTION

Equilibrium. The free-body diagram of the rigid member is shown in Fig. 4–14*b*. This problem is statically indeterminate since there are three unknowns and only two available equilibrium equations.

$$+\uparrow \Sigma F_y = 0; \qquad F_A + F_C + F_E - 15\text{ kN} = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_C = 0; \quad -F_A(0.4\text{ m}) + 15\text{ kN}(0.2\text{ m}) + F_E(0.4\text{ m}) = 0 \qquad (2)$$

Compatibility. The applied load will cause the horizontal line ACE shown in Fig. 4–14*c* to move to the inclined line $A'C'E'$. The displacements of points A, C, and E can be related by similar triangles. Thus the compatibility equation that relates these displacements is

$$\frac{\delta_A - \delta_E}{0.8\text{ m}} = \frac{\delta_C - \delta_E}{0.4\text{ m}}$$

$$\delta_C = \frac{1}{2}\delta_A + \frac{1}{2}\delta_E$$

Using the load–displacement relationship, Eq. 4–2, we have

$$\frac{F_C L}{(30\text{ mm}^2)E_{st}} = \frac{1}{2}\left[\frac{F_A L}{(50\text{ mm}^2)E_{st}}\right] + \frac{1}{2}\left[\frac{F_E L}{(50\text{ mm}^2)E_{st}}\right]$$

$$F_C = 0.3F_A + 0.3F_E \qquad (3)$$

Solving Eqs. 1–3 simultaneously yields

$$F_A = 9.52\text{ kN} \qquad \textit{Ans.}$$

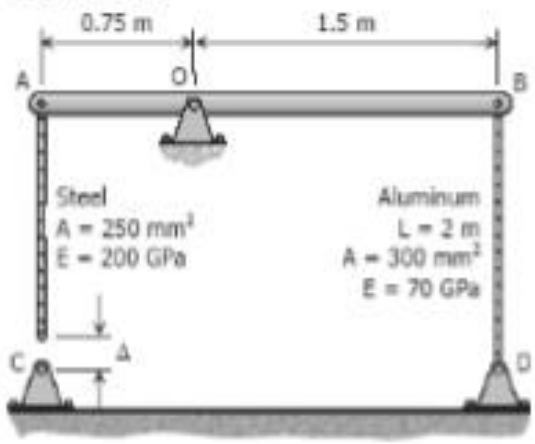
$$F_C = 3.46\text{ kN} \qquad \textit{Ans.}$$

$$F_E = 2.02\text{ kN} \qquad \textit{Ans.}$$

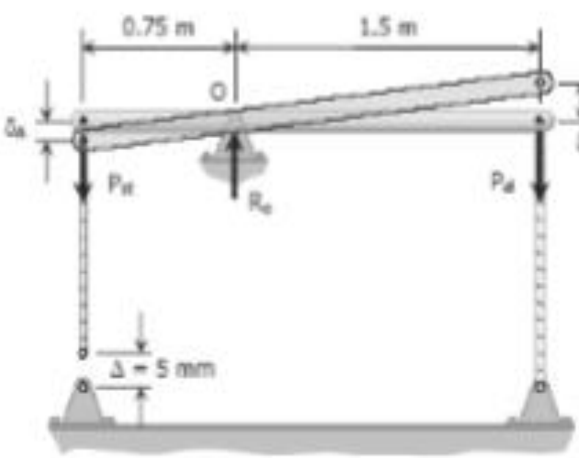
Problem 242

The assembly in Fig. P-242 consists of a light rigid bar AB, pinned at O, that is attached to the steel and aluminum rods. In the position shown, bar AB is horizontal and there is a gap, Δ = 5 mm, between the lower end of the steel rod and its pin support at C. Compute the stress in the aluminum rod when the lower end of the steel rod is attached to its support.

Figure P-242



Solution 242



$\Sigma M_O = 0$

$0.75P_{st} = 1.5P_{al}$

$P_{st} = 2P_{al}$

$\sigma_{st} A_{st} = 2(\sigma_{al} A_{al})$

$\sigma_{st} = \dfrac{2(\sigma_{al} A_{al})}{A_{st}}$

$\sigma_{st} = \dfrac{2[\sigma_{al}(300)]}{250}$

$\sigma_{st} = 2.4\sigma_{al}$

$\delta_{al} = \delta_B$

By ratio and proportion:

$\dfrac{\delta_A}{0.75} = \dfrac{\delta_B}{1.5}$

$\delta_A = 0.5\delta_B$

$\delta_A = 0.5\delta_{al}$

Problem 257

Three bars AB, AC, and AD are pinned together as shown in Fig. P-257. Initially, the assembly is stressfree. Horizontal movement of the joint at A is prevented by a short horizontal strut AE. Calculate the stress in each bar and the force in the strut AE when the assembly is used to support the load W = 10 kips. For each steel bar, A = 0.3 in.2 and E = 29 × 106 psi. For the aluminum bar, A = 0.6. in.2 and E = 10 × 106 psi.

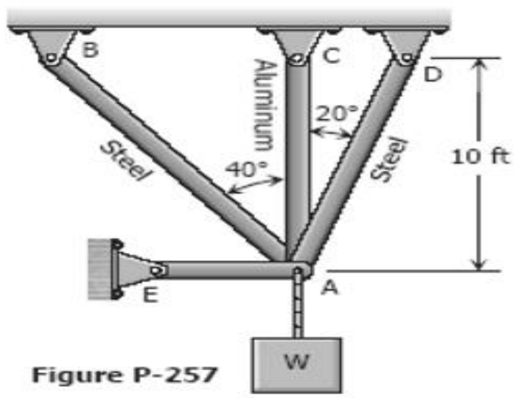
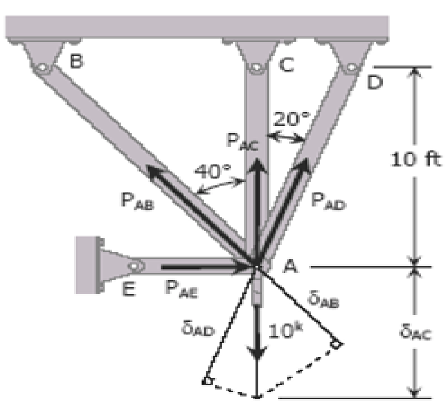


Figure P-257

Solution 257

$\cos 40° = 10 / L_{AB};\ L_{AB} = 13.05$ ft
$\cos 20° = 10 / L_{AD};\ L_{AD} = 10.64$ ft



$\Sigma F_V = 0$

$P_{AB} \cos 40° + P_{AC} + P_{AD} \cos 20° = 10(1000)$

$0.7660 P_{AB} + P_{AC} + 0.9397 P_{AD} = 10\,000 \quad \rightarrow (1)$

$\delta_{AB} = \cos 40°\ \delta_{AC} = 0.7660\ \delta_{AC}$

$$\left(\frac{PL}{AE}\right)_{AB} = 0.7660\left(\frac{PL}{AE}\right)_{AC}$$

$$\frac{P_{AB}(13.05)}{0.3(29\times10^6)} = 0.7660\left[\frac{P_{AC}(10)}{0.6(10\times10^6)}\right]$$

$P_{AB} = 0.8511 P_{AC} \quad \rightarrow (2)$

$\delta_{AD} = \cos 20°\ \delta_{AC} = 0.9397\ \delta_{AC}$

$$\left(\frac{PL}{AE}\right)_{AD} = 0.9397\left(\frac{PL}{AE}\right)_{AC}$$

$$\frac{P_{AD}(10.64)}{0.3(29\times10^6)} = 0.9397\left[\frac{P_{AC}(10)}{0.6(10\times10^6)}\right]$$

$P_{AD} = 1.2806 P_{AC} \quad \rightarrow (3)$

Substitute P_{AB} of (2) and P_{AD} of (3) to (1)
$0.7660(0.8511P_{AC}) + P_{AC} + 0.9397(1.2806P_{AC}) = 10\ 000$
$2.8553P_{AC} = 10\ 000$
$P_{AC} = 3\ 502.23$ lb

$P_{AB} = 0.8511(3\ 502.23)$ → from (2)
$P_{AB} = 2\ 980.75$ lb

$P_{AD} = 1.2806(3\ 502.23)$ → from (3)
$P_{AD} = 4\ 484.96$ lb

Stresses:
$\sigma = P/A$
$\sigma_{AB} = 2980.75/0.3 =$ **9 935.83 psi**
$\sigma_{AC} = 3502.23/0.6 =$ **5 837.05 psi**
$\sigma_{AD} = 4484.96/0.3 =$ **14 949.87 psi**

$\sum F_H = 0$
$P_{AE} + P_{AD} \sin 20° = P_{AB} \sin 40°$
$P_{AE} = 2\ 980.75 \sin 40° - 4\ 484.96 \sin 20°$
$P_{AE} =$ **382.04 lb**

Problem 239

The rigid platform in Fig. P-239 has negligible mass and rests on two steel bars, each 250.00 mm long. The center bar is aluminum and 249.90 mm long. Compute the stress in the aluminum bar after the center load P = 400 kN has been applied. For each steel bar, the area is 1200 mm2 and E = 200 GPa. For the aluminum bar, the area is 2400 mm2 and E = 70 GPa.

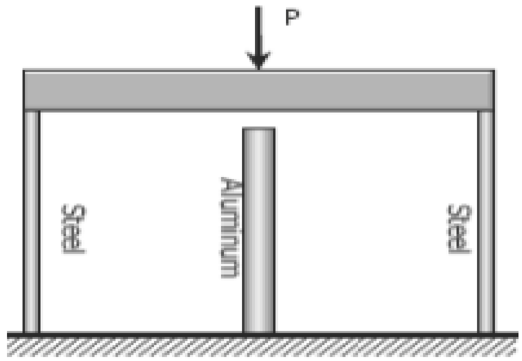
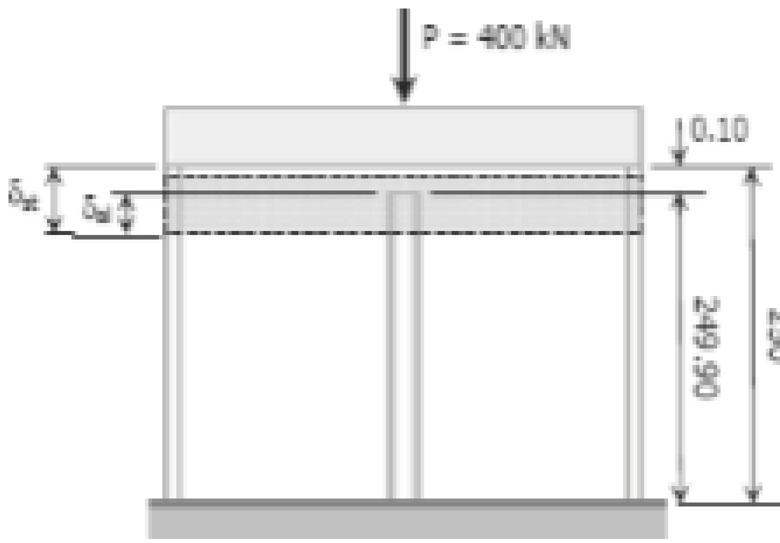


Figure P-239



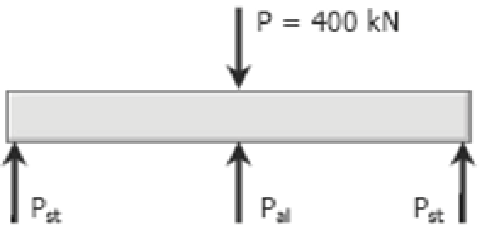
$$\delta_{st} = \delta_{al} + 0.10$$

$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{al} + 0.10$$

$$\frac{\sigma_{st}(250)}{200000} = \frac{\sigma_{al}(249.90)}{70000} + 0.10$$

$$0.00125\sigma_{st} = 0.00357\sigma_{al} + 0.10$$

$$\sigma_{st} = 2.856\sigma_{al} + 80$$



$$\sum F_V = 0$$

$$2P_{st} + P_{al} = 400\ 000$$

$$2\sigma_{st}A_{st} + \sigma_{al}A_{al} = 400\ 000$$

$$2(2.856\sigma_{al} + 80)1200 + \sigma_{al}(2400) = 400\ 000$$

$$9254.4\sigma_{al} + 192\ 000 = 400\ 000$$

$$\sigma_{al} = \mathbf{22.48\ MPa}$$

Thermal stresses

Thermal stresses or strains, in simple bar, may be found out as discussed below: -

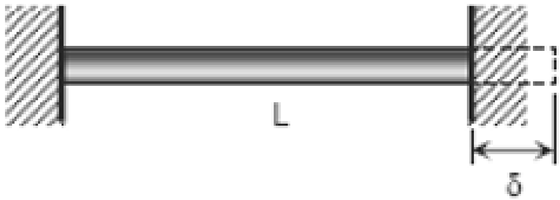
1- Calculate the amount of deformation due to change of temperature with the assumption that the bar is free to expand or contract.
2- Calculate the load (or force) required to bring the deformed bar to the original length.
3- Calculate the stresses and strains in the bar caused by this load.

$$\delta T = \alpha L(Tf - Ti) = \alpha L \Delta T$$

where α is the coefficient of thermal expansion in m/m°C, L is the length in meter, and Ti and Tf are the initial and final temperatures, respectively in °C.

258(Singer): A steel rod 2.5m long is secured between tow walls. If the load on the rod is zero at 20°C, compute the stress when temperature drops to (-20°C). The cross-sectional area of the rod is 1200mm^2 α=11.7μm/(m.°C) and E=200 GPa. Solve assuming (a)that the wall is rigid and (b)that the wall spring together a total distance of 0.500mm as the temperature drops.

Solution



(a) $\delta T = \delta P$

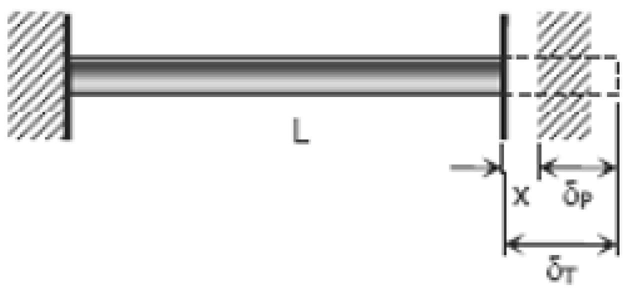
$\alpha L \Delta T = \frac{PL}{AE}$; $\sigma = \frac{P}{A}$

$\sigma = E\alpha\Delta T = (200 * 10^9)(11.7 * 10^{-6})(40)$

$\sigma = 93.6 * 10^6 \ \frac{N}{m^2}$

*Note that Cancels out of the above equations, indicating that the stress is independent of the length of the rod.

(b)



$\delta T = \delta P + X(yield)$

$\alpha L \Delta T = \frac{PL}{AE} + x$

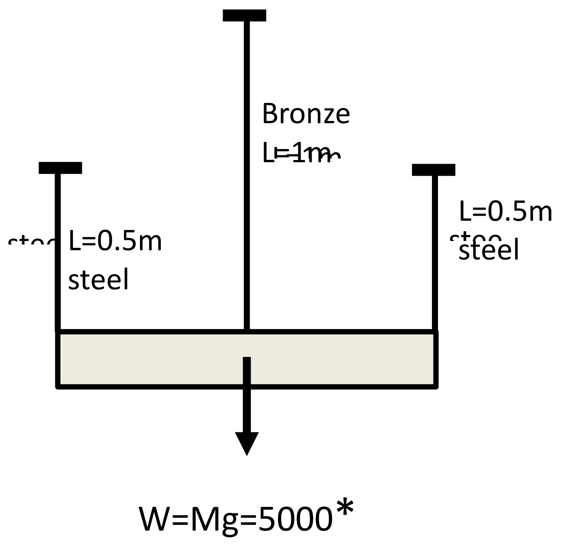
$(11.7 * 10^{-6})(2.5)(40) = \frac{\sigma(2.5)}{200 * 10^9} + (0.5 * 10^{-3})$

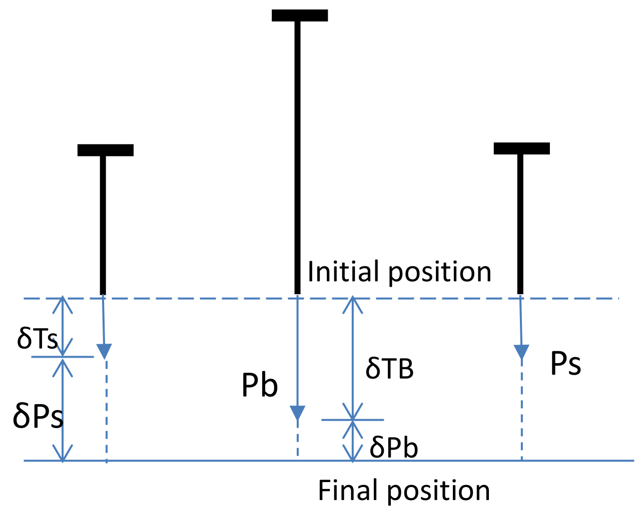
$\sigma = 53.6 \ MN/m^2$

$*$ Note that the yield of the wall reduces the stress considerably , and also that the length of the rod does not canceled out as in part a.

259(Singer) : A rigid block having a mass of 5 Mg is supported by three rods symmetrically placed as shown in figure. Determine the stress in each rod after a temperature raise of 40°C. Use the data in the accompanying table.

	Each steel rods	Bronze rod
Area(mm^2)	500	900
E(N/m^2)	$200*10^9$	$83*10^9$
α (μm/	11.7	18.9





Solution

$$\delta Ts + \delta Ps = \delta Tb + \delta Pb$$

$$((\alpha L \Delta T)s + \left(\frac{PL}{AE}\right)s = (\alpha L \Delta T)b + \left(\frac{PL}{AE}\right)b$$

$$(11.7 * 10^{-6})(0.5)(40) + \frac{Ps(0.5)}{(500 * 10^{-6})(200 * 10^{9})}$$

$$= (18.9 * 10^{-6})(1)(40) + \frac{Pb(1)}{(900 * 10^{-6})(83 * 10^{9})}$$

$Simplify\ the\ above\ equation\ ,$

$$Ps - 2.68Pb = 104 * 10^{3} \quad N \ldots\ldots\ldots\ldots\ldots (a)$$

$$\sum Fy = 0$$

$$2Ps + Pb = 5000(9.81) = 49.05 * 10^3\ N \text{.................(b)}$$

Solving Eq.s (a) and (b)

$Ps = 37.0 \quad kN$

$Pb = 25.0 \quad kN$

The negative sign for Pb means that the load Pb acts oppositely to that assumed: That is bronze rod is actually in compression and suitable position must be made to prevent bukling.

The stresses are

$$\left[\sigma = \frac{P}{A}\right] \qquad \sigma s = \frac{37 * 10^3}{500 * 10^{-6}} = 74.0 \quad \frac{MN}{m^2} \ (tension) \quad Ans.$$

$$\sigma b = \frac{25 * 10^3}{900 * 10^{-6}} = 27.8 \quad \frac{MN}{m^2} \ (compression) \quad Ans.$$

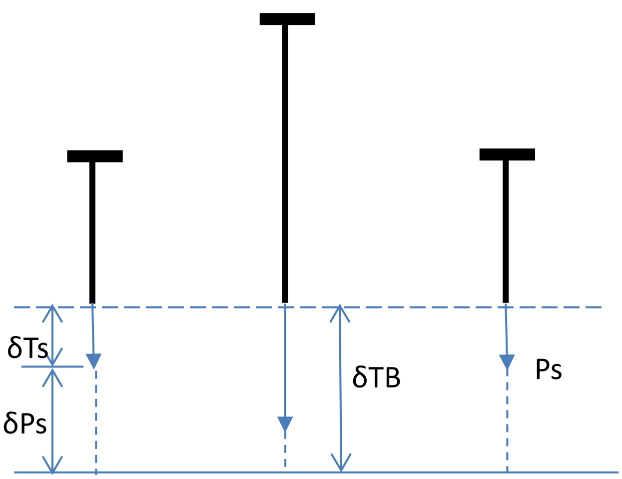
260.(Singer) :Using the data in example 259,determine the temperature rise necessary to cause all the applied load to be supported by the steel rods.

Solution

$$Ps = \frac{1}{2}(5000)(9.81) = 24.53 \quad kN$$

$$\delta Tb = \delta Ts + \delta Ps$$

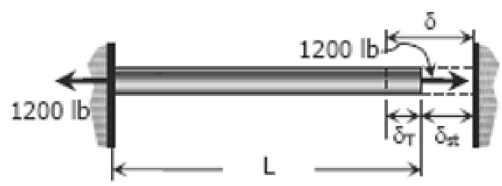
$$(\alpha L \Delta T)b = (\alpha L \Delta Ts) + \left(\frac{PL}{AE}\right)s$$



$$(18.9 * 10^{-6})(1)(\Delta T) = (11.7 * 10^{-6})(0.5)(\Delta T) + \frac{(24.53)(10^3)(0.5)}{(500 * 10^{-6})(200 * 10^{-9})}$$

$\Delta T = 9.4°\text{C}$

roblem 261 A steel rod with a cross-sectional area of 0.25 in2 is stretched between two fixed points. The tensile load at 70°F is 1200 lb. What will be the stress at 0°F? At what temperature will the stress be zero? Assume α = 6.5 × 10-6 in / (in·°F) and E = 29 × 106 psi.
Solution



For the stress at 0°C:

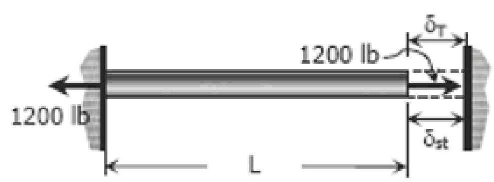
$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T) + \frac{P\cancel{L}}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$\sigma = (6.5 \times 10^{-6})(29 \times 10^{6})(70) + \frac{1200}{0.25}$$

$$\sigma = 17\,995 \text{ psi} = \mathbf{18 \text{ ksi}}$$



For the temperature that causes zero stress:

$$\delta_T = \delta_{st}$$

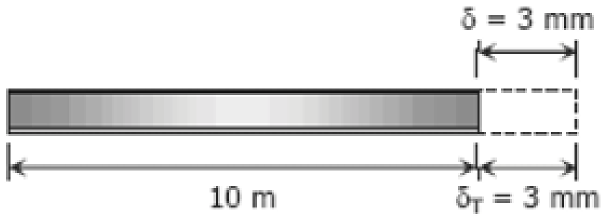
$$\alpha \cancel{L}(\Delta T) = \frac{P\cancel{L}}{AE}$$

$$(6.5 \times 10^{-6})(T - 70) = \frac{1200}{0.25(29 \times 10^{6})}$$

$$T = \mathbf{95.46°C}$$

Problem 263

Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature, will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume α = 11.7 µm/(m·°C) and E = 200 GPa.



Temperature at which $\delta_T = 3$ mm:

$$\delta_T = \alpha L(\Delta T)$$

$$\delta_T = \alpha L(T_f - T_i)$$

$$3 = (11.7 \times 10^{-6})(10\,000)(T_f - 15)$$

$$T_f = \mathbf{40.64°C}$$

Required stress:

$$\delta = \delta_T$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T)$$

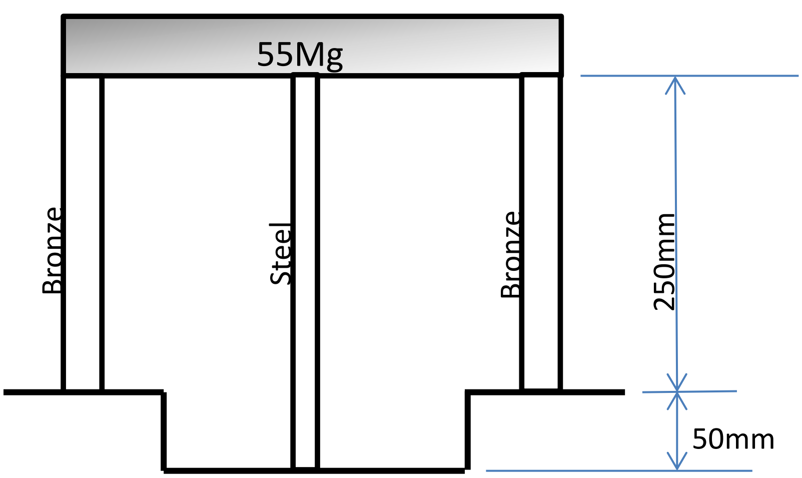
$$\sigma = \alpha E(T_f - T_i)$$

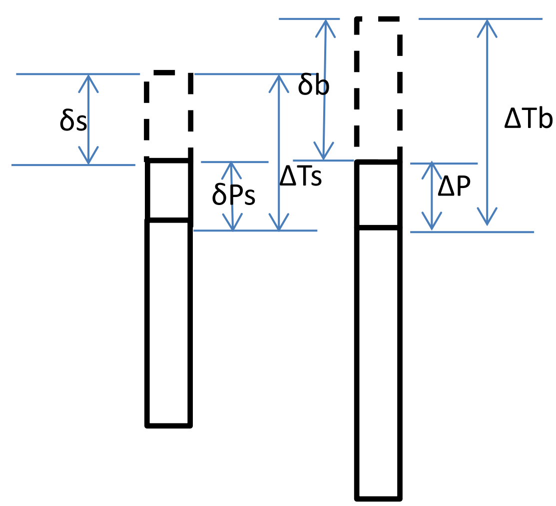
$$\sigma = (11.7 \times 10^{-6})(200\,000)(40.64 - 15)$$

$$\sigma = \mathbf{60\ MPa}$$

266(Singer): At 20° C, a rigid slab having a mass of 55 Mg placed upon two bronze rods and one steel rod as shown. At what temperature will the stress in the steel rod be zero ? For the steel rod A=600mm^2 ,E=200*10^{9}MPa and α=11.7 μm/(m.°C).For each bronze rods A=6000mm^2 ,E=83*10^9 N/m,and α=19.0 μm/(m.°C).

Solution





$$\delta st = \delta b \qquad (compatibility)$$

$$\delta st = \delta Ts - \delta Ps$$

$$\delta b = \delta Tb - \delta Pb$$

$$\alpha sLs\Delta T - \frac{PsLs}{AsEs} = \alpha bLb\Delta T - \frac{PbLb}{AbEb}$$

$\sum Fy = 0$ ....(equilibrium)

$$55 * 10^3 * 9.81 = 2Pb \quad ; \; Ps = 0$$

$$Pb = 27 * 10^4 N$$

$$11.7 * 10^{-6} * 0.3 * \Delta T - \frac{PsLs}{AsEs} = 19.0 * 10^{-6} * 0.25 * \Delta T - \frac{27 * 10^4 * 0.25}{6000 * 10^{-6} * 83 * 10^9}$$

$$\Delta T = 109\ ℃$$

$$\Delta T = Tf - Ti$$

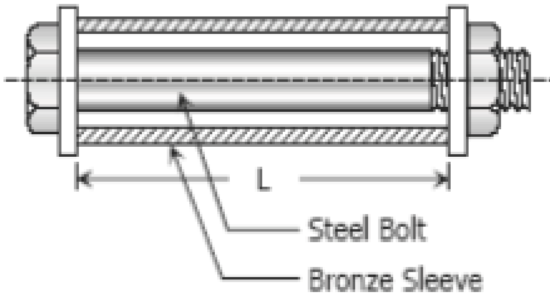
$$109 = Tf - 20$$

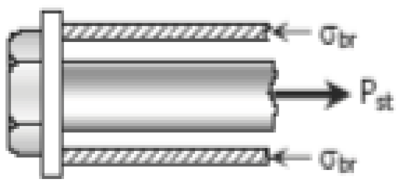
$$Tf = 129℃$$

Problem 270

A bronze sleeve is slipped over a steel bolt and held in place by a nut that is turned to produce an initial stress of 2000 psi in the bronze. For the steel bolt, A = 0.75 in2, E = 29 × 106 psi, and α = 6.5 × 10–6 in/(in·°F). For the bronze sleeve, A = 1.5 in2, E = 12 × 106 psi and α = 10.5 × 10–6 in/(in·°F). After a temperature rise of 100°F, find the final stress in each material.

Solution:





Before temperature change:

$P_{br} = \sigma_{br} A_{br}$

$= 2000(1.5)$

$= 3000$ lb compression

$\Sigma F_H = 0$

$P_{st} = P_{br} = 3000$ lb tension

$\sigma_{st} = P_{st}/A_{st} = 3000/0.75$

$= 4000$ psi tensile stress

$$\delta = \frac{\sigma L}{E}$$

$$a = \delta_{br} = \frac{2000L}{12 \times 10^6} = 1.67 \times 10^{-4}L \text{ shortening}$$

$$b = \delta_{st} = \frac{4000L}{29 \times 10^6} = 1.38 \times 10^{-4}L \text{ lengthening}$$

With temperature rise of 100°F:

(Assuming complete freedom)

$$\delta_T = \alpha L \, \Delta T$$

$$\delta_{Tbr} = (10.5 \times 10^{-6})L\,(100)$$

$$= 1.05 \times 10^{-3}L > a$$

$$\delta_{Tst} = (6.5 \times 10^{-6})L\,(100)$$

$$= 6.5 \times 10^{-4}L$$

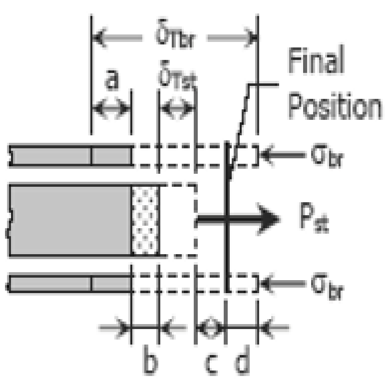
$$\delta_{Tbr} - a = 1.05 \times 10^{-3}L - 1.67 \times 10^{-4}L$$

$$= 8.83 \times 10^{-4}L$$

$$\delta_{Tst} + b = 6.5 \times 10^{-4}L + 1.38 \times 10^{-4}L$$

$$= 7.88 \times 10^{-4}L$$

$\delta_{Tbr} - a > \delta_{Tst} + b$ (see figure below)



$$\delta_{Tbr} - a - d = b + \delta_{Tst} + c$$

$$1.05 \times 10^{-3}L - 1.67 \times 10^{-4}L - \left(\frac{\sigma L}{E}\right)_{br}$$

$$= 1.38 \times 10^{-4}L + 6.5 \times 10^{-4}L + \left(\frac{PL}{AE}\right)_{st}$$

$$8.83 \times 10^{-4}L - \frac{\sigma_{br}L}{12 \times 10^{6}}$$

$$= 7.88 \times 10^{-4}L + \frac{P_{st}L}{0.75(29 \times 10^{6})}$$

$$9.5 \times 10^{-4} - \frac{P_{br}}{1.5(12 \times 10^6)} = \frac{P_{st}}{0.75(29 \times 10^6)}$$

$P_{st} = 20\ 662.5 - 1.2083P_{br}$ → Equation (1)

$\Sigma F_H = 0$

$P_{br} = P_{st}$ → Equation (2)

Equations (1) and (2)

$P_{st} = 20\ 662.5 - 1.2083P_{st}$

$P_{st} = 9356.74$ lb

$P_{br} = 9356.74$ lb

$\sigma = P/A$

$$\sigma_{br} = \frac{9356.74}{1.5} = 6237.83 \text{ psi compressive stress}$$

$$\sigma_{st} = \frac{9356.74}{0.74} = 12\ 475.66 \text{ psi tensile stress}$$

Problem 275

A rigid horizontal bar of negligible mass is connected to two rods as shown in Fig. If the system is initially stress-free. Calculate the temperature change that will cause a tensile stress of 90 MPa in the brass rod. Assume that both rods are subjected to the change in temperature.

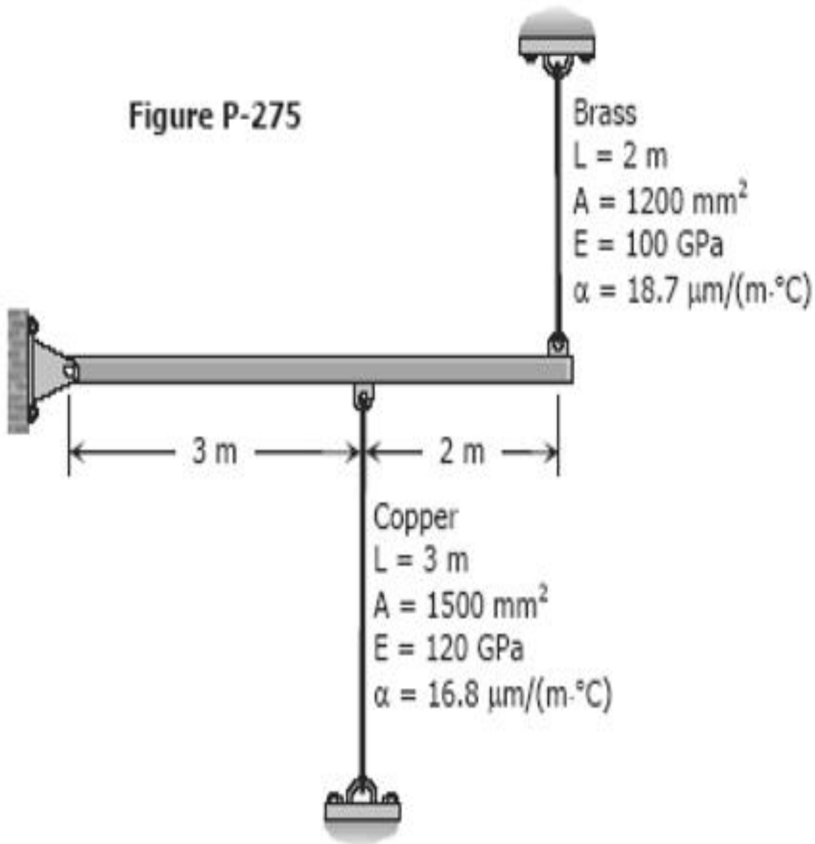
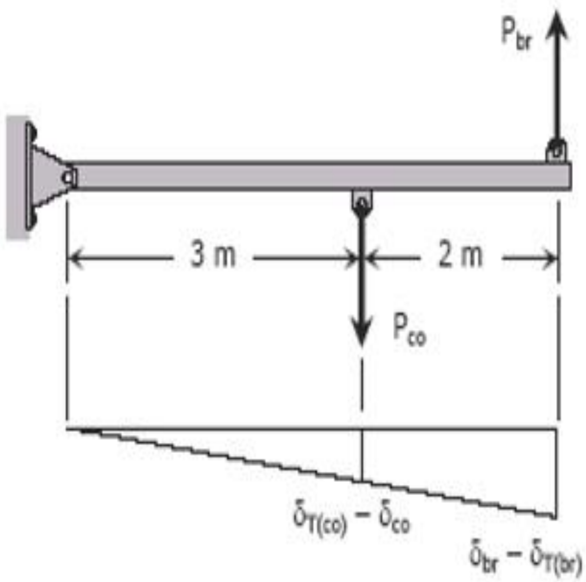


Figure P-275

Solution 275



$\Sigma M_{\text{hinge support}} = 0$

$5P_{br} - 3P_{co} = 0$

$5\sigma_{br}\,A_{br} - 3\sigma_{co}\,A_{co} = 0$

$5(90)(1200) - 3\sigma_{co}\,(1500) = 0$

$\sigma_{co} = 120 \text{ MPa}$

$\delta = \sigma L / E$

$\delta_{br} = 90(2000) / 100\,000 = 1.8 \text{ mm}$

$\delta_{co} = 120(3000) / 120\,000 = 3 \text{ mm}$

$$\frac{\delta_{T(co)} - \delta_{co}}{3} = \frac{\delta_{br} - \delta_{T(br)}}{5}$$

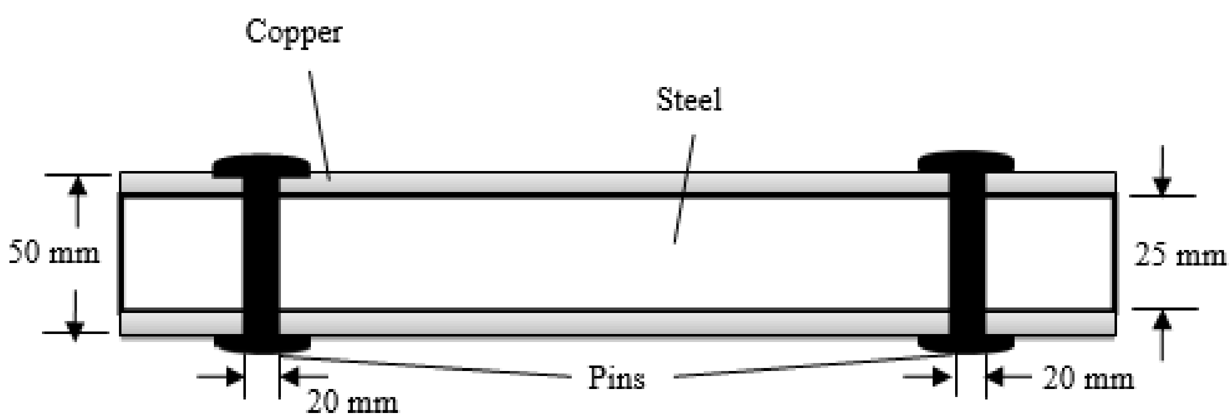
$$5\delta_{T(co)} - 5\delta_{co} = 3\delta_{br} - 3\delta_{T(br)}$$

$$5(16.8\times10^{-6})(3000)\ \Delta T - 5(3)$$
$$= 3(1.8) - 3(18.7\times10^{-6})(2000)\ \Delta T$$

$$0.3642\ \Delta T = 20.4$$

$$\Delta T = 56.01°\text{C drop in temperature}$$

Example: A composite bar is constructed from a steel rod of (25 mm) diameter surrounded by a copper tube of (50 mm) outside diameter and (25 mm) inside diameter . The rod and tube are joined by tow (20 mm) diam.pins as shown in figure . Find the shear stress set up in the pins if , after pinning , the temperature is raised by 50◦C . For steel E= 210 GN/m^2 and α = 11*10^{-6} per ◦C , for copper E= 105 GN/m^2 and α = 17*10^{-6} per ◦C .



In this case the copper attempts to expand more than the steel, thus tending to shear the pins joining the two

Let the stress set up in the steel be x, then, since

$$\text{force in steel} = \text{force in copper}$$

$$x \times \tfrac{\pi}{4} \times 25^2 \times 10^{-6} = \sigma_c \times \tfrac{\pi}{4}(50^2 - 25^2)10^{-6}$$

i.e.

$$\text{stress in copper } \sigma_c = \frac{x \times 25^2}{(50^2 - 25^2)} = 0.333x = \frac{x}{3}$$

Now the extension of the steel from its freely expanded length to its forced length in the compound bar is given by

$$\frac{\sigma L}{E} = \frac{xL}{210 \times 10^9}$$

where L is the original length.

Similarly, the compression of the copper from its freely expanded position to its position in the compound bar is given by

$$\frac{\sigma L}{E} = \frac{x}{3} \times \frac{L}{105 \times 10^9}$$

Now the extension of steel + compression of copper

$$= \text{difference in "free" lengths}$$

$$= (\alpha_2 - \alpha_1)(T_2 - T_1)L$$

$$\therefore \quad \frac{xL}{210 \times 10^9} + \frac{xL}{3 \times 105 \times 10^9} = (17 - 11)10^{-6} \times 50 \times L$$

$$\frac{3x + 2x}{6 \times 105 \times 10^9} = 6 \times 10^{-6} \times 50$$

$$5x = 6 \times 10^{-6} \times 50 \times 6 \times 105 \times 10^9$$

$$x = 37.8 \times 10^6 = 37.8 \text{ MN/m}^2$$

$$\therefore \quad \text{load carried by the steel} = \text{stress} \times \text{area}$$

$$= 37.8 \times 10^6 \times \tfrac{\pi}{4} \times 25^2 \times 10^{-6}$$

$$= 18.56 \text{ kN}$$

The pins will be in a state of double shear (see §1.15), the shear stress set up being given by

$$\tau = \frac{\text{load}}{2 \times \text{area}} = \frac{18.56 \times 10^3}{2 \times \frac{\pi}{4} \times 20^2 \times 10^{-6}}$$

$$= \mathbf{29.5\ MN/m^2}$$