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STRUCTURAL DESIGN

PART I :-

STRUCTURAL DESIGN IN CONCRETE

Syllabous (ACI Code is followed)

ACI : American Concrete institute

1. Simply Supported & Continuous beams.
2. Two-way edge supported slabs.
3. Two-way Column-supported slabs (flat slabs).
4. One-way & Two-way ribbed slabs, (for large spans)
5. Circular slabs (edge supported)
6. Openings in slabs.
7. stair cases.
8. Corbles (or brackets).
9. structural frames in concrete (slabs, beams & columns)
10. Prestressed Concrete beams. (Pre)

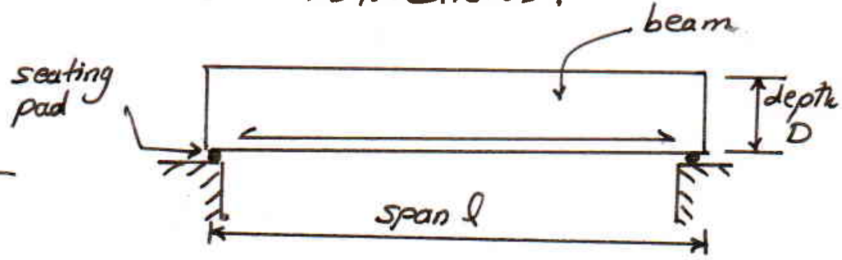
* $f'_c = 21$ MPa for (1:2:4)
water 50% from weight of cement

SIMPLY SUPPORTED & CONTINUOUS BEAMS

Simply Supported Beams :-

Simply supported beams are used frequently in precast constructions (floor & bridge beams.). This type of beams is not economic because of high bending moments at mid-span & high deflections.

Note: it used because to ease to constructions.



Design Procedures :-

1. Specify the imposed loads (the surface loads, live loads, ...). The self weight is not given.

2. Specify the dimensions of the beam (width & depth). These are required to estimate the self-weight of the beam.

* For the depth, Consult ACI Code (Table 9.5(a)). This gives $D = \frac{l}{16}$ for deflection control.

The width is usually specified as $\frac{D}{3}$ to $\frac{2D}{3}$ or given by judgment.

3. Compute the ultimate load

$W_u = 1.4\, W_d + 1.7\, W_\ell$ (by ACI code)

4. Compute the maximum positive bending moment (at mid-span)

$$M_u = \frac{W_u \cdot l^2}{8} \quad (M_u = \text{ultimate moment})$$

5. Thue find the required reinforcement A_s.

i) use $M_u = \phi \rho f_y \left(1 - 0.59 \dfrac{\rho f_y}{f'_c}\right) . bd^2$

where $\rho = \dfrac{A_s}{bd}$ the steel ratio & $\phi = 0.90$

The effective depth d is (d = total depth − concrete cover)
(to center of steel)

This formula gives a second degree equation. Use the formula to find (ρ)

ii) Use curves of $\dfrac{M_u}{\phi b d^2} \longrightarrow \rho$ for different f_y & f'_c

iii) Use tables of $R = \dfrac{M_u}{\phi f'_c b d^2}$ against $\omega = \dfrac{\rho f_y}{f'_c}$

Here both R & ω are dimensionless. From ω find

$$\rho = \frac{\omega f'_c}{f_y}$$

check $\rho_{min} < \rho < \rho_{max}$ where $\rho_{min} = \dfrac{1.4}{f_y}$ in (SI)

& $\rho_{max} = 0.75\rho_b$

Cases

1. If $\rho < \rho_{min}$

Here either use ($\rho = \rho_{min}$) or decrease the width (b) or depth (d) or both to get higher (ρ).

Repeat the calculations for W_u & M_u. ?? why?

2. If $\rho > \rho_{max}$, then increase the width (b) or the depth (d) or both to get lower ρ. If the dimensions of the section are specified (not allowed to change) then use double reinforcement (in compression)

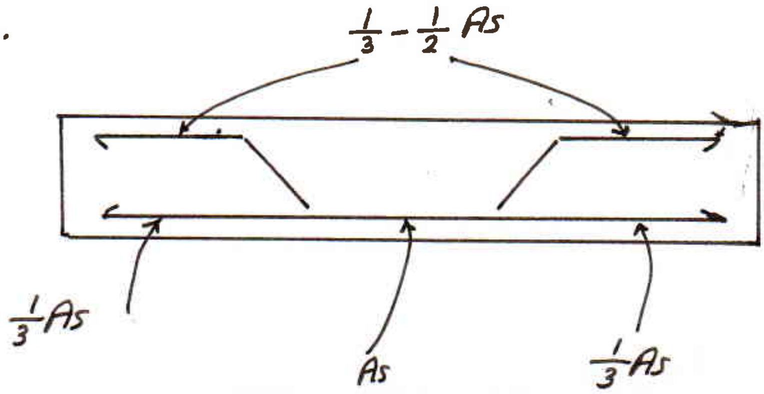
$$\rho' = \rho - \rho_{max}.$$

when steel area A_s is determined, find the steel bars. Draw the details of reinforcement.

Note:-

According to ACI Code, not less than $\frac{1}{3}$ reinforcements should be straight bars & continuing to the ends.

Also, negative moment reinforcement ($\frac{1}{3} - \frac{1}{2}$ of positive moment reinforcement is needed).



6. check for deflections (at mid-span)

$$\Delta = \frac{5}{384} \cdot \frac{W l^4}{E_c I_e}$$

where $W = W_d + W_l$ (service load)

$E = 4700 \sqrt{f'_c}$ (Modulus of elasticity for concrete) (in N/mm^2 or MPa)

$I_e \simeq I_g$ (for gross area), approximatly.

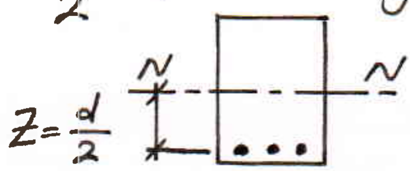
For improved of I_c, consult ACI Code. (equation 9.7)

Also Consult ACI Code for maximum allowable deflections (Table 9.5 b).

7. Calculate the cracking moment M_{cr} use $\frac{M}{I} = \frac{\sigma}{Z}$

where here (I) is for uncracked transformed section or roughly $I = I_g$ for approximate calculations & $\sigma = f_r$ $= 0.7\sqrt{f'_c}$ → is the modulue of rupture (maximum tensile stress in Concrete) & f_r and f'_c in MPa (or N/mm^2).

Also $Z = D - C$ the distance of the tension face from the neutral axis. Roughly use $Z = \frac{d}{2}$ for rectangular sections.



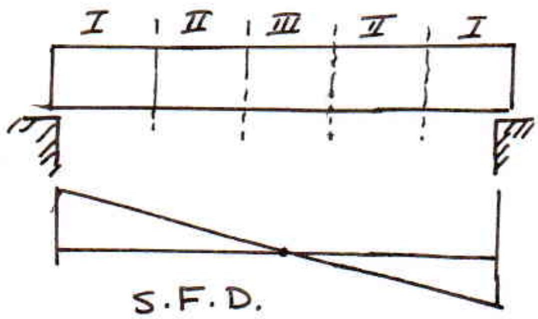
8. Calculate the service moment M_s

$$M_s = \frac{w \cdot l^2}{8} \quad \text{where} \quad w = w_d + w_l$$

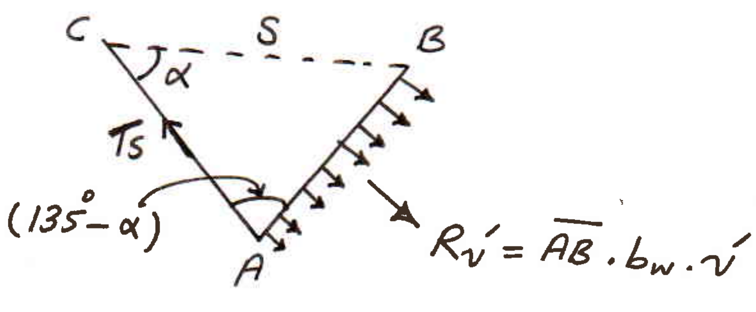
Compare M_s with M_{cr}.

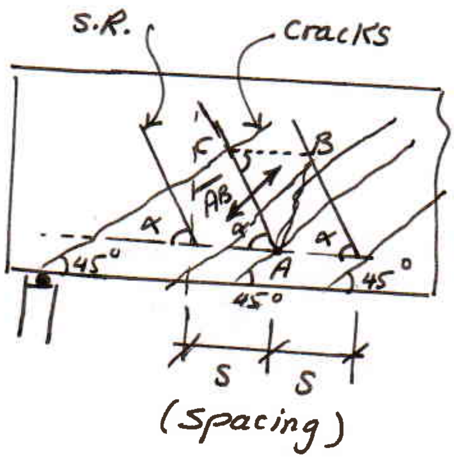
9. Design for web (or transverse shear) reinforcement. Use vertical stirrups (with 2 legs). Specify bar diameter & find the spacing s between bars.

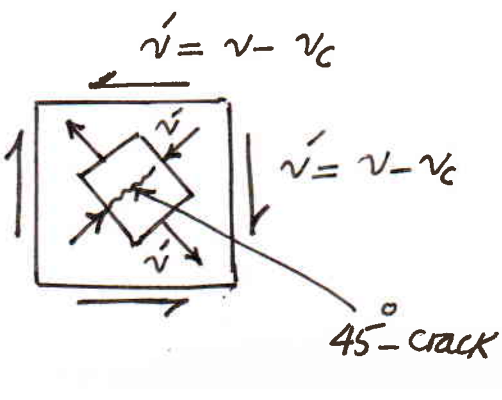
Divide the span into 3 regions:



Shear Reinforcements In Beams:-







Consider a portion of beam under high shearing force V. The bending moment M is neglected. Therefore, shear cracks at 45^{0} will occur. The shearing stress on the section is:

$$v = \frac{V}{b_w \cdot d} \qquad \text{(by ACI code)}$$

The concrete can take a shearing stress v_c, here

$$v_c = \frac{1}{11}\sqrt{f'_c} \quad \text{(MPa) (in working stress)}$$

$$v_c = \frac{\phi}{6}\sqrt{f'_c} \quad \text{(MPa) (in ultimate strength)}, \ \phi = 0.85$$

<u>Notice</u> :-

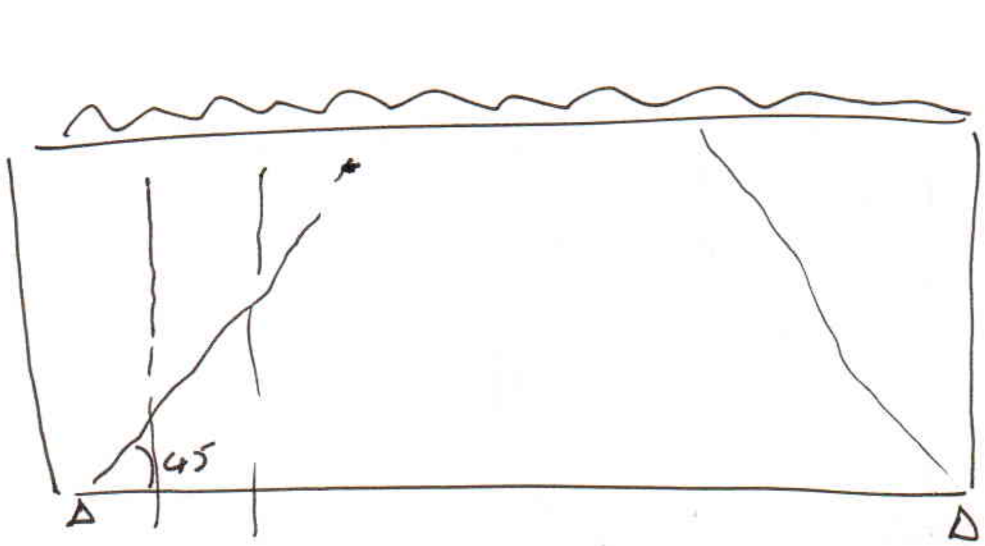
$V_c = v_c \cdot (b_w d)$ is the shearing force taken by the concrete section.

<u>Cases</u> :-

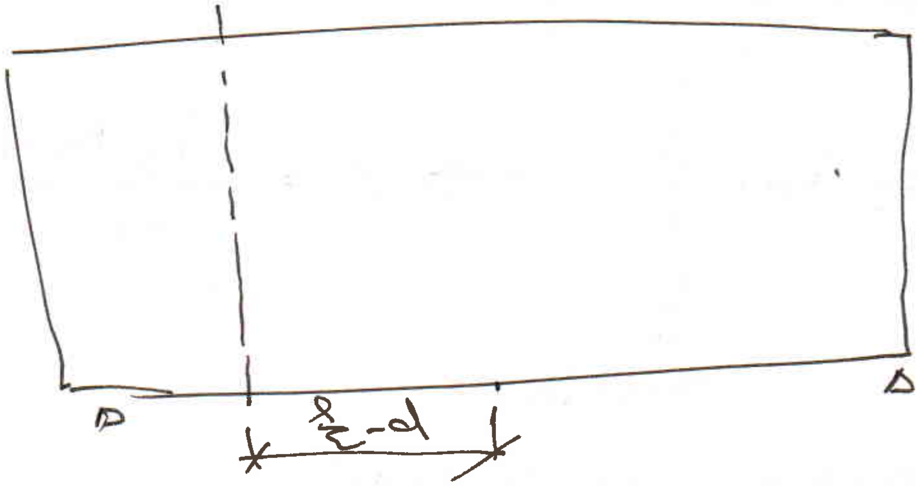
1. If $v_c > v$ use minimum shear reinforcement
2. If $v_c < v$, then $v' = v - v_c$ must be taken by the shear reinforcement.

The tension in steel $T_s = A_v \cdot f_v$ must have component that can balance the excess tension $R_{v'} = \overline{AB} \cdot b_w \cdot v'$ in concrete. First find the length $\overline{AB}$ by rule of sines.

$$\overline{AB} = \frac{S \sin\alpha}{\frac{1}{\sqrt{2}}(\cos\alpha + \sin\alpha)} \qquad \text{So, } R_{v'} = \frac{v' \cdot b_w \cdot S \cdot \sin\alpha}{\frac{1}{\sqrt{2}}(\cos\alpha + \sin\alpha)}$$



V_u (critical)



$V_u = 1.15\, w_u \left(\frac{\ell}{2} - d \right)$ In Iraq $\frac{\ell}{2} - \frac{d}{2}$

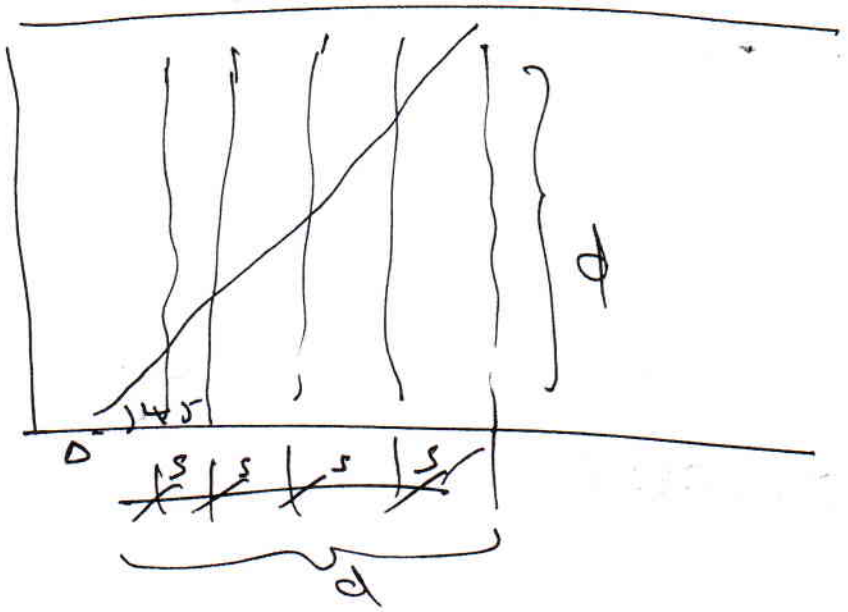
$V_c = \frac{0.85}{6} \sqrt{f'_c}\; b_w\, d$ (MPa = N/mm²)

If $V_c > V_u$ no shear reinforcement

But ACI code uses min. shear reinf.

If $V_c < V_u$ shear reinforcement is needed.

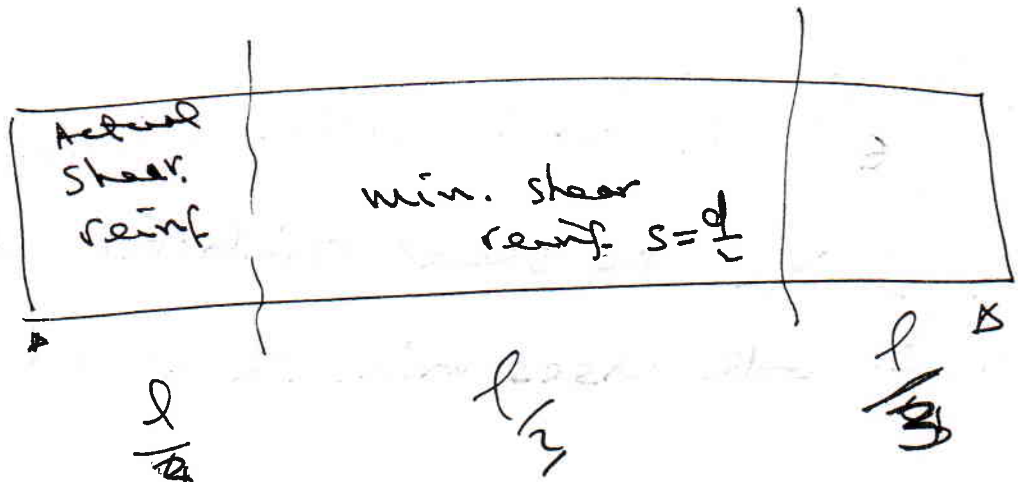
$\underbrace{V_u = V_c}_{\text{conc.}} + \underbrace{V_s}_{\text{steel}} \longrightarrow V_s = V_u - V_c$



$$V_s = f_y A_v \cdot \frac{d}{s} \longrightarrow s = \frac{f_y A_v d}{V_s}$$

number

A_v = area × no. of legs



Use vertical equilibrium, then $T_s \sin\alpha = R_{v'} \cdot \sin 45^\circ$

But $T_s = A_v \cdot f_v$, then:

$$A_v f_v \sin\alpha = \frac{v' \cdot b_w \cdot S \cdot \sin 45^\circ \cdot \sin\alpha}{\frac{1}{\sqrt{2}}(\cos\alpha + \sin\alpha)} \Rightarrow S = \frac{A_v \cdot f_v \cdot (\cos\alpha + \sin\alpha)}{v' \cdot b_w}$$

For best shear reinforcement, Use $\alpha = 45^\circ$ (normal to shear cracks) this is difficult in practice

For ease in practice use $\alpha = 90^\circ$ (vertical stirrups). Then the spacing becomes:

$$S = \frac{A_v f_v}{v' \cdot b_w} \quad \& \quad v' = v - v_c$$

For Ultimate strength:

$$v_u = \frac{V_u}{b_w \cdot d} \quad \text{(actually on the section)}$$

$$v_c = \frac{\phi}{6}\sqrt{f'_c} \quad \text{(MPa) (shear strength of Concrete)}$$

So use $v' = v'_u = v_u - v_c$

Also use $f_v = f_y$ (yield strength of steel)

In rectangular sections, Use ($b_w = b$).

R.C. Beams

ultimate moment (M_n)

A reduction Factor ($\phi = 0.9$) is used to get

($M_u = \phi M_n$)

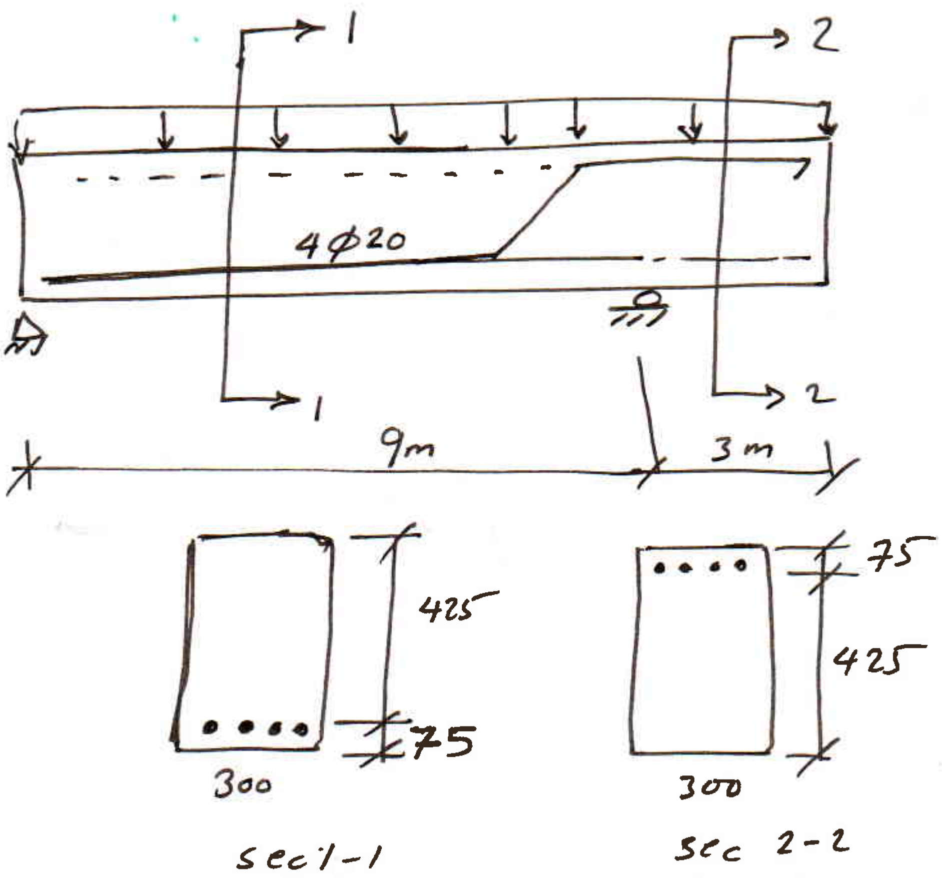
$$\boxed{M_u = \phi \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right) b d^2}$$

EX1- For The R.C. beam find W_{live}?

given for concrete : $f'_c = 25 \ N/mm^2$

$\gamma_c = 25 \ kN/m^3$

for steel $f_y = 410 \ N/mm^2$



Solution :

$A_s = 4 - \phi 20 = 314 \times 4 = 1256 \ mm^2$

$\rho = \frac{A_s}{bd} = \frac{1256}{300 \times 425} = 0.009851 > \rho_{min} = \frac{1.4}{410} = 0.0034$

$M_u = 0.9 * 0.009851 \times 410 \left(1 - 0.59 \frac{0.009851 \times 410}{25}\right) 300 \times (425)^2$

$M_u = 1.782 \times 10^8 \ N.mm = 1.782 \times 10^8 \times 10^{-6} = \underline{178.2} \ kN.m$

For tension & Comp. sec 1-1, sec. 2-2

$\Sigma M_B = 0$

$9R_A - w_u * 12 * 3$

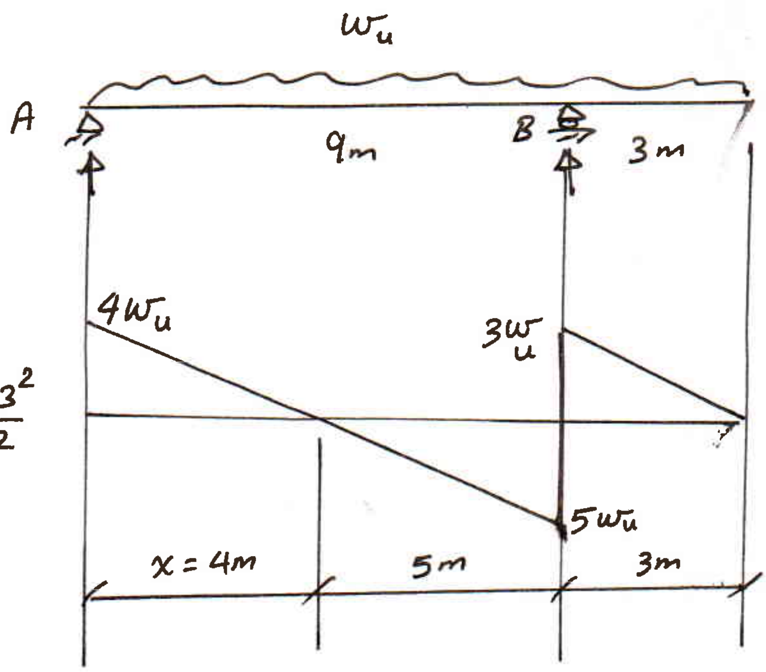
$R_A = 4w_u$

or $+ R_A(9) - w_u * \frac{9^2}{2} + w_u * \frac{3^2}{2} = 0$

$\Rightarrow R_A = \frac{\frac{81}{2} w_u - \frac{9}{2} w_u}{9}$

$= \frac{72}{18} w_u$

$= 4w_u$



$R_B = 8w_u$

$4w_u = w_u x \Rightarrow (x = 4m)$

$$M^{+}_{u\,max.} = 4W_u * \frac{4}{2} = 8W_u \ KN.m \quad \text{(control)}$$

or $$M_u^{+} = 4W_u * 4 - W_u * 4 * \frac{4}{2} = 8W_u \ KN.m$$

$$M^{-}_{u\,max.} = W_u * 3 * \frac{3}{2} = 4.5W_u \ KN.m$$

$$8W_u = 178.2$$

$$W_u = 22.275 \ KN/m$$

$$W_u = 1.4W_d + 1.7WL$$

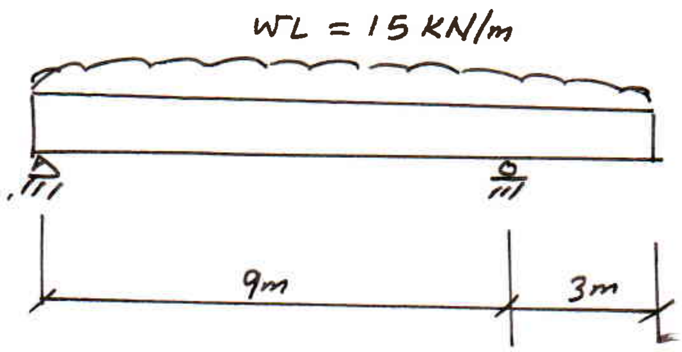
$$W_d = 0.3 \times 0.5 \times 25 = 3.75 \ KN/m$$

$$22.275 = 1.4 Wd + 1.7 WL$$

$$\cancel{W_d =} \ 22.275 = 1.4 * 3.75 + 1.7 W_L$$

$$\Rightarrow W_L = 10.014 \ KN/m$$

EX 2 :-



Design the R.C. Beam shown below . if

$f_c' = 21\ N/mm^2$

$\gamma_c = 25\ KN/m^3$

$F_y = 410\ N/mm^2$

$W_\ell = 15\ KN/m$

① $h = \dfrac{9000}{18.5} = 486.48\ mm$

or $h = \dfrac{3000}{8} = 375\,mm$

use $h = 600\ mm$, $d = 525\ mm$

② $b = \dfrac{1}{3} - \dfrac{2}{3}\ h$, use $b = 300\,mm$

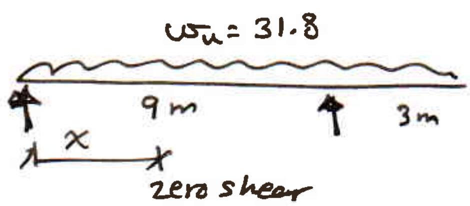
③ $W_d = 25 \times 0.3 \times 0.6 \times 1 = 4.5\ KN/m$

$W_u = 15 * 1.7 + 4.5 * 1.4 = 31.8\ KN/m$

④ Calculate the B.M.

$R_A = \dfrac{31.8 \times 12 \times 3}{9} = 127.2\ kN$ ↑

$x = 4m$



$R_B = 254.4\ kN$ ↑

$M_{u\,max}^{+} = \dfrac{127.2 \times 4}{2} = 254.4\ kN.m$

or $127.2(4) - 31.8 \times 4 \times \frac{4}{2} = M^+ = 254.4\ kN.m$

$$M_u^- = 31.8 \times \frac{3^2}{2} = 143.1 \ kN.m$$

$d = 600 - 75 = 525 \ mm$ $\quad$ $\left[= 600 - 40 - 16 - \frac{16}{2} - 10 \approx 525 \ mm\right]$

Design for +ve B.M.

$M_u^+ = 254.4 \ \cancel{kN.m}$ kN.m

$$R = \frac{254.4 \times 10^6}{0.9 \times 21 \times 300 \times 525^2} = 0.\cancel{16}278$$

from Tables $\omega = 0.1825$

$$\cancel{P} \quad \rho = \omega \frac{f'_c}{F_y} = 0.1825 * \frac{21}{410} = 0.0093476$$

$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{410} = 0.0034146 < 0.0093476 \quad \underline{o.k.}$$

$$\rho_{max.} = 0.75 \left[0.85 * 0.85 * \frac{21}{410} \ \frac{600}{600+410}\right] = 0.01649$$

> 0.0093476 $\underline{o.k.}$

$$A_s = \rho bd = 0.0093476 \times 300 \times 525$$
$$= 1472.247 \ mm^2$$

Use : $6 \phi 18 = 6 \times 255 = 1530 \ mm^2$

$$A_s = 2 \times \underset{\phi 18}{255} + 5 \times \underset{\phi 16}{201} = 1515 \ mm^2 > 1472.24 \ mm^2$$

Design for negative B.M.

$M_u^- = 143.1 \ kN.m$

$$R = \frac{143.1 \times 10^6}{0.9 \times 21 \times 300 \times 525^2} \simeq 0.09156 \Rightarrow \omega \doteq 0.097$$

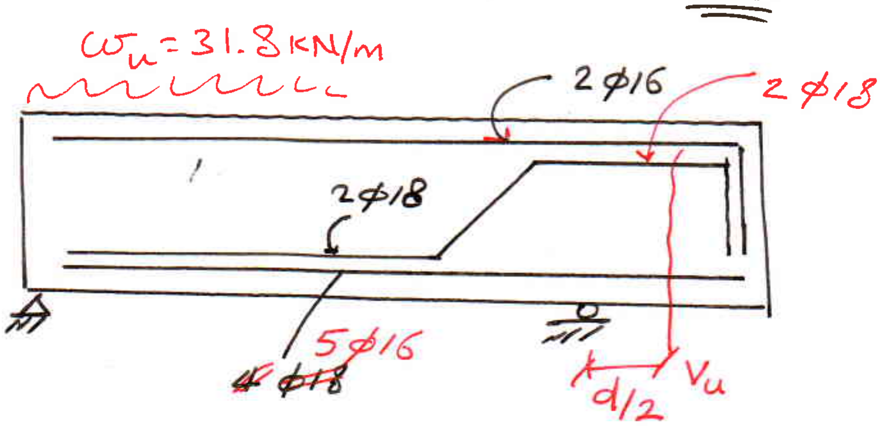
$\rho = 0.097 \times \frac{21}{410} = 0.00497 > \rho_{min}$

$< \rho_{max}$. <u>O.K.</u>

$As = 0.00497 \times 300 \times 525 = 782.775\ mm^2$

$As = 2\phi16 + 2\phi18 = 2 \times 201 + 2 \times 255 = 852 > 782.775$

<u>O.K.</u>



<u>Check for shear</u> @ $d/2$ from supports is :-

RB

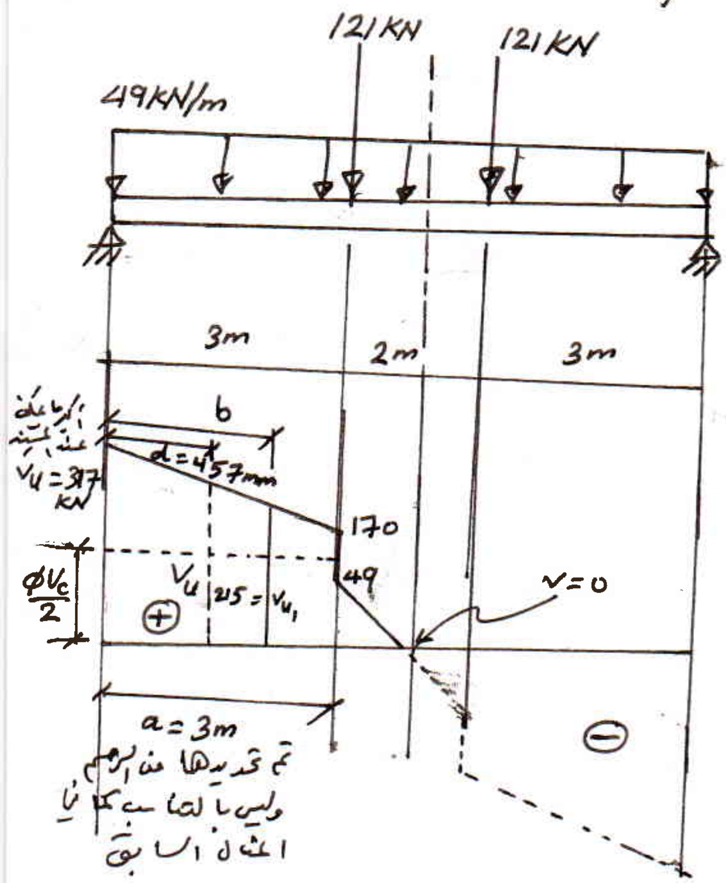
$V_u = 254.4 - 31.8 * \frac{0.525}{2} = 246.052$ KN (ultimate shear)

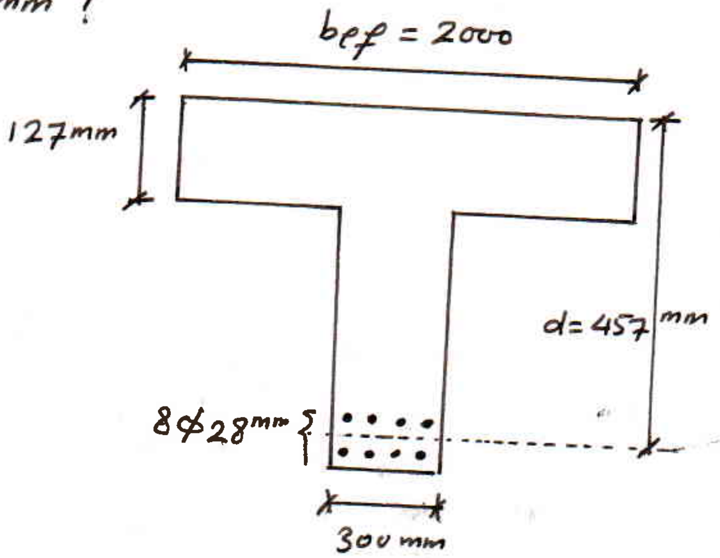
$v_u = \frac{V_u}{bd} = \frac{246.052 \times 10^{3}}{300 \times 525} = 1.56\ N/mm^2$

$v_c = \frac{\phi}{6}\sqrt{f_c'} = \frac{0.85}{6}\sqrt{21} = 0.6492\ N/mm^2$

$v_u > v_c$ shear reinforcement is needed.

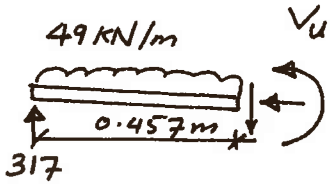
EX: Design the shear reinforcement for the beam shown in Fig. below, $f'_c = 27.6$ MPa, $F_y = 414$ MPa, $b_w = 300$ mm, $d = 457$ mm, $b_{ef} = 2000$ mm ?





$$R = \frac{49*8 + 121*2}{2} = 317 \text{ kN}$$

① To Find (V_u) بالشبه والتناسب , المثلثات



$$V_u = 317 - 0.457 * 49 = 294.6 \text{ kN}$$

max. shear. @ d

② لمعرفة هل أن الكونكريت كاف لتحمل قوى القص أم لا ؟ أي أننا نحتاج (stirrups) أشاير لزيادة التحمل للقص لعدم كفاية مقاومة الكونكريت لوحده ، لذلك نقارن وكما يلي :-

$$\phi V_c = \phi\, 0.17 \sqrt{f'_c}\, b_w d = 104 \text{ kN}$$

(ϕ = 0.85) مقاومة الكونكريت (Concrete strength)

But $V_u = 294.6$ kN, then we need the shear reinf.

since ($V_u > \phi V_c$) then web reinforcement is required.

③ $\frac{\phi V_c}{2} = 52$, $\phi V_s = V_u - \phi V_c$ or $V_s = \frac{294.6 - 104}{0.85} = 224$ KN

$\phi = 0.85$

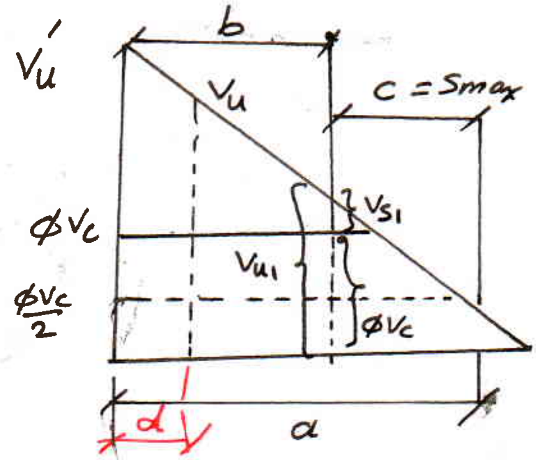
Ultimate shear force (max.)

④ $\underset{224}{\underset{\parallel}{V_s}} \leqslant \frac{2}{3}\sqrt{f'_c}\, b_w d = 480\ kN$ (the section is o.k.)

لاحظ المقارنة

⑤ بعد معرفة أن المقطع جيد نبدأ باستخراج المسافات لحديد التسليح
في هذه المسافات نستخرج أولاً (S_o) :-

$$V_{u_1} = \phi V_{s_1} + \phi V_c$$



لاحظ الرسم المنقط (c) تسليح بمقدار $(S_{max}.)$.

$$\frac{1}{3}\sqrt{f'_c}\, b_w d = 240$$

Note ① V_{u_1} @ (b) distance from s.
② V_u @ (d) distance from s.

, since $\underset{224}{\underset{\parallel}{V_s}} < 240$

$$\therefore S_{max} = \frac{d}{2} = 457/2 = \underline{228.5\ mm}$$

(Control)

~~or S = 600 mm~~

~~or~~ $S = \frac{3 A_v f_y}{b_w} = \frac{3\times 2\times 79\times 414}{300} = 654\ mm \rightarrow$ use $S = \underline{600\ mm}$

for safety ⇒ use $S_{max} = 220\ mm$ (Control)

*استخرج (S_o) بالرجوع للمعادلة :-

Find S_o, Use the equation $S_o = \frac{A_v f_y d}{V_s}$

$$= \frac{2\times 79\times 414\times 457}{224\times 10^3}$$

V_s

$$= 133.5\ mm$$

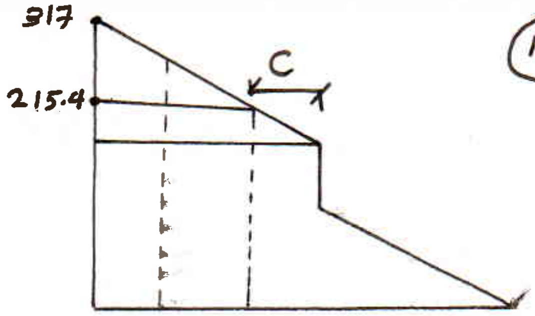
use $S_o = 130\ mm$

{ * نلاحظ إستخدام (S_o) لتحديد أماكن التسليح في العشب، لكن في حالة $S_o > S_{max}$ فلا داعي لإكمال الحل حيث نستخدم (S_{max}) كحد أعلى لوضع حديد القص. }

$$* \quad V_s = \frac{A_v f_y d}{s}$$

$V_{u_1} = \phi V_{s_1} + \phi V_c \rightarrow$, $V_{s_1} = \frac{0.85\times 2\times 79\times 414\times 457}{228\times 10^3}$ ← S_{max} (Control)

$$= 111.4\ kN$$



$$V_{u1} = \phi V_{s1} + \phi V_c$$

$$= 111.4 + 104$$

$$= 215.4 \text{ kN}$$

تحدد هذه القيمة في الرسم أيضاً وبالتالي
إيجاد المسافة (b) على الرسم.

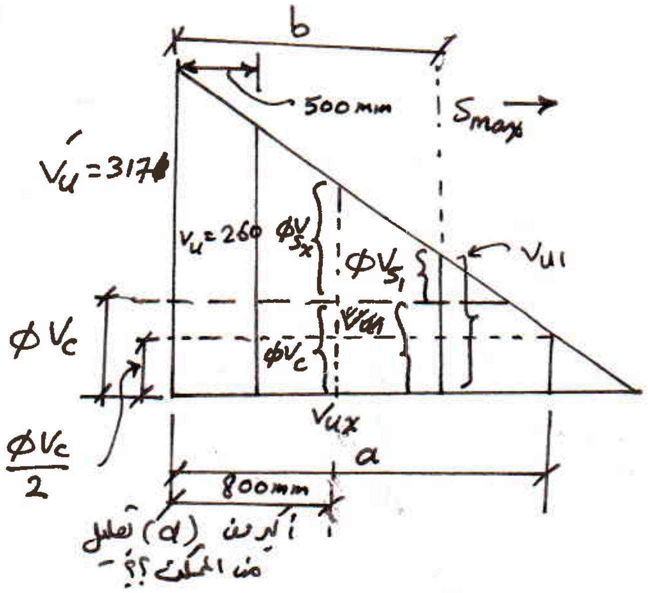
تسلح بـ (S_{max}) $(a - b = c)$.

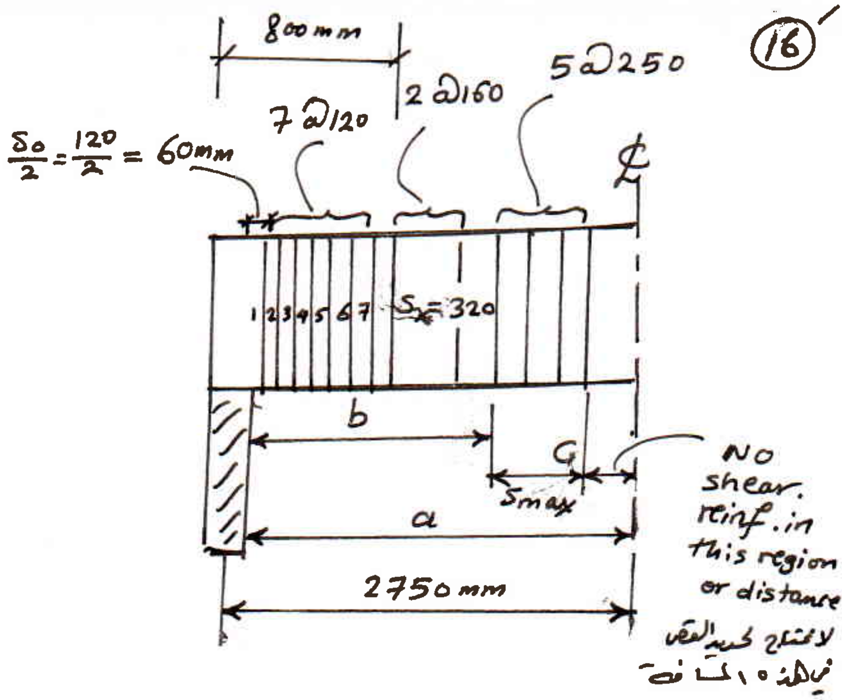
$$\frac{317 - 170}{(3 = a)} = \frac{317 - V_{u1}}{b}$$

$$\Rightarrow b = 2074 \text{mm}$$

$$(a - b = c)$$

تسلح هذه المسافة بـ (S_{max})





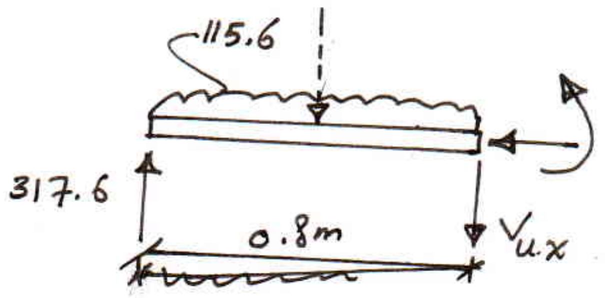
$S_o = 120$, $\frac{S_o}{2} = 60$

$$\boxed{V_u = \phi V_{sx} + \phi V_c}$$

$$\phi V_{sx} = \frac{\phi A_v f_y d}{S_x} \Rightarrow \boxed{S_x = \frac{A_v f_y d}{V_{sx}}}$$

find V_{ux} at distance (800 mm) from support :-

يتم إيجاد (V_{ux}) على مسافة (800mm) :-



$$\uparrow^{+} \sum F_y = 0$$

$$V_{ux} = 317.6 - 0.8 \times 115.6$$

$$\Rightarrow V_{ux} = 225.12 \text{ kN}$$

$$V_{ux} = \phi V_c + \phi V_{sx}$$

$$V_{sx} = \frac{V_{ux} - \phi V_c}{\phi} = \frac{225.12 - 113.9}{0.85} = 131 \text{ kN}$$

$$S_x = \frac{A_v f_y d}{V_{sx}} = \frac{2 \times 79 \times 276 \times 500}{131}$$

$$= 166 \text{ mm}$$

$$\boxed{\text{use } S_x = 160 \text{ mm}}$$

$800 - 60 = 740$, $\therefore \frac{740}{120} = 6.2$ use $7 @ 120$

عدد الإسارات موزعة على المسافة

$7 \times 120 = 840\ mm$

$S_o = 120$

$b = 1123 - 900 = 223$; $\frac{223}{160} = 1.39$ use $2 @ 160 = S_x$

$c = 2260 - (840 + 320 + 60) = 1040$

S_{max} ??

(Zone C) ?

$\frac{1040}{250} = 4.16$ use $5 @ 250$

* لا تُهمل المنطقة (C) تسلح بـ ($S_{max.}$)

<u>Note</u>

① $V_u - V_c = \frac{d}{s} A_v f_y$ (There are, two unknowns S & A_v)

② usually spacing, A_v & final (S). check by ACI code.

ACI code (11.5) P. 148

* If $V_u > V_c$ shear reinf. is needed.

$$S = \frac{d A_v f_y}{V_u - V_c}$$

* If $\frac{1}{2} V_c < V_u < V_c$ use nominal shear reinf.

$$S_{max.} = \frac{d}{2} \leq 600\ mm$$

* If $V_u < \frac{1}{2} V_c$ no shear reinf.

CONTINUOUS R.C. BEAMS

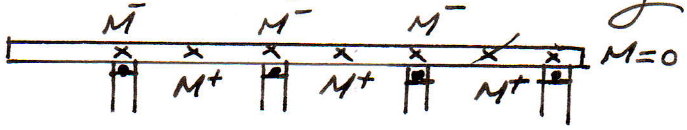
These beams have the advantage of smaller values of bending moments (in positive or negative regions). Also the deflections at mid-spans are smaller. But there is difficulty in practice to align the supports.

لوجود
العزم السالب
يقل الانحراف ؟

Calculations of positive & negative bending moments:

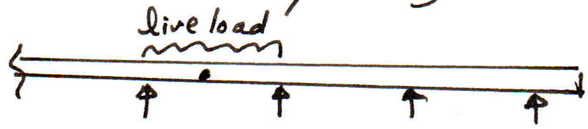
1. First calculate the positive bending moments (at mid-spans) & negative bending moments (over supports) due to dead loads only (the selfweight & surfacing loads).

دائماً في
التصميم تأخذ
أسوأ
احتمال



* use the equation of three moments or the slope-deflection method or the moments distribution method.

2. For the positive bending moment at any span, due to live load, assume the live load only at that span (or on that span & the alternate spans).

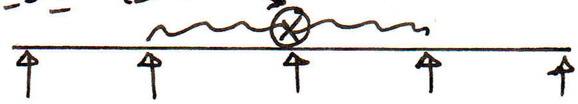


Use any method in structure (mentioned in 1). Add the values from the dead & live loads to get the final values of positive moment in that span, Repeat for other spans.

نجمع القيم الموجبة
للعزوم الناتجة من
الاحمال الحية لنحصل
على القيمة النهائية
بالتناوب وإعادة
الكل بقدر عدد
الـ (spans)

3. For the negative moment over any support due to live load, assume the live load only at the two spans of the support.

الموقع الذي نريد إيجاد العزم فيه ؟؟

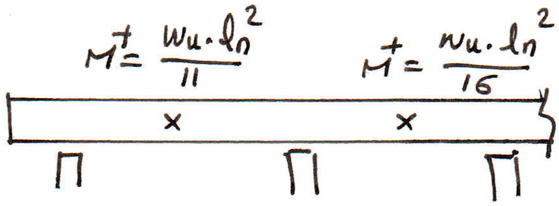


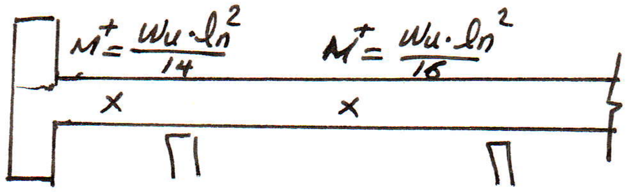
Add this to the value from the dead loads to get the final value of the negative at that support. Repeat for other supports.

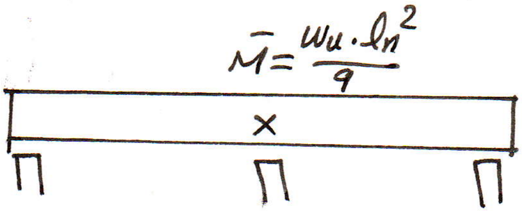
4. Under certain conditions, use moment coefficients by ACI code (8.3.3) P.81

simply supported = unrestrained end
نهاية غير مقيدة

l_n = clear span



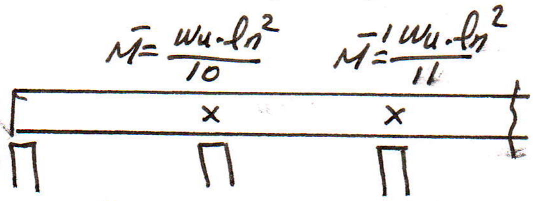


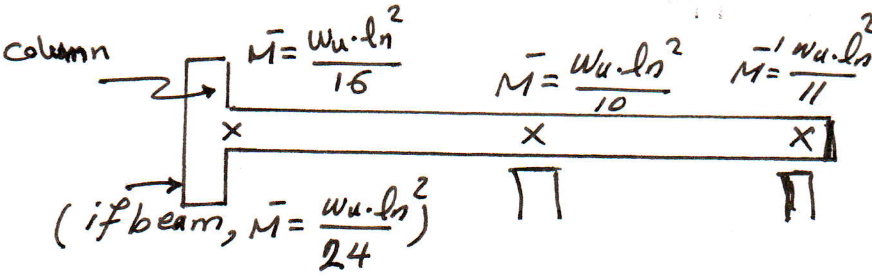


(2 spans)

Note: في الحياة العملية ولسرعة العمل في المشاريع نستخدم المعامل (over 10) لسهولة الحفظ وتوزيع الحديد وتفاصيله (steel detailing).

In Practice & for the ease of steel detailing use: $\boxed{M^+ = M^- = \frac{w_u \cdot l_n^2}{10}}$

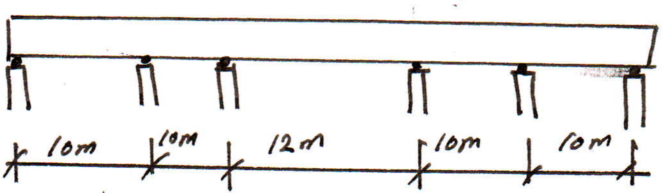




Design Procedures:-

1. Specify the live load & estimate the dead load (self weight & surfacing loads). Try a depth from ACI Code (Table 9.5a).
2. Calculate the ultimate load W_u & then M^+ & M^-.
3. Find steel ratio ρ & then steel A_s, detail this steel.
4. Design for shear reinforcement.

<u>Example:-</u> (R.C. Continuous beam).

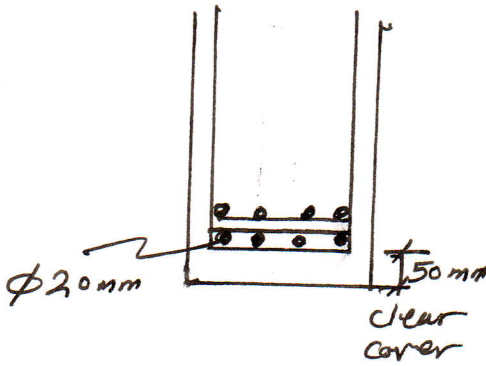


A R.C. beam (shown above) is to be designed. The beam will have a rectangular section of width 400mm, the concrete mix is $1:1\frac{1}{2}:3$ with w/c = 50% giving strength $f'_c = 27$ MPa & density $\gamma_c = 24.5$ kN/m^3. The steel has yield $f_y = 410$ MPa. The surfacing load comes from asphalt layer of 75mm & density $\gamma = 22$ kN/m^3. The specified live load is 40 kN/m^2. Calculate the depth & the reinforcement? for flexure (bending) & shear & detail of reinforcement? (ACI P.96)? h = 600mm?

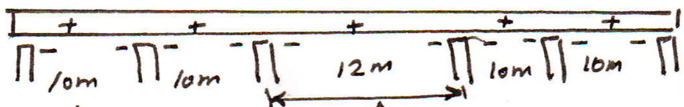
Note:

Effective Cover = 50+20+10 = 80mm

∴ depth d = 600 − 80 = 520mm



(see. Winter)?



(moment dist. هنا يؤخذ معدل العزم أو حسب

out of balance

R.C. SLABS

R.C. Slabs have several types:-

1. One-Way Slabs:-

These are wide rectangular beams spanning in one direction. Flexural (or bending) reinforcement is needed in this span. For the other direction, distribution steel (حديد توزيع) (for shrinkage & temperature / about $\rho = 0.002$) is needed. مثال * (hair cracks) تشققات شعرية بسبب وجود الحديد في الخلطة

Two-way slabs occur in:

i. Rectangular slabs supported on two opposite edges & free in the other two edges.
The effective span is the distance between the two supported (or fixed) edges.

ii. Rectangular slabs supported (or fixed) on all four edges but has length more than twice the width. The effective span is in the short direction (the width). Flexural reinforcement is needed in this short direction. Distribution steel in the other (long) direction.

2. Two-Way edge Supported slabs:-

These are rectangular slabs with all four edges supported or fixed & have length less than twice the width. Flexural (or bending) reinforcements are needed in both direction. The short direction requires more steel. التسليح باتجاهين والقصير يحتاج حديد أكثر ؟؟

3. Two-Way Column Supported Slabs (also called Flat Slab):-

These slabs have no edge beams. لاتوجد حافات مسنده

Flexural reinforcements are needed in both directions. The long direction requires more steel.

يستند على الاعمدة فقط دون عتبات beams
وهنا الاتجاه الطولي يحتاج حديد تسليح أكثر ؟؟

<u>Note</u>:- All Types of Slabs Usually Need No Shear Reinforcement.

دائما : لا يحتاج الى (S.R.) لماذا ؟
لأن سمك السقف ؟ يكفي لتحمل القص ؟

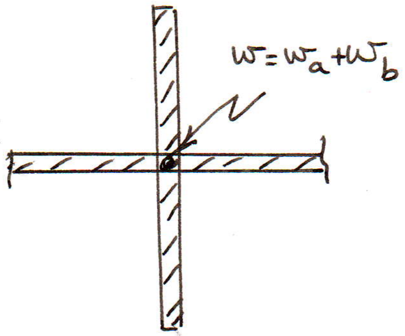
TWO_WAY EDGE SUPPORTED SLABS

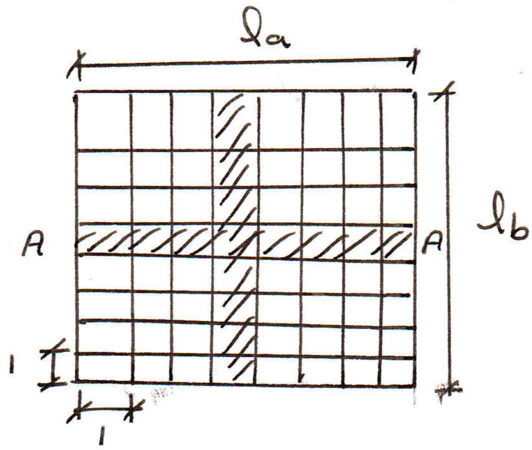
Foundamental Theory:- (2-way) النظرية الرئيسية للاسلوب الـ

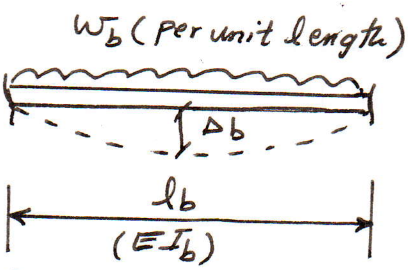
Consider an edge-supported rectangular slab under load w (Per unit area). This slab is assumed to act as a grid of beams in both directions. This width of each beam is 1 unit.

Take the central beams for purpose of analysis. Let the load w (per unit area) be divided into w_a in the short direction (span l_a) & w_b in the long direction (span l_b).

Basic Concept :-

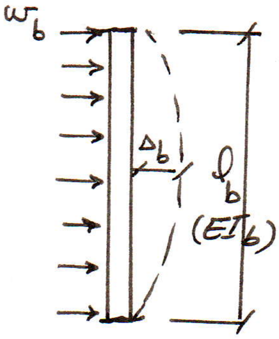


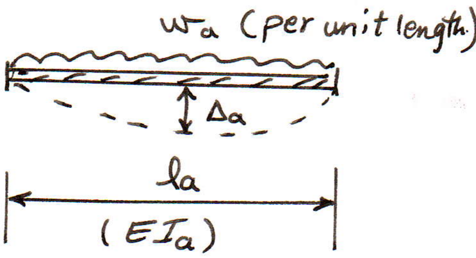




(beam in long direction)

or





(beam in short direction)
(Beam A-A)

The deflection Δ_a at midspan of short span is :

$$\Delta_a = \alpha \frac{w_a \cdot l_a^4}{EI_a}$$

The deflection Δ_b at midspan of long span is :

$$\Delta_b = \alpha \frac{w_b \cdot l_b^4}{EI_b}$$

where, $\alpha = \frac{5}{384}$ if both ends are simply supported

and $\alpha = \frac{1}{384}$ if both ends are fixed.

يوجد إنحراف خمسة أضعاف للعتبة البسيطة نسبة للمثبتة الطرفين

Also $E = E_c$ (for Concrete in both spans)

$$I_a = I_b = \frac{1}{12} \cdot 1 \cdot h^3 = I_g \quad \text{(for both spans).}$$

use the Condition, $\Delta_a = \Delta_b$, then

$$\alpha \frac{w_a \cdot l_a^4}{E_c I_g} = \alpha \frac{w_b \cdot l_b^4}{E_c I_g}$$

or $$\boxed{\frac{w_a}{w_b} = \left(\frac{l_b}{l_a}\right)^4}$$

If $l_b = l_a$ (square slab).

then, $w_a = w_b$ (equal).

If $l_b = 2\,l_a$ (rectangular slab).

Then $w_a = 16 w_b$, But $w = w_a + w_b$ then,

$$w_a = \frac{16}{17} w = 0.94 w$$
$$= 94\% \text{ of } w$$ بالاتجاه القصير

$$w_b = \frac{1}{17} w = 0.06\, w = 6\% \text{ of } w \text{ (very small)}$$
الاتجاه الطويل لا يأخذ ثقل كبير

* This is the limit for the 2-way action.

Design of two-way slabs:-

1. The live load w_l (per unit area) is specified. نسبة الاستعمال

2. Estimate the dead load (the self weight & the surfacing loads) w_d (per unit area). For total thickness h use ACI Code

$$h = \frac{\text{perimeter}}{180} \quad \text{(simplified)}$$

* ($h > 200$ mm)

3. Calculate the Ultimate load $w_u = 1.4 w_d + 1.7 w_l$ (ACI Code)

4. Calculate the positive bending moment M^+ at mid-span in the short direction, use ACI Code (Method 3):

$$M_a^+ = C_{A.DL}\,(1.4 w_d).\,l_a^2 + C_{A.LL}\,(w_l \cdot 1.7)\,l_a^2$$

(per unit width)

The coefficients $C_{A.DL}$ & $C_{A.LL}$ are given in tables.

5. Calculate the negative moment M_a^- in the edges of short directions:

$$M_a^- = C_{A.neg.} \cdot w_u \cdot l_a^2 \quad \text{(per unit width)}$$

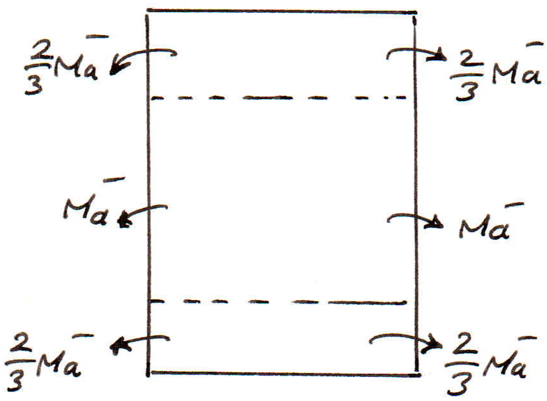
short dir.

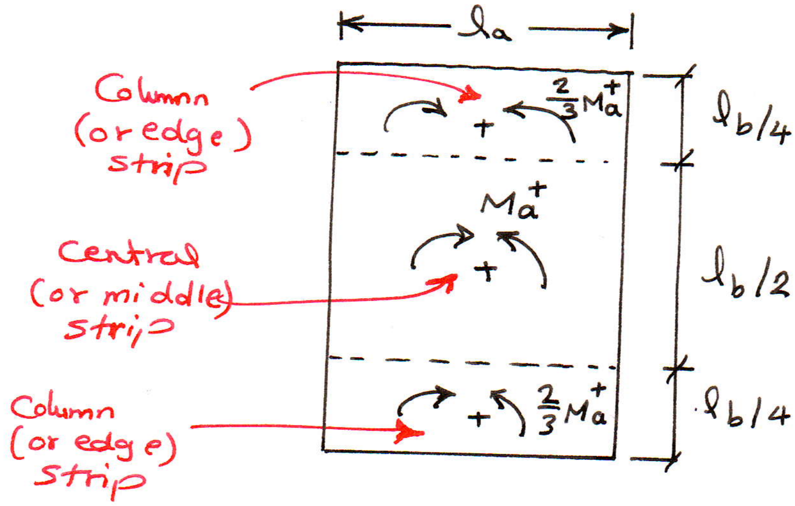
6. Repeat M_b^+ & M_b^- for the long direction

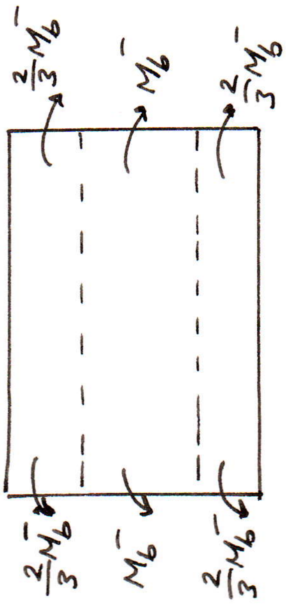
$$M_b^+ = C_{B.DL} \cdot (1.4 w_d) \cdot l_b^2 + C_{B.LL} \cdot (1.7\, w_l) \cdot l_b^2 \quad \text{(per unit width)}$$

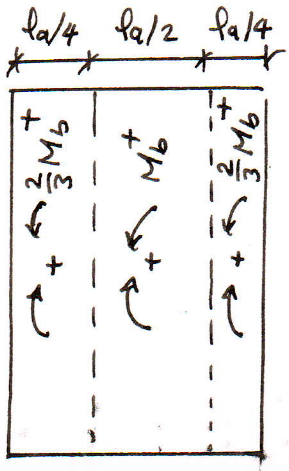
$$M_b^- = C_{B.neg.} \cdot w_u \cdot l_b^2 \quad \text{(per unit width)}$$

7. Assume $\frac{2}{3}$ of these moments in the column (or edge) strips.







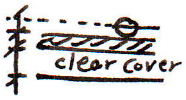


8. Specify the clear cover (usually 20 mm) and then the effective cover. Specify bar size (usually ϕ 8 mm), ϕ10, ϕ12, ϕ14, ϕ16, ϕ18 mm).

9. Start with short direction and find steel ratio ρ and then A_s (per unit width) for M_a^+ & M_a^-. Use

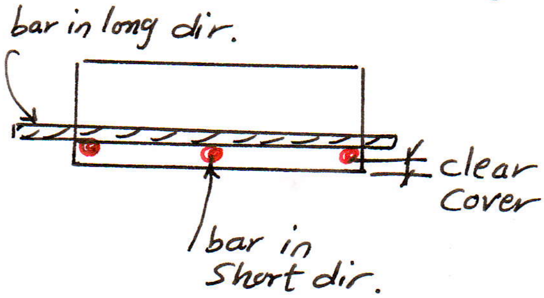
$$M_u = \phi \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) b d^2 \quad , \text{ where } b = (1\,m) \text{ or}$$

$$\frac{M_u}{\phi b d^2} \rightarrow \rho \quad \text{or} \quad R = \frac{M_u}{\phi f_c' b d^2} \rightarrow \omega = \frac{\rho f_y}{f_c'}$$



10. Repeat with long direction. Remember that the effective cover in the long direction is equal to (clear cover + bar in the short direction + $\frac{1}{2}$ bar in the long dir.). This is only for M_b^+.

eff. cover = (clear cover + bar) in short direction + $\frac{1}{2}$ bar in long dir.

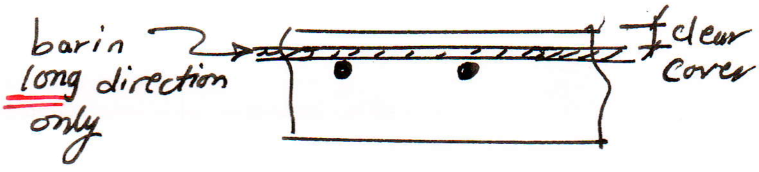


Note:-

For M_b^-, effective cover

= clear cover + $\frac{1}{2}$ bar diam.

* لعدم وجود حديد معارض في حالة M^{-}



* Minimum ratios of temperature and shrinkage reinforcement in slabs :- (ρ_{min}) :-

slabs where deformed bars with $f_y = 276$ or 345 MPa	0.0020
slabs where reinforced with $f_y = 414$ MPa	0.0018
slabs where reinforced with $f_y > 414$ MPa	$\frac{0.0018 * 414}{f_y}$

$A_s = \rho_{min}\, b\, h$ (for temp. & shrinkage reinforcement).

The max. distance between the bars $\leq$ 5h or 45 cm

* Minimum thickness h of nonprestressed (typical) one-way slabs :-

simply supported	$l/20$
one end continuous	$l/24$
Both end continuous	$l/28$
Cantilever	$l/10$

Corrections :-

1. For concrete having $w_c = 15-20\ kN/m^3$ the values should be multiplied by $(1.65 - 0.0003\, w_c) \geq 1.09$
2. For reinforcement having $f_y \neq 414$ MPa the values should be multiplied by $(0.4 + f_y/700)$.

One-way slabs

Ex 1 Design the slab shown in fig. below, L.L. = 6 kN/m²

Use ϕ 10mm , $f_y = 414\ N/mm^2$

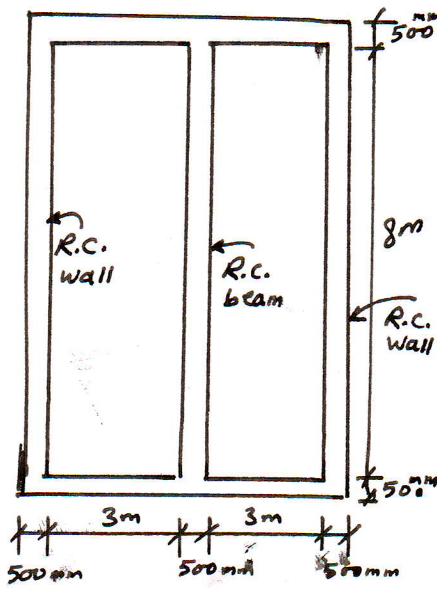
$f'_c = 25\ N/mm^2$

$\gamma_c = 25\ kN/m^3$, total cover (25mm).

① $\frac{L}{S} = \frac{8.0}{3} = 2.67 > 2$ one-way slab.

② $h = \frac{L}{24} = \frac{3500}{24} = 145.8\ mm$, Use $h = 150mm$

③ $w_d = 25 * 0.15 = 3.75\ kN/m^2$



$w_u = 1.4 w_d + 1.7 w_l$

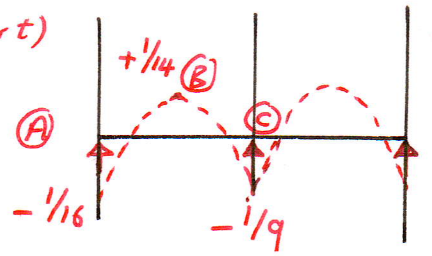
$= 1.4(3.75) + 1.7(6) = 15.45\ kN/m^2$

④ Calculations of bending moments:

$M_A^- = \frac{1}{16} * 15.45 * (3.5)^2 = 11.83\ kN.m$ (At exterior support)

$M_B^+ = \frac{1}{14} * 15.45 * (3.5)^2 = 13.52\ kN.m$ (at midspan)

$M_C^- = \frac{1}{9} * 15.45 * (3.5)^2 = 21.03\ kN.m$ (At interior support)



* $\rho_{min} = 0.0018$

$A_s = \rho b d = 0.0018 * 150 * 1000$ ← h (total depth) or thickness

$= 270\ mm^2$

$d = 150 - 25 = 125\ mm$

$M_u^- = 11.83$ (ext. supp.) , $R = \frac{11.83 * 10^6}{0.9 * 25 * 1000 * 125^2} = 0.03365 \Rightarrow \omega = 0.0345$

$\rho = 0.0345 * \frac{25}{414} = 0.0020833$

$A_s = 0.0020833 * 1000 * 125 = 260\ mm^2 < A_{s\,min}$, Use $A_s = 270\ mm^2 = A_{s\,min}$.

$M_B^+ = 13.52$, $R = 0.03846 \rightarrow \omega = 0.0394$
(mid-span)
supp.

$\rho = 0.0394 * \frac{25}{414} = 0.00238$

$As = 0.00238 * 1000 * 125 = 297.5\ mm^2$

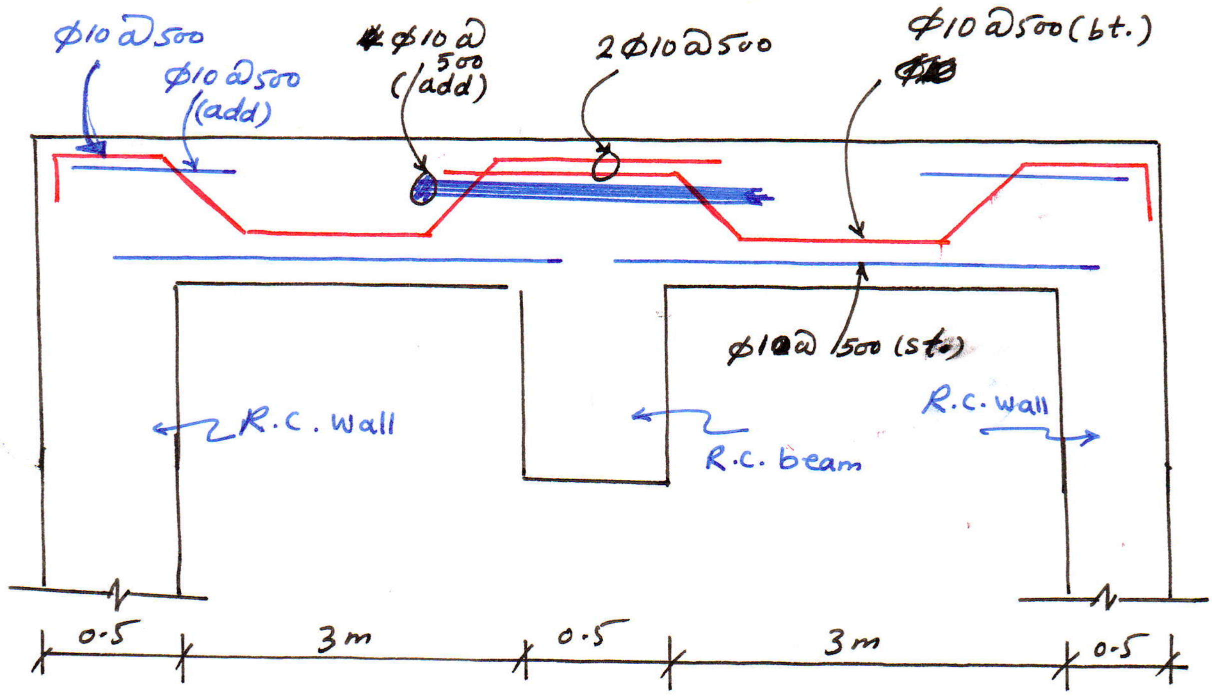
$M^- = 21.03\ kN.m$
(interior supp.)

$R = 0.0598$

$\omega = 0.062$

$\rho = 0.062 * \frac{25}{414} = 0.003744$

$As = 0.003744 * 1000 * 125 = 468\ mm^2$



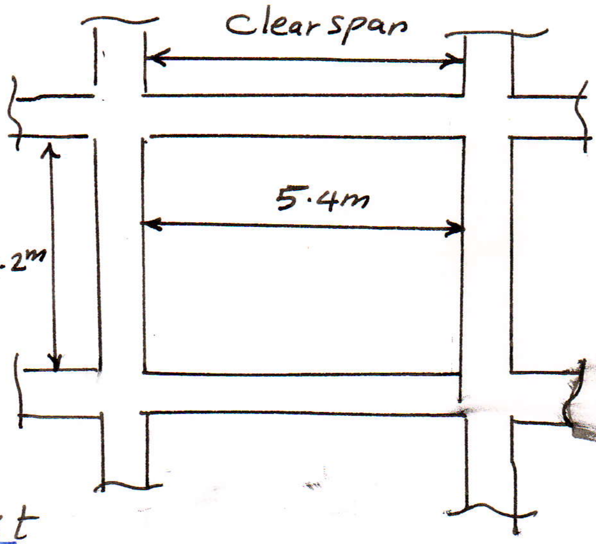
Example:- An interior 2-way slab (4.2m * 5.4m) is continuous over edge beams. The slab carries a live load of $6\ kN/m^2$ & tiling load $1.2\ kN/m^2$. Design this slab?

For concrete: $f'_c = 21\ MPa$

$\gamma_c = 24.5\ kN/m^3$

For steel: $f_y = 310\ MPa$

use bars of ϕ 10mm. Use clear cover = 20mm ?



Solution:

First estimate the total thickness:

$$h = \frac{2(5.4+4.2)*1000}{180}$$

$= 107mm$; Use $h = 125mm$ (nearest to next 25mm)

$$h = \frac{\text{Perimeter}}{180}$$

self weight $= 0.125 * 1 * 1 * 24.5 = 3.0625\ kN/m^2$

total dead load $w_d = 3.0625 + 1.2 = 4.2625\ kN/m^2$

tiling

live load $w_\ell = 6\ kN/m^2$

ultimate total load $= 1.4\, w_d + 1.7\, w_\ell$

$= 1.4(4.2625) + 1.7(6) = 16.1675\ kN/m^2$

where;

Impo. $w_{u\ell} = 6 * 1.7 = 10.2\ kN/m^2$, $w_{ud} = 1.4(4.2625) = 5.9675\ kN/m^2$

clear spans $\ell_a = 4.2m$ (short span)

$\ell_b = 5.4$ (long span)

start with short span :-

Effective Cover $= 20 + 5 = 25\,mm$

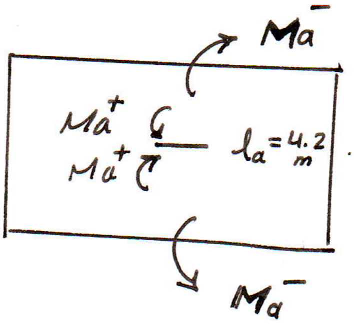
Effective depth $d = 125 - 25 = 100\,mm$

calculate the positive M_a^+ & the negative M_a^- (in the short direction). Use ACI - code (method three)

$$M_a^+ = C_{a\ \text{of live load}} * 1.7\, W_l * l_a^2 + C_{a\ \text{of dead load}} * 1.4 W_d * l_a^2$$

From table ; ratio $m = \dfrac{l_a}{l_b} = \dfrac{4.2}{5.4} = 0.777$

Here, case 2 is used :

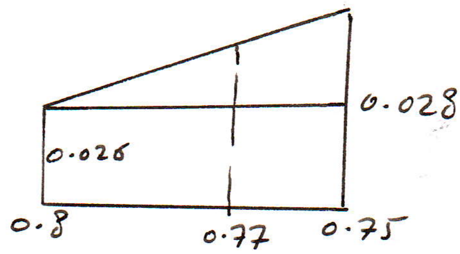


$C_{A\,D.L} = 0.026$ for $m = 0.8$

$C_{A\,D.L} = 0.028$ for $m = 0.75$

$\boxed{\therefore\ C_{A\,D.L} = 0.027 \text{ for } m = 0.775}$

Also $C_{A\,l.l} = \begin{cases} 0.041 & \text{for } m = 0.8 \\ 0.045 & \text{for } m = 0.75 \end{cases}$



$\boxed{\text{Use } C_{A\,L.L} = 0.043}$ ~~for m = 0.~~ $\boxed{\text{for } m = 0.775}$

$$M_a^+ = 0.043 * 10.2 * (4.2)^2 + 0.027 * (5.9675) * (4.2)^2$$
$$= 10.579 \text{ KN.m/m (width)}$$

$$M_a^- = C_{a\,neg.} * w_u * l_a^2$$

$$M_a^- = 0.067 * 16.1675 * (4.2)^2$$
$$= 19.108 \text{ KN.m/m (width)}$$

$C_{A\,neg.} = \begin{cases} 0.065 & \text{for } m = 0. \\ 0.069 & \text{for } m = 0.7 \end{cases}$

$\therefore$ use $C_{a\,neg.} = 0.067$

$\boxed{m = \dfrac{4.2}{5.4} = 0.777}$

Find the steel reinforcements in short direction:-

For $M_a^+ = 10.579$ kN.m/m
$= 10.579 * 1000$ N.mm/mm
$= 10.579 * 1000 * 1000$ N.mm/m

Use $R = \dfrac{M_a}{\phi f'_c b d^2}$; $\rho = \omega f'_c / f_y$:-

$$R = \frac{10.579 * 1000 * 1000 \text{ (N.mm/m)}}{\underbrace{0.90}_{\phi} * \underbrace{21 \text{ (N/mm}^2)}_{f'_c} * \underbrace{1000 \text{ (mm)}}_{b} * \underbrace{(100^2) \text{ (mm}^2)}_{d^2}}$$

$= 0.05597$

This gives : $\omega = 0.058$ for $R = 0.0560$

The steel ratio $\rho = \omega f'_c / f_y$; when $\omega = \rho f_y / f'_c$

$= 0.058 * 21/310$
$= 0.003929$

steel Area $A_s = \rho b d$
$= 0.003929 * 1000 * 100$
$= 392.9$ mm²/m (width)
$= 3.929$ cm²/m

∴ Use $\phi 10$ mm @ 200 mm → $A_s = 3.93$ cm²/m

(Check 7.6.5 in Code)

For $M_a^- = 19.108$ kN.m/m
$= 19.108 * 1000$ N.mm/mm
$= 19.108 * 1000 * 1000$ N.mm/m

Thus $R = \dfrac{19.108 * 1000 * 1000}{0.90 * 21 * 1000 * (100)^2}$

$= 0.1011$

This gives : $\omega = 0.108$

So, $\rho = \omega f'_c / f_y$

$= 0.108 * 21 / 310 = 0.007316$

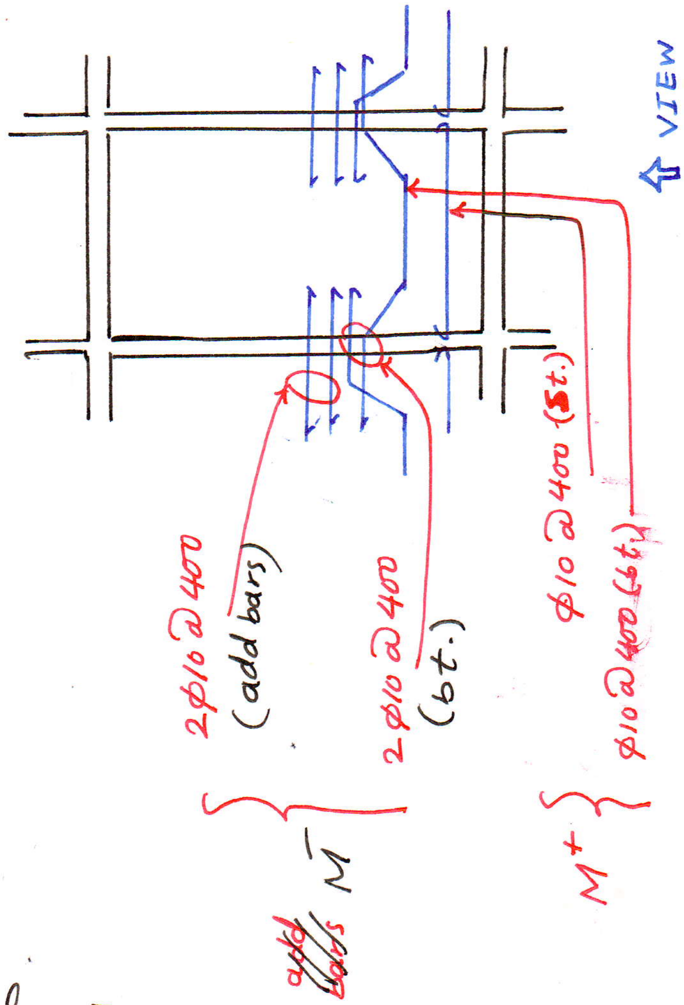
$A_s = \rho b d = 0.007316 * 1000 * 100$

$= 731.6 \text{ mm}^2/\text{m} = 7.316 \text{ cm}^2/\text{m}$

Use ϕ 10 mm @ 100 mm $\rightarrow A_s = 7.85 \text{ cm}^2/\text{m}$.

check area of steel ??

Show the details of this reinforcement :-



Return to the long span $l_b = 5.4$ m

& find M_b^+ & M_b^-. Here (ACI/method 3)

$M_b^+ = C_{B.DL} \cdot (1.4 w_d) \cdot l_b^2 + C_{B.LL} \cdot (1.7 w_l) \cdot l_b^2$

For $m = \frac{4.2}{5.4} = 0.777$, then

$C_{B.DL} = \begin{cases} 0.011 & \text{for } m = 0.80 \\ 0.009 & \text{for } m = 0.75 \end{cases}$

اقرأ ما بين الأرقام ؟

Use $C_{B.DL} = 0.010$ for $m = 0.775$

Also $C_{B.LL} = \begin{cases} 0.017 & \text{for } m = 0.80 \\ 0.014 & \text{for } m = 0.75 \end{cases}$

Use $C_{B.LL} = 0.0155$ for 0.775

Then $M_b^+ = 0.010 * (1.4 * 4.2625) * 5.4^2$

$+ 0.0155 * (1.7 * 6) * 5.4^2$

$= 6.350$ kN.m/m (width)

دائماً في (2-way) في الاتجاه القصير عزم أكبر ⊖ من الاتجاه الطويل

Also; $M_b^- = C_{B.neg.} \cdot w_u \cdot \ell_b^2$

Here; $C_{Bneg.} = \begin{cases} 0.027 & \text{for } m = 0.8 \\ 0.022 & \text{for } m = 0.75 \end{cases}$

Use $C_{B.neg.} = 0.0245$ for $m = 0.775$

Then: $M_b^- = 0.0245 * 16.1675 * 5.4^2$

$= 11.315$ kN.m/m (width)

11.55

Find the steel in the long direction

for $M_b^+ = 6.350$ kN.m/m

$= 6.350$ N.mm/mm $* 1000$

$= 6.350 * 1000 * 1000$ /m

Use

$$R = \frac{6.350 * 1000 * 1000}{0.90 * 21 * 1000 * 90^2}$$

(as $d = 125 - 35$
$= 90$ mm)

في حالة العزوم الموجبة (+) في الاتجاه الطويل فقط ؟

$= 0.04148 \simeq 0.0415$

This gives $\omega = 0.0425$

Then,

$\rho = \omega f_c' / f_y$

$= 0.0425 * 21/310$

$= 0.002879$

$A_s = \rho b d$

$= 0.002879 * 1000 * 90$

$= 259.1\ mm^2/m$

$= 2.591\ cm^2/m$

$= 2.591\ cm^2/m \rightarrow$ Use $\phi 10\ mm$ @ $300\ mm$ (+)

$\therefore A_s = 2.62\ cm^2/m$

For $M_b^- = 11.315\ kN.m/m$

$= 11.315 * 10^6\ N.mm/m$

Use $R = \dfrac{11.315 * 10^6}{0.9 * 21 * 1000 * 100^2}$ (as $d = 100\ mm$)

لايوجد مايفيد بعمق لذا
يكون $(d=100)$

$= 0.05986$

This gives; $\omega = 0.0622$

So $\rho = 0.0622 * 21 / 310$

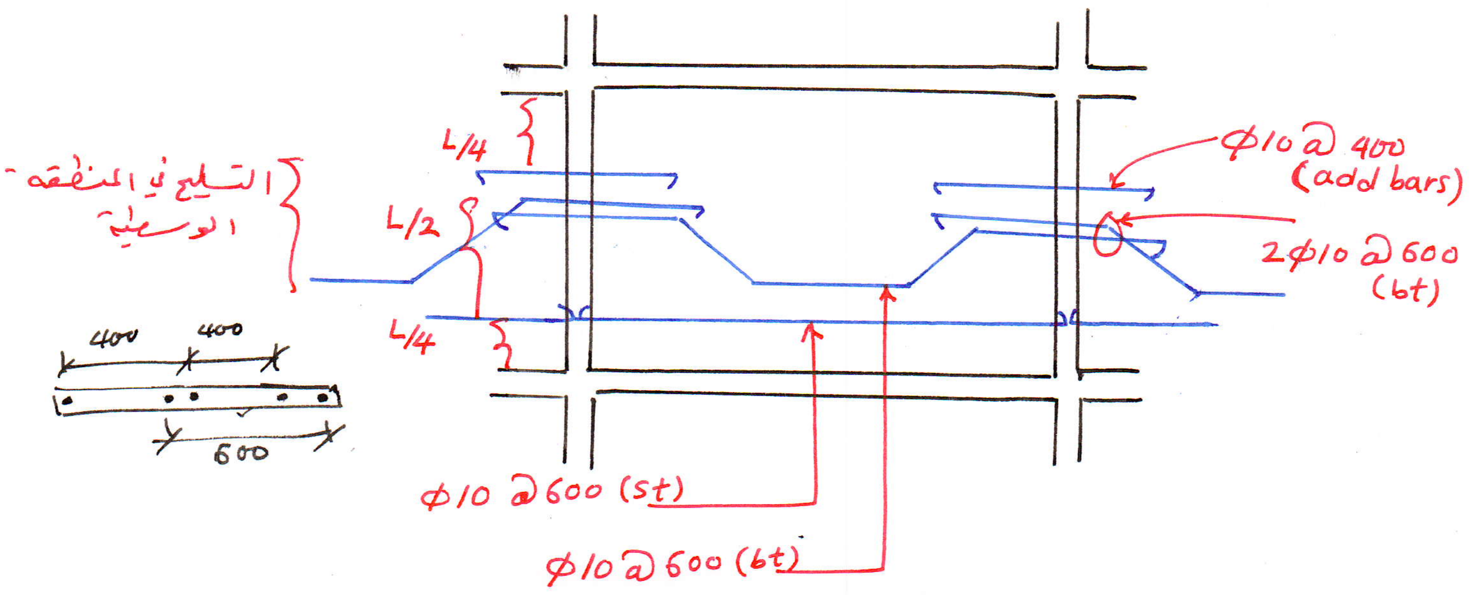
$= 0.004214$

$A_s = 0.004214 * 1000 * 100 = 421.4\ mm^2/m$

$= 4.214\ cm^2/m$

Use $\phi 10mm$ @ $175\ mm \rightarrow A_s = 4.49\ cm^2/m$ (−)

The details must be shown



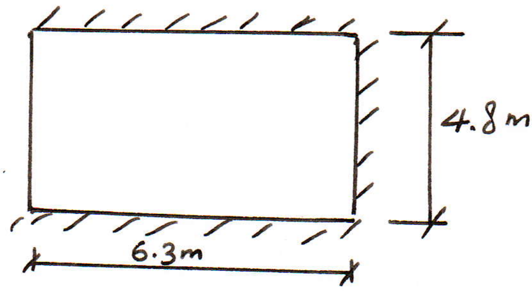
(VIEW)

Note: These steel spacings are the central (or middle) strips (in the short or long directions). For the column (or edge) strips use spacings $= \frac{3}{2} * \text{spacing}$ in central strips.

* العزوم الطرفية أقل من العزوم الوسطية بمقدار ؟ نستخدم مسافات التسليح هي spacing $*(\frac{3}{2})$ in central

Example (2-way slab):-

An edge slab in a floor has clear spans $l_a = 4.8$ m (in short direction) & $l_b = 6.3$ m (in long direction). The slab is continuous over 3 edges & discontinuous over one edge (over short span). The imposed live load $w_l = 7.0\ kN/m^2$. The tiling load is $1.1\ kN/m^2$ use concrete ($1 : 1\frac{1}{2} : 3$) mix giving $f'_c = 27$ MPa & $\gamma_c = 24.5\ kN/m^3$ use $\phi 12$ mm steel bars of $f_y = 350$ MPa. Let the clear cover be 20 mm?



(edge slab)

Solution:-

Estimate the self weight of the slab by estimating the total thickness:

$$h = \frac{2(4.8 + 6.3) * 1000}{180} = 123.3 \text{ mm}$$

use $h = 125$ mm. The self weight will be:

$0.125 * 1 * 1 * 24.5 = 3.0625 \ kN/m^2$

Total dead load,

$$w_d = 3.0625 + 1.1 = 4.1625 \ kN/m^2$$

live load

$$w_l = 7.0 \ kN/m^2 \quad \text{(given)}$$

The ultimate load: $w_u = 1.4 * 4.1625 + 1.7 * 7$
$= 17.7275 \ kN/m^2$

start with the reinforcement in the short direction ($l_a = 4.8m$).

Here the aspect ratio, $m = \frac{4.8}{6.3} = 0.7619$

Use ACI method 3:

$$M_a^+ = C_{A.DL} \cdot (1.4 \, w_d) . l_a^2 + C_{A.LL} \cdot (1.7 \, w_l) . l_a^2$$

Here,

$$C_{A.DL} = \begin{cases} 0.029 & \text{for } m = 0.8 \\ 0.031 & \text{for } m = 0.75 \end{cases}$$

use $C_{A.DL} = 0.0307$

Also

$$C_{A.LL} = \begin{cases} 0.042 & \text{for } m = 0.80 \\ 0.046 & \text{for } m = 0.75 \end{cases}$$

Use $C_{A.LL} = 0.045$

Then

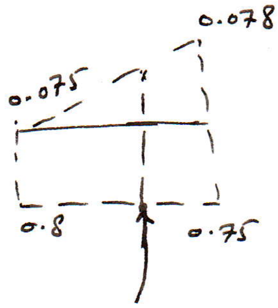
$$M_a^+ = 0.0307 * (1.4 * 4.1625) * 4.8^2 + 0.045 * (1.7 * 7.0) * 4.8^2$$

$$= 16.4598 \ kN.m/m \ \text{(width)}$$

Also

$$\cancel{C_{A.LL}} = M_a^- = C_{A.neg.} \cdot w_u \cdot l_a^2$$

Here $C_{A.neg.} = \begin{cases} 0.075 & \text{for } m = 0.8 \\ 0.078 & \text{for } m = 0.75 \end{cases}$



use $C_{A.neg} = 0.0774$ for $m = \frac{4.8}{6.3} = 0.7619$

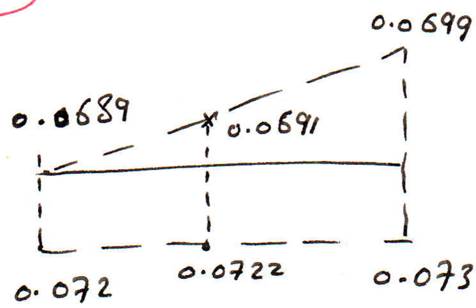
so, $M_a^- = 0.0774 * 17.7275 * (4.8)^2$
$= 31.6133$ kN.m / m (width)

The effective depth $d = 125 - 20 - \frac{12}{2} = 99$ mm

use $R = \frac{M_u = M^+}{\phi f'_c \, b d^2} = \frac{16.4598 * 10^6 \ (N.mm/m)}{0.9 * 27 \ (N/mm^2) * 1000 \ (mm) * 99^2 \ (mm^2)}$

$= 0.0691$

This gives $\omega = 0.0722$



The steel ratio:

$\rho = \omega f'_c / f_y$

$= 0.0722 * \frac{27}{350} = 0.005570$

Check: $\rho_{min} \leq \rho < \rho_{max}$ (Here o.k.) $\rightarrow$ $\rho_{min} = 0.002$ (S. & T.)

ρ_{max} = large value

Steel area: $A_s = \rho b d$

$= 0.00557 * 1000 * 99$
$= 551.4$ mm^2/m (width)
$= 5.514$ cm^2/m (width)

Use $\phi 12$ mm @ 200 mm $\rightarrow A_s = 5.65\ cm^2/m$ (width)

For the negative moment $M_a^- = 31.6133$ kN.m/m (width)

Use $$R = \frac{31.6133 * 10^6}{0.90 * 27 * 1000 * 99^2} = 0.1327$$

this gives $\omega = 0.145$

so;

$$\rho = 0.145 * \frac{27}{350} = 0.011186$$

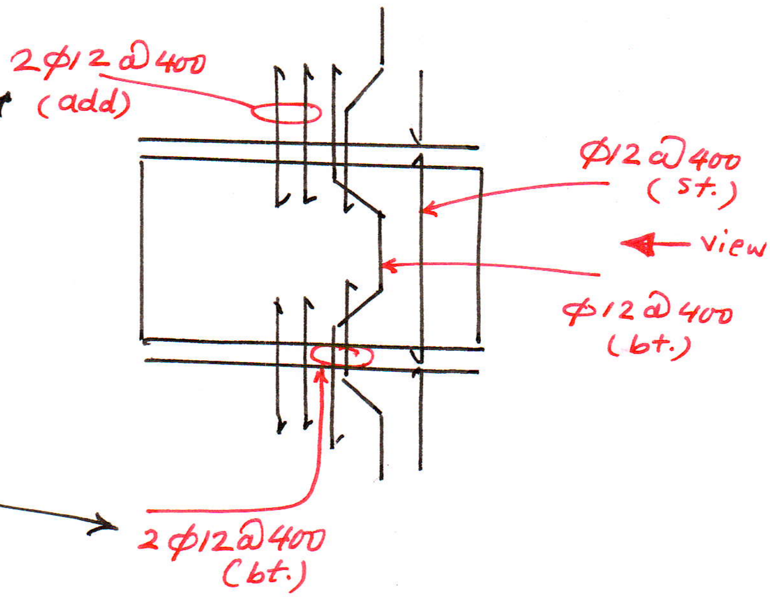
$$A_s = 0.011186 * 1000 * 99 = 1107.4\ mm^2/m$$
$$= 11.074\ cm^2/m \text{ (width)}$$

أو بطريقة
النسبة يمكن ملاحظة
الفرق الناتج من
التقريب ؟

Notice that : $$A_s = \frac{5.514}{16.4598} * 31.6133 = 10.59\ cm^2/m$$

Use $\phi 12$ @ 100 mm $\rightarrow A_s = 11.31\ cm^2/m$

<u>Note</u> : For the edge (or column) strips use spacings ($\frac{3}{2}$ spacings) in central strip.



لأننا نحتاج أربعة (bars) كل (100^{mm})
واحد في منطقة العمود بالي

| | | | |

100mm & so on . . . .

Long direction $\quad l_b = 6.3m$ to find M_b^+ & M_b^-

$$M_b^+ = C_{BDL}(1.4w_d)\,l_b^2 + C_{BLL}(1.7w_l)\,l_b^2$$

$$m = \frac{l_a}{l_b} = \frac{4.8}{6.3} = 0.7619$$

$$C_{B.DL.} = \begin{cases} 0.01 & \text{for } m=0.8 \\ 0.007 & \text{for } m=0.75 \end{cases}$$

$\therefore$ use $\quad C_{BDL.} = 0.0076$

$$C_{B\,LL.} = \begin{cases} 0.017 & \text{for } m=0.8 \\ 0.013 & \text{for } m=0.75 \end{cases}$$

$\therefore$ use $\quad C_{B\,LL.} = 0.01380$

$$\therefore\ M_b^+ = 0.0076\,(1.4 * 4.1625) * (6.3)^2 + (0.01380)(1.7 * 7) * (6.3)^2$$

$M_b^+ = 8.276$ kN.m/m (width)

$$M_b^- = C_{Bnegative} * w_u * l_b^2$$

$$C_{Bnegative} = \begin{cases} 0.017 & \text{for } m=0.8 \\ 0.014 & \text{for } m=0.75 \end{cases}$$

$\therefore$ use $C_{Bnegative} = 0.0146$

$$M_b^- = 0.0146 * 17.7275 * (6.3)^2$$

$M_b^- = 10.273$ kN.m/m (width)

For steel reinforcement, use $M_b^+ = M_u^{+} = \underline{8.276}$

$$R = \frac{M_u = M_b^+}{\phi f_c' b d^2} = \frac{8.276 * 10^6}{0.9 * 27 * 1000 * (87)^2} = 0.045$$

where $d = 125 - (20 + 12 + \frac{12}{2}) = 87$

$\omega = 0.046$

$$\rho = \frac{\omega f'_c}{f_y} = \frac{0.046 * 27}{350} = 0.003548$$

$A_s = \rho b d \Rightarrow A_s = 0.003548 * 1000 * \cancel{87}\ 87 = 308.67\ mm^2/m$ (width)

∴ use $\phi 12\ mm$ @ $350\ mm$ $\Rightarrow A_s = 323\ mm^2/m$ (width)

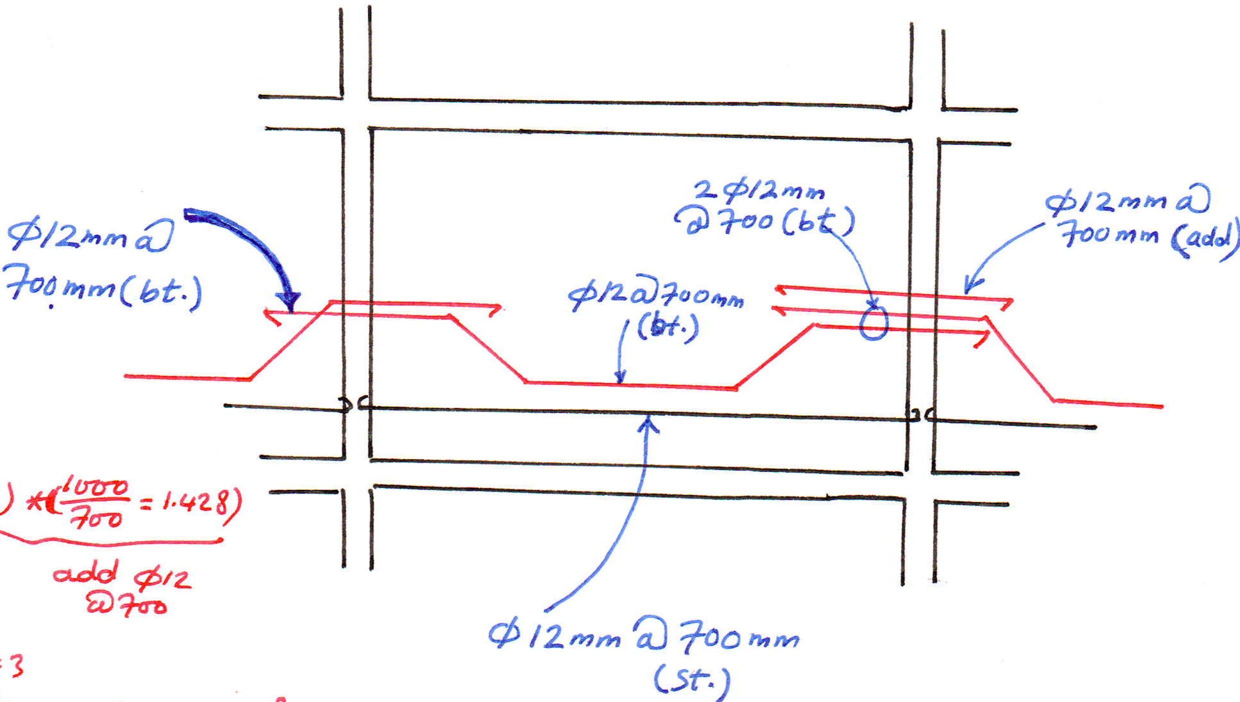
for $M_b^- = 10.273\ kN.m/m$ (width)

$$R = \frac{10.273 * 10^6}{0.9 * 27 * 1000 * (99)^2} = 0.0431 \Rightarrow \omega = 0.044$$

$$\rho = \frac{\omega f'_c}{f_y} = \frac{0.044 * 27}{\cancel{410}\ 350} = 0.003394$$

∴ $A_s = \rho b d \Rightarrow A_s = 0.003394 * 1000 * 99 = 336\ mm^2/m$ (width)

∴ use $\phi 12$ @ $300\ mm$ $\Rightarrow A_s = 377\ mm^2/m$ (width) $= 3.77\ cm^2/m$ (width)



check area

$\bar{A} = \underbrace{2(113)}_{\text{bt. (-) @ 700 mm}} + \underbrace{1(113) * (\frac{1000}{700} = 1.428)}_{\text{add } \phi 12 \text{ @ } 700}$

$\bar{A} = 226 + 161.43$
$= 387.43\ mm^2$ o.k $> 377\ mm^2$

Note:- Use $\frac{3}{2} * 700 = 1050$ mm for spacing in the edge (or column) strips.

Example (Analysis of 2-way slab):-

An a 2-way R.C. slab, with thickness $h = 150$ mm, clear cover $= 20$mm, For concrete; $f'_c = 22$ MPa

$\gamma_c = 24$ kN/m^3

For steel; $f_y = 310$ MPa

The tiling load is for (40mm) thick., take $\gamma_{tiling} = \gamma_c = 24$ kN/m^3

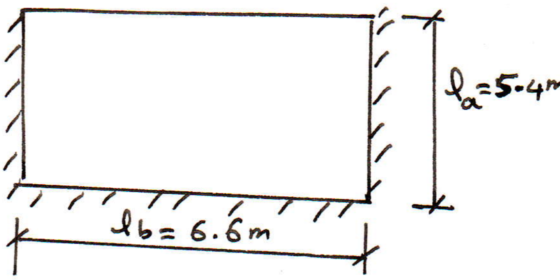
; Find w_l ?

For $M_a^+ = \phi 12$mm @ 200mm

$M_a^- = \phi 12$mm @ 150mm

For $M_b^+ = \phi 10$mm @ 200mm

$M_b^- = \phi 10$mm @ 100mm



Solution:-

Start with the short direction:

التحليل ؟ كما هو موجود واقع حال
وهو يختلف عن التصميم حيث هنالك
حرية في إيجاد النتائج نوعاً ما !

For M_a^+: $A_s = 565.0$ mm^2/m (width)

$$\rho = \frac{A_s}{bd} = \frac{565}{1000 * 124} = 0.004556$$

where $d = 150 - 20 - \frac{12}{2} = 124$ mm

Find $\omega = \rho \dfrac{f_y}{f'_c} = 0.004556 * \dfrac{310}{22} = 0.0642$

This gives $R = \begin{cases} 0.0616 & \text{for } \omega = 0.064 \\ 0.0625 & \text{for } \omega = 0.065 \end{cases}$

use $R = 0.0618$ for $\omega = 0.0642$

Then M_a^+ is obtained from $R = \dfrac{M_a^+}{\phi f'_c bd^2}$ or

; $M_a^+ = R(\phi f'_c bd^2)$

So,

$$M_a^+ = 0.0618 * (0.9 * 22 * 1000 * 124^2)$$
$$= 18.815 * 10^6 \text{ N.mm / m (width)}$$
$$= 18.815 \text{ kN.m / m (width)}$$

Use ACI code, Method (3):

$$M_a^+ = C_{A.DL} * (1.4 w_d) \cdot l_a^2 + C_{A.LL} * (1.7 w_l) \cdot l_a^2$$

Here $m = \dfrac{l_a}{l_b} = \dfrac{5.4}{6.6} = 0.818$

Thus

$C_{A.DL} = \begin{cases} 0.029 & \text{for } m = 0.85 \\ 0.032 & \text{for } m = 0.8 \end{cases}$ (case 8)

interpolation Use $C_{A.DL} = 0.0308$ for $m = 0.82$

Also

$C_{A.LL} = \begin{cases} 0.040 & \text{for } m = 0.85 \\ 0.044 & \text{for } m = 0.80 \end{cases}$

use $C_{A.LL} = 0.0424$ for $m = 0.82$

The dead load is:

$$w_d = \frac{(150 + 40)}{1000} * 24$$
$$= 4.560 \text{ kN/m}^2$$

substitute :

$$18.815 = 0.0308 * (1.4 * 4560) * 5.4^2 + 0.0424 * (1.7\, w_l) * 5.4^2$$

This gives $w_l = \underline{6.224}$ kN/m^2

<u>Repeat for Ma^-</u> :

$$A_s = 754.0\ mm^2/m \text{ (width)}$$

$$\rho = \frac{754}{1000 * 124} = 0.006081$$

$$\omega = 0.006081 * \frac{310}{22} = 0.0857$$

$$R = \begin{cases} 0.0807 & \text{for } \omega = 0.085 \\ 0.0816 & \text{for } \omega = 0.086 \end{cases}$$

Use $R = 0.0814$ for $\omega = 0.0857$

0.08133

Then $Ma^- = 0.0814 * (0.9 * 22 * 1000 * 124^2)$
$= 24.782 * 10^6$ N.mm/m (width)
$= 24.782$ kN.m/m (width)

Use ACI method 3 (case 8) where here $m = 0.818$
$\simeq 0.82$.

Thus

$$C_{A\,neg.} = \begin{cases} 0.049 & \text{for } m = 0.85 \\ 0.055 & \text{for } m = 0.8 \end{cases}$$

Use $C_{A\,neg.} = 0.053$ for $m = 0.82$

So, from $Ma^- = C_{A.neg.} \cdot w_u \cdot l_a^2$

Then $Ma^- = 24.782 = 0.053 * w_u \cdot l_a^2$

or $24.782 = 0.053 * w_u * 5.4^2 \rightarrow w_u = 16.035$ kN/m^2

Then $16.035 = 1.4 * 4.56 + 1.7 * w_l$

This gives $w_l = 5.677\ kN/m^2$

Start again with the long span:

For M_b^+:

$$d = 150 - 20 - 12 - \frac{10}{2} = 113mm$$

* لاحظ ماه (d) هنا ، عند حساب: (M_b^+)

As befor:

$$A_s = 393\ mm^2/m\ (width)$$

$$\rho = \frac{393}{1000 * 113} = 0.003478$$

$$\omega = 0.003478 * \frac{310}{22} = 0.049$$

This gives $R = 0.0476$

Thus

$$M_b^+ = 0.0476 * (0.9 * 22 * 1000 * 113^2)$$
$$= 12.034\ kN.m/m\ (width)$$

From tables (13.5 & 13.6 / case 8)

$$M_b^+ = C_{B.DL.} \cdot (1.4 w_d) l_b^2 + C_{B.LL.} \cdot (1.7 w_l) \cdot l_b^2$$

Here $m = 0.82$ thus

$$C_{B.DL.} = \begin{cases} 0.017 & \text{for } m = 0.85 \\ 0.015 & \text{for } m = 0.8 \end{cases}$$

Use $C_{B.DL.} = 0.0158$ for $m = 0.82$

Also

$$C_{B.LL} = \begin{cases} 0.022 & \text{for } m = 0.85 \\ 0.019 & \text{for } m = 0.8 \end{cases}$$

الفرق 0.03

الفرق 0.05

Use $C_{B.LL.} = 0.0202$ for $m = 0.82$

Then

$$12.034 = 0.0158 * (1.4 * 4.56) * 6.6^2$$
$$+ 0.0202 * (1.7 * w_l) * 6.6^2$$

This gives $w_l = 5.107\ kN/m^2$

Repeat for M_b^- : $A_s = 785\ mm^2/m$ (width)

$$\rho = \frac{785}{1000 * 124} = 0.006331$$

?

$$\omega = 0.006331 * \frac{310}{22}$$
$$= 0.0892$$

This gives :

$$R = \begin{cases} 0.0843 & \text{for } \omega = 0.089 \\ 0.0852 & \text{for } \omega = 0.090 \end{cases}$$

Use $R = 0.0845$ for $\omega = 0.0892$

Then $M_b^- = 0.0845 * (0.9 * 22 * 1000 * 124^2)$
$= 25.725\ kN.m/m$ (width)

Use ACI method 3 (case 8 & $m = 0.82$)

From table 13.4 :

$$M_b^- = C_{B.neg.} * w_u * l_b^2$$

Here $C_{B.neg.} = \begin{cases} 0.046 & \text{for } m = 0.85 \\ 0.041 & \text{for } m = 0.8 \end{cases}$

use $C_{B.neg.} = 0.043$ for $m = 0.82$

Then $25.725 = 0.043 * w_u * 6.6^2$

This gives $w_u = 13.734\ kN/m^2$

Then $13.734 = 1.4 * 4.560 + 1.7 * w_l$

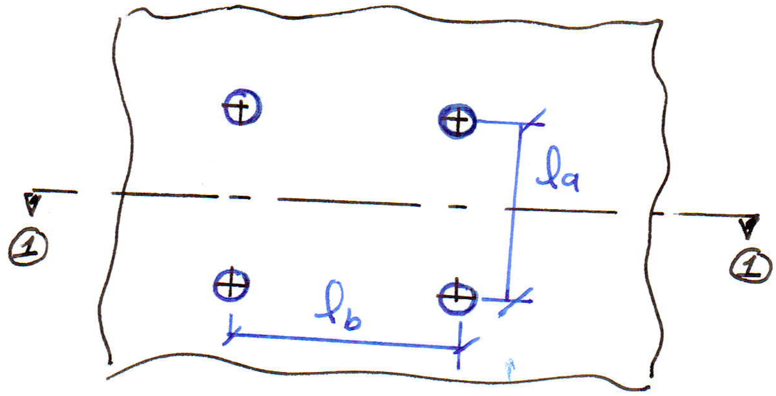
Thus $\Rightarrow w_l = 4.323\ kN/m^2$

Use the smallest $w_l = 4.323\ kN/m^2$.

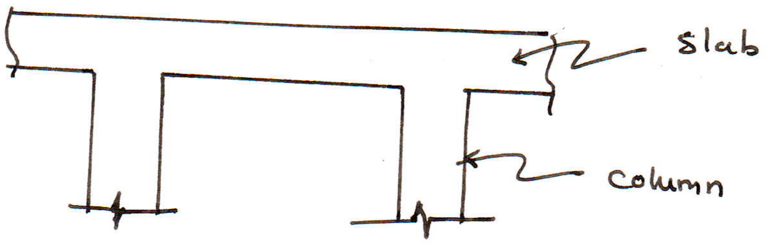
* معدل وزن الإنسان (70kg)

FLAT SLABS

A flat slab is supported directly on Columns (without edge beams). It is also 2-way slab because curvatures and bending moments are acting in both directions. Here, the long direction is dominant in design and analysis.

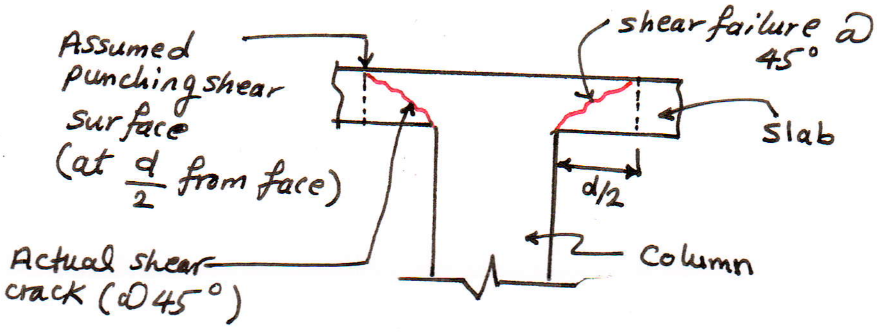


section 1 – 1 :-



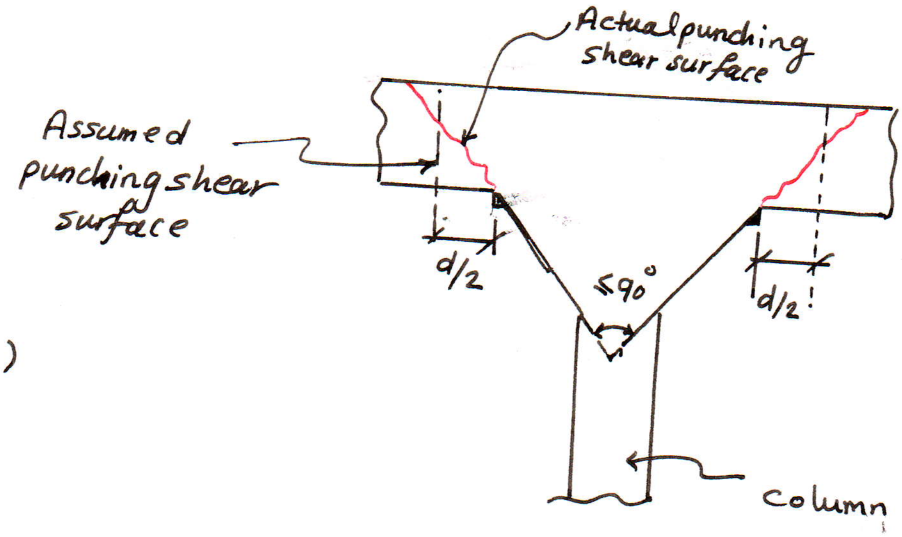
The most danger of failure in flat slabs is from the Punching shear from columns. In most cases, the thickness of the slab must be determined to resist the punching shear. No shear reinforcements are used.

1. Flat plate :-



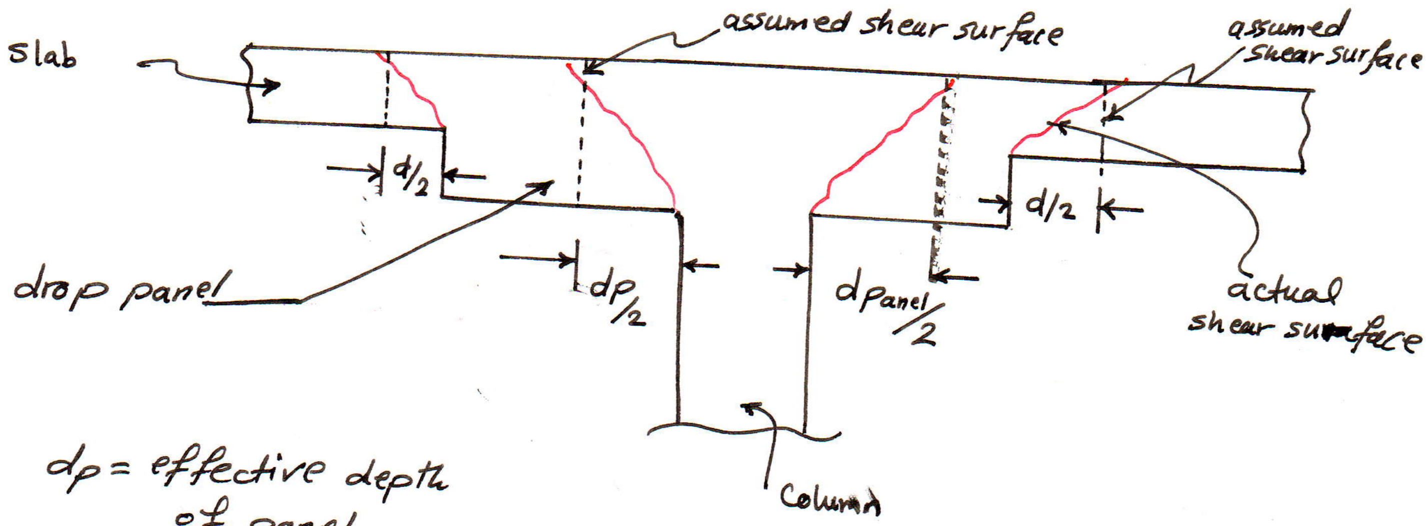
Thus slab has no enlarged capital at top of column or drop panels in the slab around the columns.

2. Flat Slab with column Capitals :-



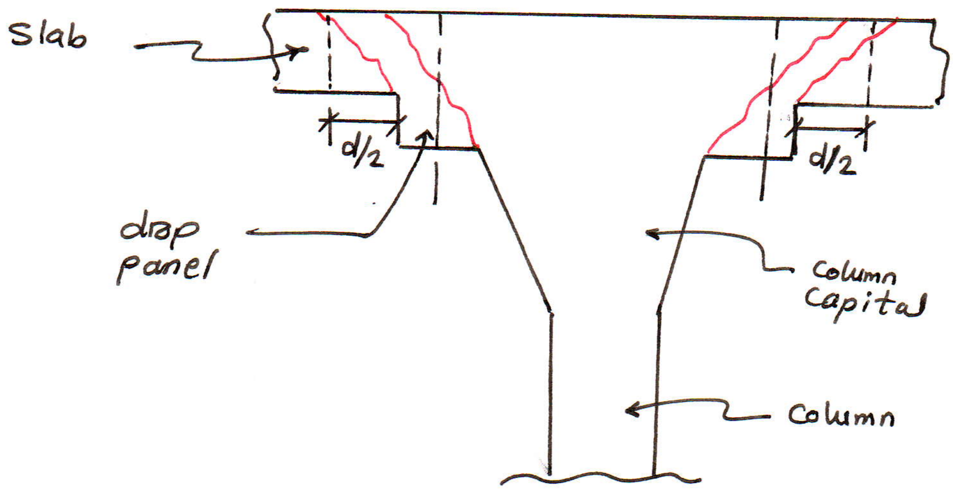
(∡ ≤ 90° or 30°–60°)

3. Flat slab with drop panels



dp = effective depth of panel

4. Flat slab with both column Capital and drop panel (for heavy loads),



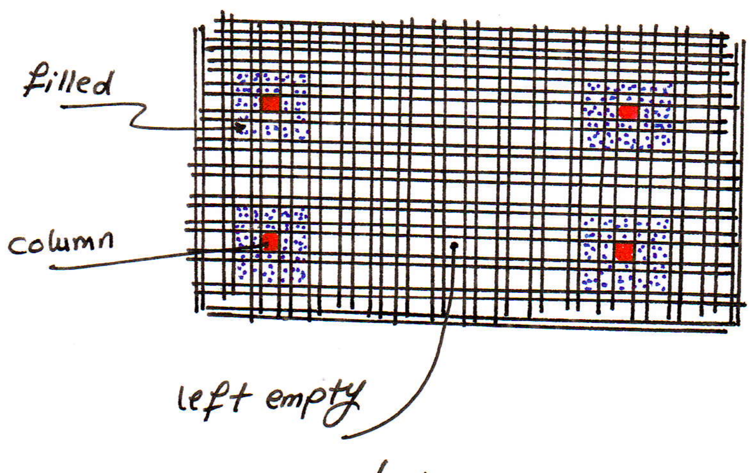
5. Two-way ribbed flat slab:-

This consists of intersecting ribs (in both directions). The spaces in the middle region of the slab is left empty but filled in regions around the columns (to behave as drop panels).

* This is used for very heavy load & when the space are large.

* عندما تكون الفضاءات واسعة والأحمال كبيرة يفضل إستخدام هذا النظام ؟

* الفكرة جعل ال (slab) ليس مصمتاً جداً، بل إضافة سمك محدد قرب الأعمدة فقط !!



(View From Bottom)

Design and Analysis of Flat Slabs

First: the total thickness h of the slab must be estimated. Next the flexural (or bending) reinforcements are calculated.

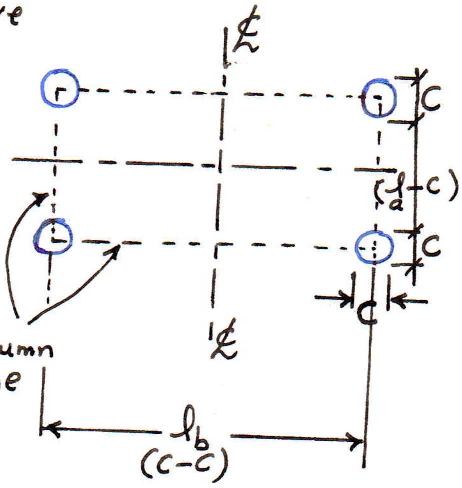
The total thickness h is estimated from the punching shear and deflection control. consult ACI Code, Table 9.5 (c)

f_y	with drop panels	without drop panels
276 MPa	$h = \frac{\ell}{40}^{*} \geq 100\,mm$	$h = \frac{\ell}{36} \geq 125\,mm$ [ℓ = c−c]
345 MPa	$h = \frac{\ell}{36} \geq 100\,mm$	$h = \frac{\ell}{33} \geq 125\,mm$
414 MPa	$h = \frac{\ell}{33} \geq 100\,mm$	$h = \frac{\ell}{30} \geq 125\,mm$

* here ℓ is the long Clear span

Positive & negative bending moments in flat slabs:

The positive bending moment M^+ occurs in the (center or centre) lines & the negative bending moment M^- in the column lines (in both directions).

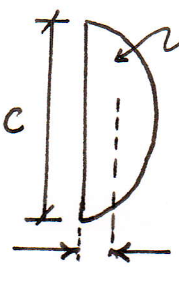


To Calculate these moments, the following notes are needed :-

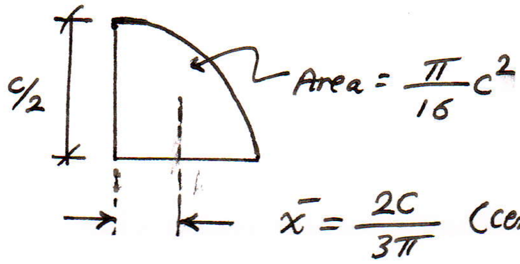
1. Area & centroid of $\frac{1}{4}$-circle & $\frac{1}{2}$-circle.

c = diameter of circle

* إيجاد المركز بواسطة التكامل (integration)

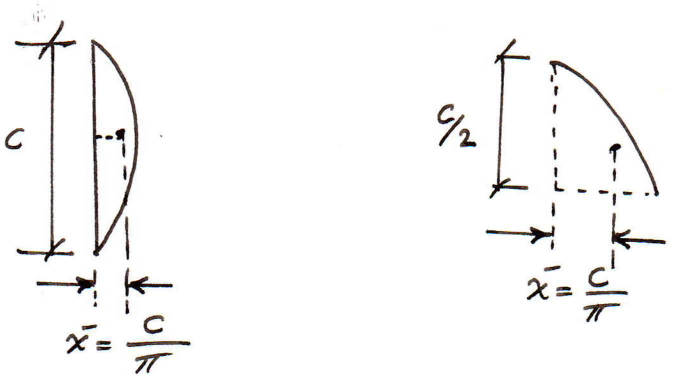


$\bar{x} = \frac{2c}{3\pi}$



$\bar{x} = \frac{2c}{3\pi}$ (centroid)

2. Centroid of $\frac{1}{4}$ - circular arc & $\frac{1}{2}$ - circular arc:



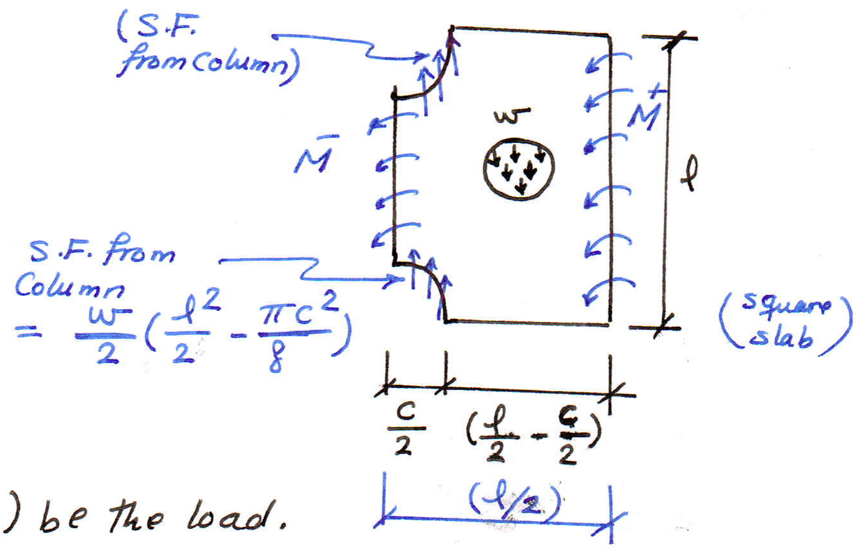
There are several methods to calculate the positive and negative bending moments in flat slabs. Here a simple but useful method is used (empirical method). نعتمد الطريقة التجريبية (بالاعتماد على التجربة والخبرة) بسيطة لكنها مفيدة))

دائماً S.F. = 0 في حالة التناظر

Consider an interior square slab. The column lines and center lines are lines of symmetry and so no shearing forces exist in these lines.

Take $\frac{1}{2}$ slab (with columns removed)

* here, (S.F.) as a reaction of column, where it is support the slab over 1/4-circle.



Let w (per unit area) be the load.

First find the centroid of this $\frac{1}{2}$ - slap (with columns removed).

واستخراج الـ (centroid) من المساحات واعتبار الفراغ مساحة سالبة ؟؟

$$\left(l \cdot \frac{l}{2}\right) \cdot \frac{l}{4} - \left(\frac{\pi}{8} c^2\right) \cdot \frac{2c}{3\pi} = \left(l \cdot \frac{l}{2} - \frac{\pi}{8} c^2\right) \cdot \bar{x}$$

This gives $$\bar{x} = \frac{3l^3 - 2c^3}{3(4l^2 - \pi c^2)}$$

$$\bar{x} = \frac{A_1 d_1 + A_2 d_2 \ldots}{\Sigma A}$$

Come back to the $\frac{1}{2}$-slab. Let

$$M_o = M^+ + M^-$$

Here M^+ is over the whole length of the center line (l) and M^- over the whole length of the column line (l). Find M_o by taking moments about the column line:

$$M^+ + M^- - w\left(\frac{l^2}{2} - \frac{\pi}{8}c^2\right) \cdot \frac{3l^3 - 2c^3}{3(4l^2 - \pi c^2)}$$

$$+ \underbrace{w\left(\frac{l^2}{2} - \frac{\pi}{8}c^2\right) \cdot \frac{c}{\pi}}_{} = 0$$

↑ رد الفعل يؤثر على خط ربع الدائرة ($\frac{1}{4}$ ~~...~~circle)
وهو قوة قص (S.F.)

Put $M_o = M^+ + M^-$ and simplify, then

$$M_o = \frac{w}{8}(4l^2 - \pi c^2)\left[\frac{3\pi l^3 - 12cl^2 + \pi c^3}{3\pi(4l^2 - \pi c^2)}\right]$$

$$= \frac{w}{24\pi}(3\pi l^3 - 12cl^2 + \pi c^3)$$

$$= \frac{Wl}{8}\left(1 - \frac{4c}{\pi l} + \frac{c^3}{3l^3}\right)$$

where $W = wl^2$ (total load)
(shear series)
reinforcement & Concrete.

The quantity in the brackets can be approximated as:

$$1 - \frac{4c}{\pi l} + \frac{c^3}{3l^3} \simeq \left(1 - \frac{2c}{3l}\right)^2 \quad \text{for } c < l$$

(دائماً: (c) أقل من l)

Then

$$M_o = 0.125\, Wl\left(1 - \frac{2c}{3l}\right)^2$$

المعادلة المشتقة بصورة نظرية

بعد الدراسة والخبرة والتجربة، تم إجراء تصحيحات على هذه المعادلة، لقيمة (M_o) ؟؟ :-
حيث تم تقليل المعامل (0.125) إلى ← (0.09) وكذلك ضربت المعادلة بعامل (F) ؟؟

Bending moment in flat slabs, $M_o = M^+ + M^-$

$$M_o = 0.125\ Wl\left(1 - \frac{2C}{3l}\right)^2$$

$W = wl^2$ (total load)

Modifications: This formula is found to be too conservative. This formula is modified for working stress method is:

$$M_o = 0.09\ Wl\left(1 - \frac{2C}{3l}\right)^2$$

A better formula (ACI 318-1963) is:

$$M_o = 0.09\ WlF\left(1 - \frac{2C}{3l}\right)^2$$

where; $F = 1.15 - \frac{C}{l} \geqslant 1$

For the ultimate strength design or analysis:

$$M_o = 0.10\ WlF\left(1 - \frac{2C}{3l}\right)^2$$

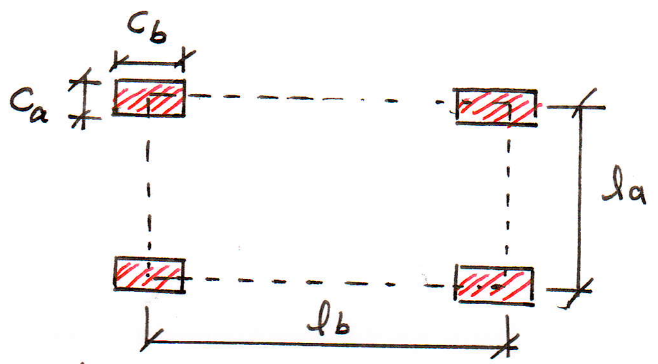
Here $W = w_u \cdot l^2$ (total ultimate load).

and $F = 1.15 - \frac{C}{l} \geqslant 1.$

* The above formulas are for round columns and square slabs.

For rectangular slabs with rectangular columns:-

Here l_a & l_b are the short and long spans (center to center of columns) and C_a and C_b are the short and long dimensions of the columns.



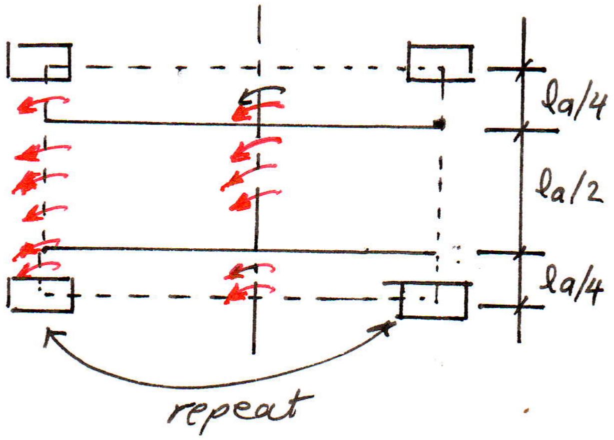
For the long span:-

$$M_o = 0.10\ Wl_b F\left(1 - \frac{2C_b}{3l_b}\right)^2$$

$$F = 1.15 - \frac{C_b}{l_b} \geqslant 1$$

and $W = w_u \cdot l_a l_b$ (total ultimate load).

This M_o is divided into positive & negative bending moments in the central strip (length $l_a/2$) & two column strips (total length $l_a/2$).



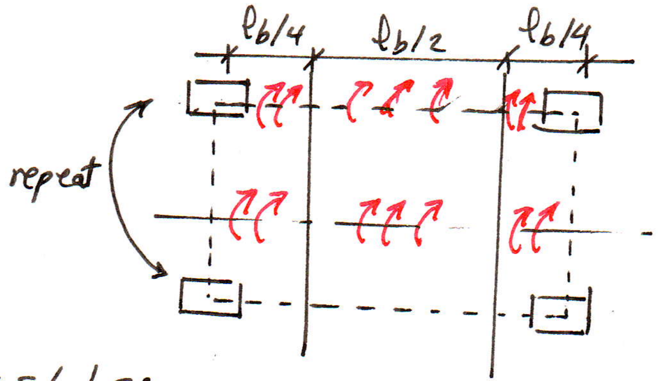
For the Short Span:

$$M_o = 0.10\, W l_a F \left(1 - \frac{2C_a}{3l_a}\right)^2$$

Here $\quad F = 1.15 - \dfrac{C_a}{l_a} \geq 1 \qquad W = w_u \cdot l_a l_b$

This M_o is divided into M^+ & M^- in the central strip (length $l_b/2$) & column strips (total length $l_b/2$).

توزيع مطلق M_o For the ~~approximate~~ apportionment of M_o see tables.



Transverse shear in flat slabs:-

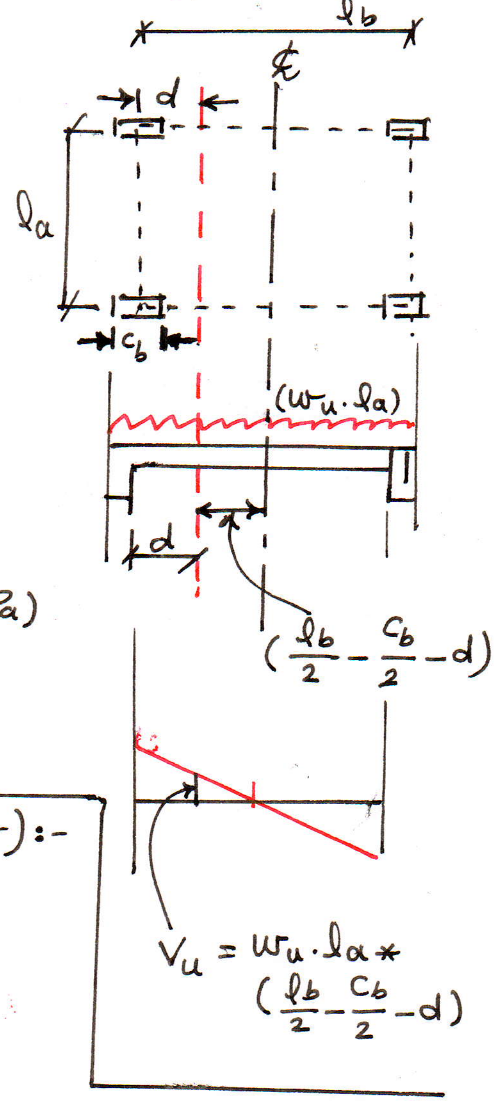
Two types of shear must be checked in flat slabs:-

1. Beam shear (or one-way shear):-

The slab is assumed as one wide beam between column lines. Always take the long span for checking (more critical):-

The ultimate shearing force V_u on the critical section (at d from face of column), is:

$$V_u = (w_u \cdot l_a) \cdot \left(\frac{l_b}{2} - \frac{c_b}{2} - d \right)$$



The critical section has area $(d \cdot l_a)$.

Thus the ultimate shearing stress on the section is:-

$$v_u = \frac{V_u}{d \cdot l_a}$$

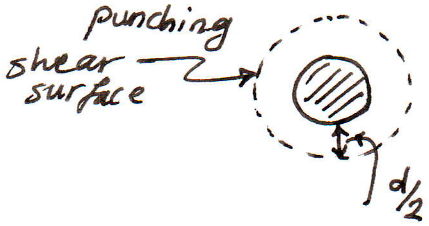
The shear strength of concrete v_c (for this type of transverse shear) is:

[عندما تحمل الكونكريت أي من القوة المسلطة عليه ؟] $v_c = \frac{\phi}{6} \sqrt{f'_c}$ (MPa)

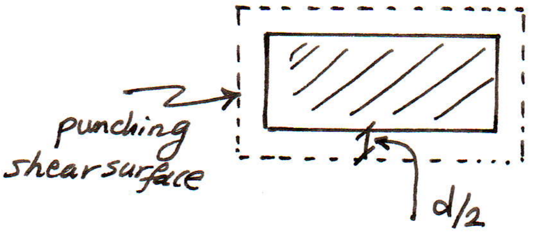
Check $v_c > v_u$

2. Punching shear (or two-way shear):-

This occurs round columns at $\frac{d}{2}$ from faces of columns.



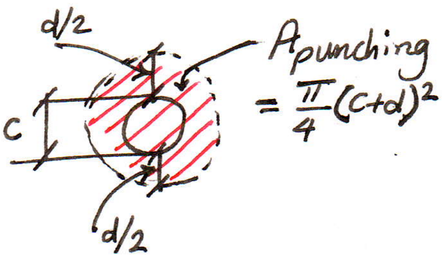
(round column)



(rectangular column)

The total punching shearing force (at ultimate load) is:

$$\boxed{V_u = w_u \cdot [l_a l_b - A_{punch}]}$$



This punching shearing force acts on a punching surface $(\Sigma_o \cdot d)$ where Σ_o is the perimeter. Thus the ultimate punching shearing stress on the punching surface is:

$$v_u = \frac{V_u}{\Sigma_o \cdot d}$$

The punching shear strength of concrete is :

$$v_c = \frac{\phi}{3}\sqrt{f'_c} \quad (MPa)$$

Check ! $v_c > v_u$

* ملاحظة : من أهم الأمور الواجب مراعاتها في التصميم هو تحديد وزن المنشأ ؟ حيث يحدد بسمك ؟
حيث يزداد السمك ليفي بالغرض في بعض الاحيان ؟ ويجري تدقيق الانبعاج والمقاومة ... إلخ
(check punching) ، أو نضع (drop panel) لتحقيق السمك المطلوب ؟ وهكذا ؟
ومن ثم يستنتج (M_o) ويوزع ⇐ ويحسب ($M_u/\phi f'_c b d^2$) ، هنا في (long dir.) نضع الحديد
في الاسفل لأنه هو الاتجاه الحرج وليس الاتجاه القصير ؟؟ أما في الاتجاه القصير يجب
طرح حالة التعارض في حالتي (+ & −) عند حساب قيمة (d) ؟ (effective depth)
في التحليل : - نشغل العزم في توزيعه في الوسطى والأطراف ؟ ... إلخ ، ونتأكد منه عشرة مرات ؟؟
للدقة ؟

Example (Design of flat slab) :-

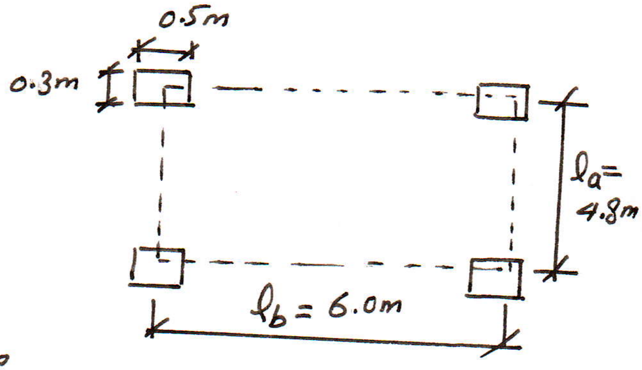
Example (Design of flat slab)

1. An interior flat slab has spans 6.0 m & 4.8m (centers to centers of columns). The columns are rectangular 500mm * 300 mm. The imposed live load is 5.0 kN/m^2
The surfacing load is 0.75 kN/m^2

For Concrete : $f_c' = 25$ MPa, $\gamma_c = 24.5$ kN/m^3

why $\phi 12$? in long dir.?

For steel : $f_y = 375$ MPa. Use ϕ12mm in the long direction & ϕ10mm in the short direction. Use clear cover 20mm?



Solution:-

Try to design the slab without drop panels. Estimate the total thickness (& the selfweight).

Here Use;

$$h = \frac{l}{32} \quad (\text{for } f_y = 375 \text{ MPa})$$

* see; choose $l/32$, $f_y = 375 > 276$, < 414

$$h = \frac{6000}{32} = 187.5 \text{ mm}$$

Try to use $h = 185$ mm (nearest to 5mm)

First check the beam & punching shear.

live load; $w_l = 5.0$ kN/m^2 (given)

self weight $= 0.185 * 24.5 = 4.5325$ kN/m^2

dead load; $w_d = 4.5325 + 0.75 = 5.2825$ kN/m^2

Ultimate load:

$$w_u = 1.4 * 5.2825 + 1.7 * 5.0$$
$$= 15.8955 \text{ kN/m}^2$$

For beam shear (in the long direction):

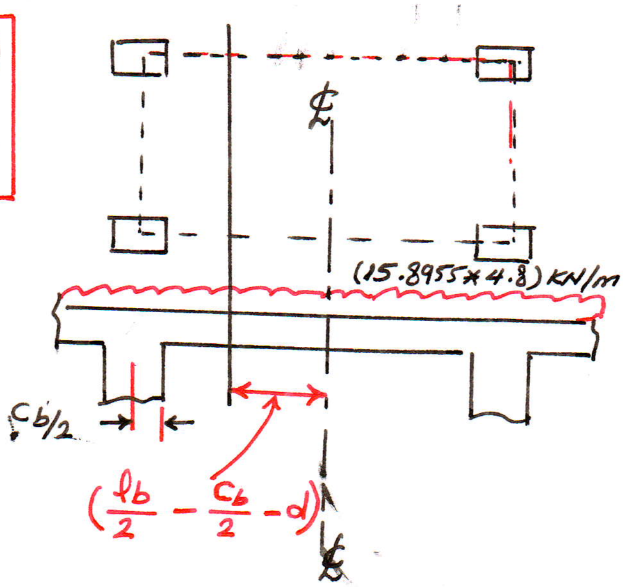
نأخذ الاتجاه الطويل

The ultimate shearing force (at the critical section) is:

$$V_u = (15.8955 * 4.8) * \left(\frac{6.0}{2} - \frac{0.5}{2} - 0.159\right)$$

This comes from;

$$\boxed{V_u = \underbrace{(w_u \cdot l_a)}_{\text{load/length}} \cdot \underbrace{\left(\frac{l_b}{2} - \frac{c_b}{2} - d\right)}_{\text{distance from ℄}}}$$



$$\left(d = 185 - 20 - \frac{12}{2} = 159\,mm\right)$$

Thus $V_u = 197.689\ kN$

The ultimate beam shear stress is:

* يقارن مع تحمل الكونكريت

$$v_u = \frac{V_u}{l_a\,d} = \frac{\text{shear force}}{\text{sec. Area.}}$$

$$= \frac{197.689}{4.8 * 0.159} = 259.026\ kN/m^2$$

$$= 0.259 * 10^6\ N/m^2 = 0.259\ MPa$$

The beam shear strength of concrete is:-

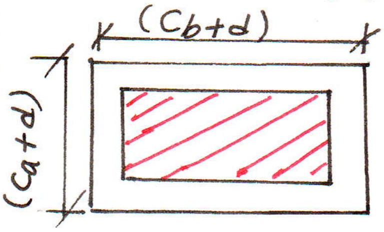
$$v_c = \frac{\phi}{6}\sqrt{f'_c}$$

$$= \frac{0.85}{6}\sqrt{25} = 0.708\ MPa$$

Thus $v_c > v_u$ (o.K.)

For the punching shear:

The punching shearing force (at ultimate load):-



$$V_u = w_u \cdot [l_a l_b - (c_a + d)(c_b + d)]$$

$$= 15.8955 * [6.0 * 4.8 - (0.459 * 0.659)]$$

$$= 452.982\ kN.$$

The ultimate punching shear stress:

القوة

$$v_u = \frac{V_u}{2\,[(c_a+d)+(c_b+d)] \cdot d}$$

مساحة المحيط المخترقة ؟!

$$v_u = \frac{452.982}{2[0.459+0.659]*0.159} = 1274.124\ kN/m^2$$
$$= 1.274\ MPa$$

The punching shearing strength of concrete:

$$v_c = \frac{\phi}{3}\sqrt{f'_c}$$
$$= \frac{0.85}{3}\sqrt{25}$$
$$= 1.416 MPa$$

Thus $v_c > v_u$ (o.k.)

No drop panels are needed.

Next find the bending moments in the slab. start with the long span ($l_b = 6.0m$ & $C_b = 0.5m$)

المعادلة التجريبية ؟

Use the empirical formula : $M_o = 0.1 W_u l_b F (1 - \frac{2C_b}{3l_b})^2$

Here :

$$W_u = w_u \cdot l_a \cdot l_b$$
$$= 15.8955 * 4.8 * 6.0$$
$$= 457.790\ kN$$

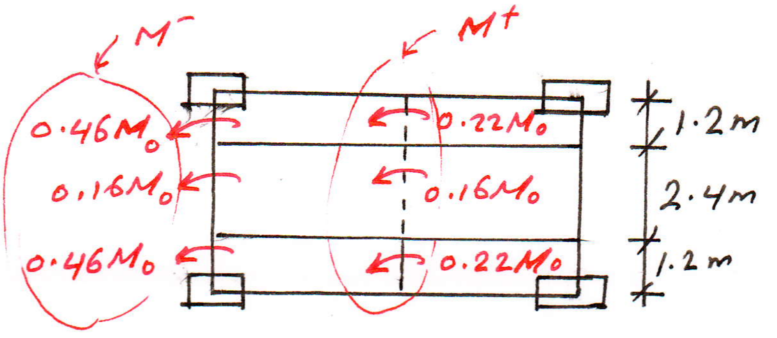
$$F = 1.15 - \frac{C_b}{l_b} = 1.15 - \frac{0.5}{6.0} = 1.06 > 1$$

Thus,

$$M_o = 0.1 * 457.790 * 6.0 * 1.06 * (1 - \frac{2*0.5}{3*6.0})^2$$
$$= 259.702\ kN.m$$

Divide this moment into M^+ & M^- in the column & middle strips:-

see Tables ?



Take the negative B.M. in the Column Strip:

$0.46 M_o = 0.46 * 259.702$
$= 119.463 \text{ KN.m} / (2.4\text{m width})$

Use $R = \dfrac{Mu}{\phi f_c' b d^2} = \dfrac{119.463 * 10^6}{0.9 * 25 * 2400 * 159^2} = 0.0875$

use tables:
$$\omega = \begin{cases} 0.092 & \text{for } R = 0.0870 \\ 0.093 & \text{for } R = 0.0879 \end{cases}$$

use $\omega = 0.0925$ for $R = 0.0875$

steel ratio: $\rho = \omega f_c' / f_y = \dfrac{0.0925 * 25}{375} = 0.006167$

$As = \rho b d$
$= 0.006167 * 2400 * 159 = 2353 \text{ mm}^2$

No. of bars ($\phi 12$mm) is , $n = \dfrac{2353}{113} = 20.8$

use $21 \phi 12$ mm; spacing ; $S = \dfrac{2400}{21} = 114.3$ mm

use (115 mm) spacing.

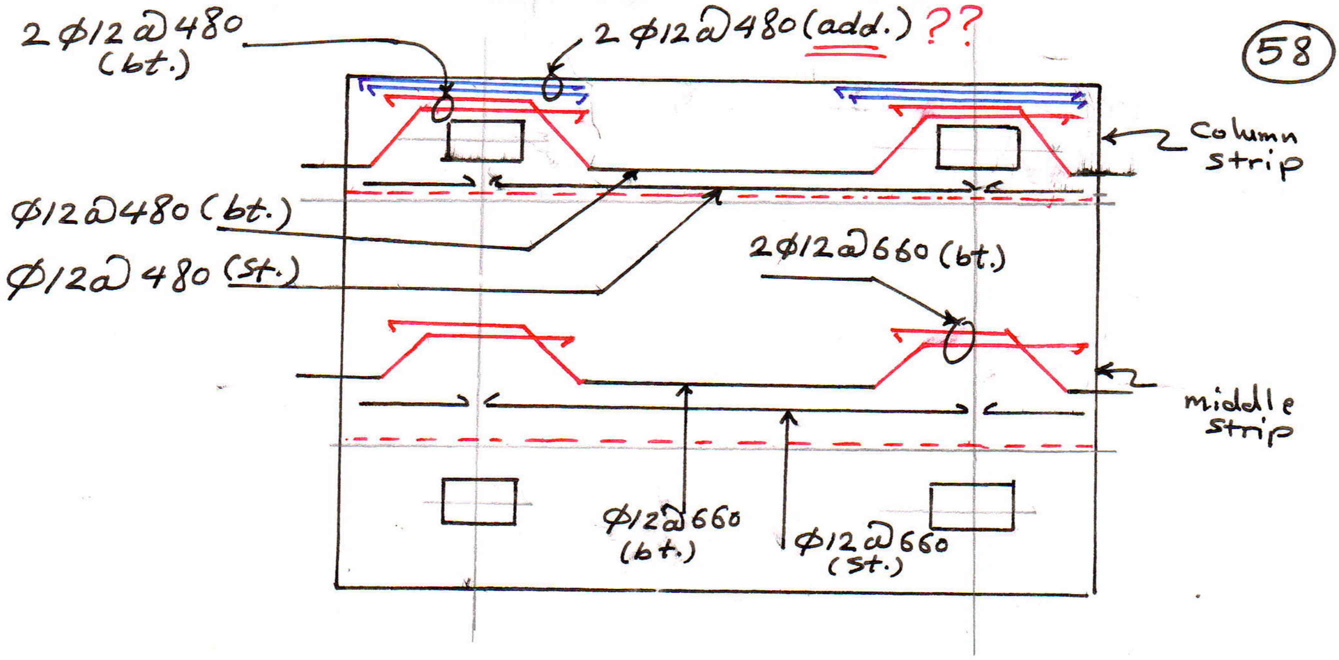
i) For the negative B.M. at middle strip, use
$S = 115 * \dfrac{0.46}{0.16} = 330$ mm

For the positive B.M. in the Column strip:

$S = 115 * \dfrac{0.46}{0.22}$
$= 240$ mm

ii) For the positive B.M. in the middle strip:

$S = 115 * \dfrac{0.46}{0.16}$
$= 330$ mm



Take the short direction ($l_a = 4.8m$ & $C_a = 0.3m$);

Find (M_o) :

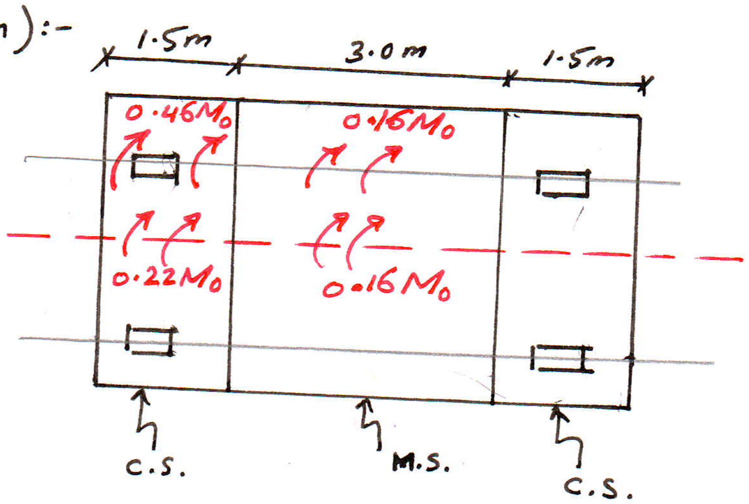
$$M_o = 0.1\, W_u \cdot l_a \cdot F \cdot \left(1 - \frac{2C_a}{3\,l_a}\right)^2 ;$$

$$F = 1.15 - \frac{C_a}{l_a} = 1.15 - \frac{0.3}{4.8} = 1.087 > 1$$

$$= 0.1 * 457.790 * 4.8 * 1.087 * \left(1 - \frac{2*0.3}{3*4.8}\right)^2$$

$$= 219.366 \text{ kN.m}$$

(For the column strip = 3.0 m):-



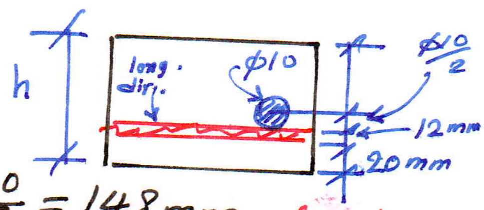
$$\bar{M} = 0.46\, M_o$$
$$= 0.46 * 219.366$$
$$= 100.9 \text{ kN.m}$$
(for 3.0m width)

Use $R = \dfrac{M_u}{\phi f_c' \, b d^2}$

$$= \frac{100.9 * 10^6}{0.9 * 25 * 3000 * 148^2}$$

$$= 0.06824$$

where here d is:



$$185 - 20 - 12 - \frac{10}{2} = 148\text{mm}$$

for short dir. ??

From the table: $\omega = \begin{cases} 0.071 & \text{for } R = 0.0680 \\ 0.072 & \text{for } R = 0.0689 \end{cases}$

use $\omega = 0.0712$ for $R = 0.0682$

Then $\rho = \omega f'_c / f_y = 0.0712 * \frac{25}{375} = 0.004747$

The steel A_s will be:

$$A_s = \rho b d$$
$$= 0.004747 * 3000 * 148$$
$$= 2108 \text{ mm}^2$$

No. of $\phi 10$ mm bars $= \frac{2108}{79}$

$= 26.6 = 27$ bars, (for 3m width)

If spacing is needed:

$$S = \frac{3000}{27} = 111 \text{ mm} \simeq 110 \text{ mm}$$

For M^+ (in the Column strip / width 3.0m)

$M^+ = 0.22 * 219.366$

$= 48.260$ kN.m (for 3m width)

Use proportions:

No. of $\phi 10^{mm}$ bars $= \frac{27}{0.46 M_o} * 0.22 M_o$

$= 13$ bars (for 3m width)

or $S = \frac{111}{0.22} * 0.46 = 230$ mm

For the middle strip (width 3.0m):

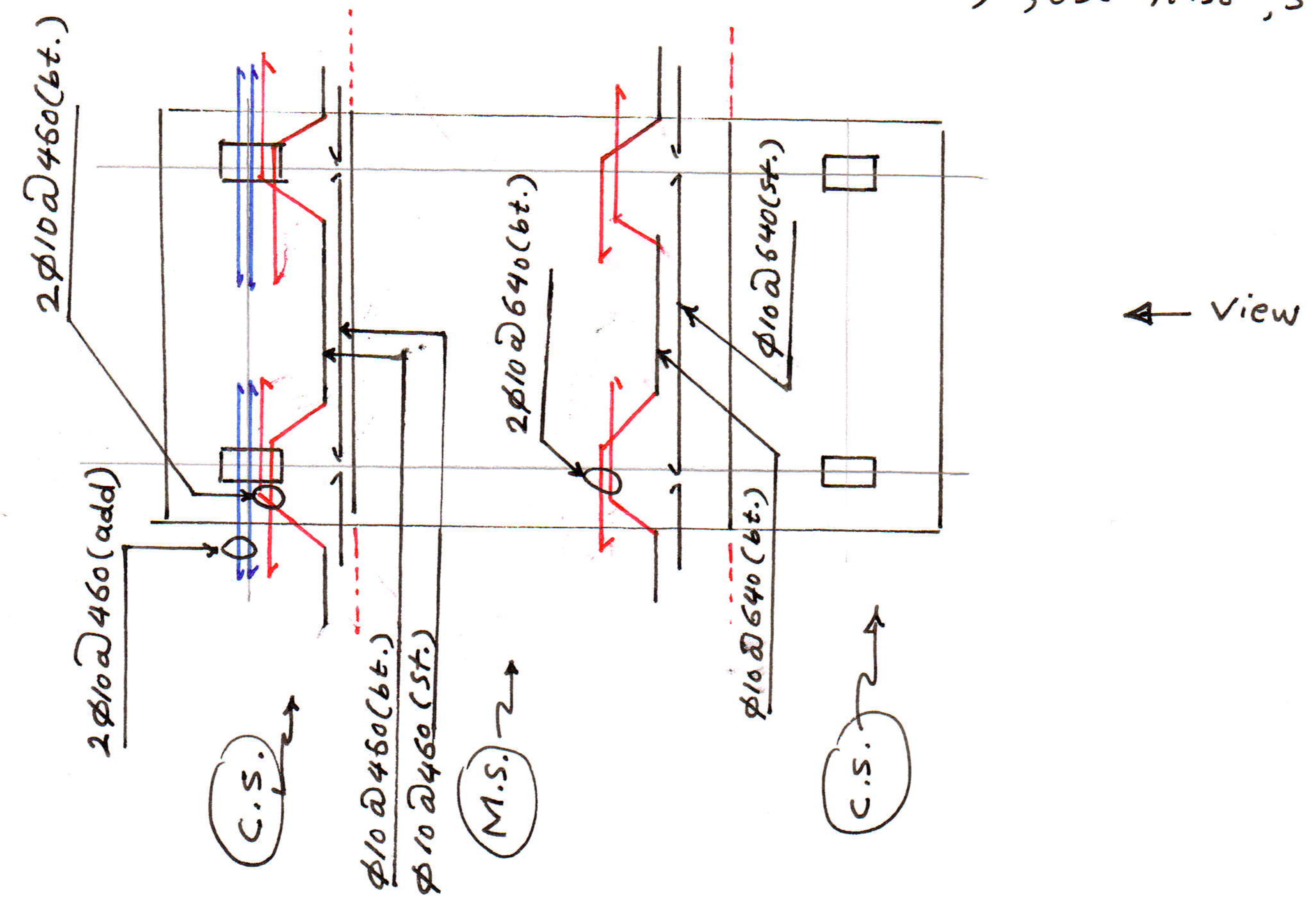
$M^- = 0.16 M_o = 0.16 * 219.366 = 35.09$ kN.m (for 3m width)

Use proportions: $S = \frac{111}{0.16} * 0.46 = 319 \text{ mm} \simeq 320 \text{mm}$

Also

$M^{+} = 0.16\, M_o$

$= 35.09\ kN.m$ (for 3m width), Use Also, $S = 320^{mm}$



2. (Flat slab with drop panels)

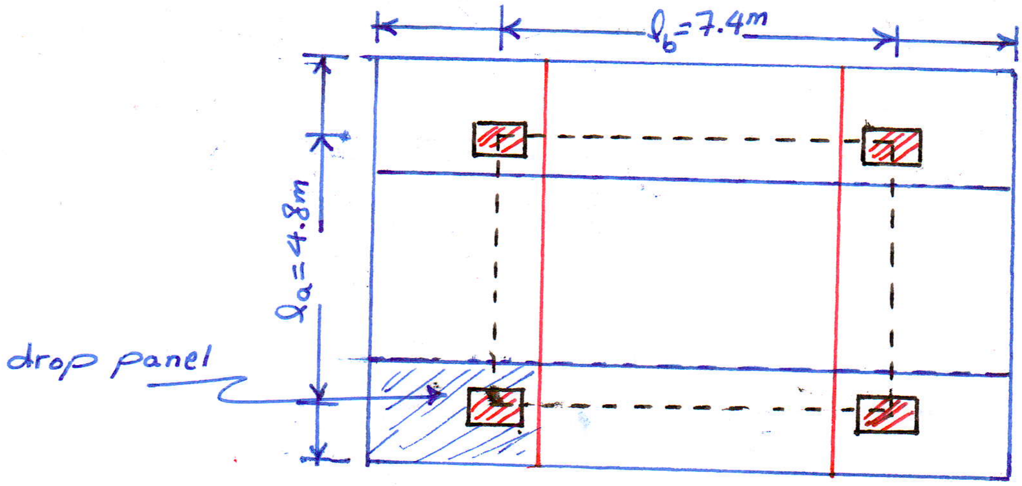
An interior flat slab has spans 7.4m & 4.8m (Centers to centers of columns). The columns are rectangular (0.6m * 0.4m). The slab thickness is 150mm & strengthened by drop panels to 200mm. The drop panels are 3.7m * 2.4m (occuping the column strips).

For Concrete : $f'_c = 27.5$ MPa.

$\gamma_c = 24.5\ kN/m^3$

For Steel $f_y = 350$ MPa

Use $\phi 12$mm for reinforcement & use a clear cover of 20mm.



i) Check for punching in the drop panels.

ii) Calculate the required steel for the positive & negative moments in the column strip in the long direction.

The imposed live load is $8\ kN/m^2$ & the surfacing load $1.0\ kN/m^2$.

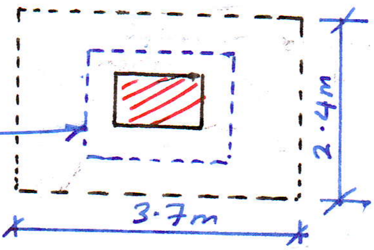
Solution:-

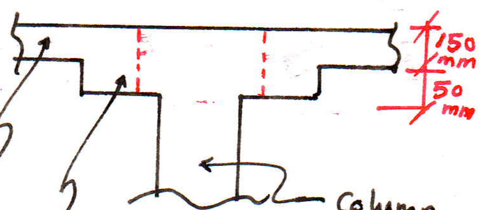
First calculate the selfweight. The total selfweight is:

$$24.5 * [(7.4 * 4.8 * 0.150) + (3.7 * 2.4 * 0.050)]$$
$$= 141.414 \text{ KN (total load)}.$$

Thus ; $\frac{141.414}{7.4 * 4.8} = 3.981 \text{ KN/m}^2$

The self weight per unit area.





Note:-

If the weight of the drop panel is neglected, then the selfweight per unit area becomes:

$$24.5 * 0.150 = 3.675 \text{ KN/m}^2$$

(This is not in big error with 3.981 KN/m^2)

في حالة إهمال السمك القليل لل (panel) في بعض
الحالات فإن الخطأ ليس كبيراً ؟

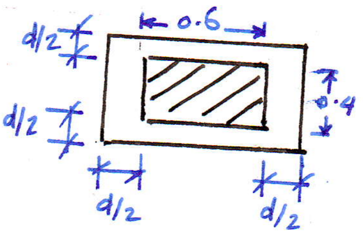
The ultimate load becomes:

$$w_u = 1.4(3.981 + 1.0) + 1.7 * 8.0$$
$$= 20.573 \text{ KN/m}^2$$

Calculate the punching shearing force. Find the effective depth (in the drop panel).

$$d = 200 - 20 - \frac{12}{2} = 174 \text{ mm}$$

The punching shearing force ;



$$V_u = 20.573 * [(7.4 * 4.8) - \underbrace{(0.6 + 0.174) * (0.4 + 0.174)}_{\text{punching area}}]$$

$$V_u = 721.6129 \ kN$$

The ultimate punching shearing stress :

$$v_u = \frac{721.612}{2[(0.6+0.174)+(0.4+0.174)] * 0.174} = 1538.277 \ kN/m^2$$

$$= 1.538 \ MPa$$

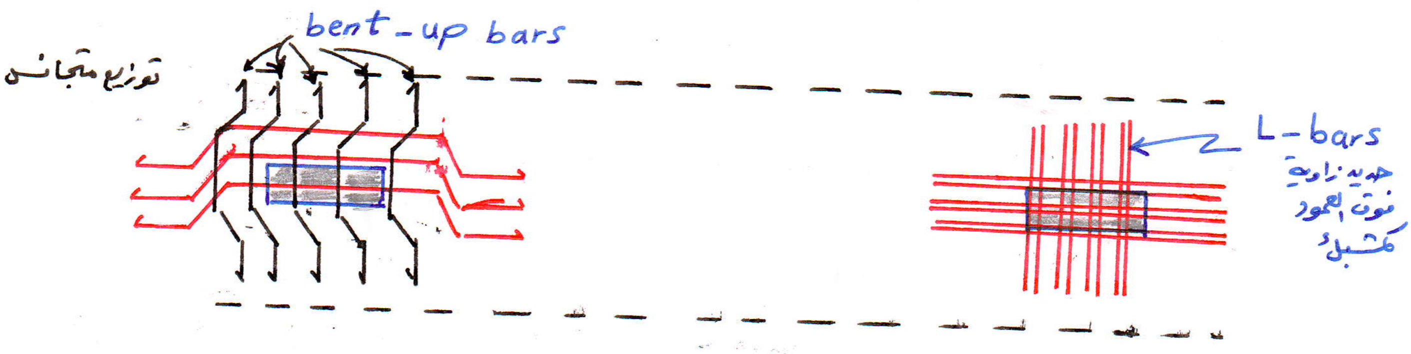
The punching shearing strength of concrete is :

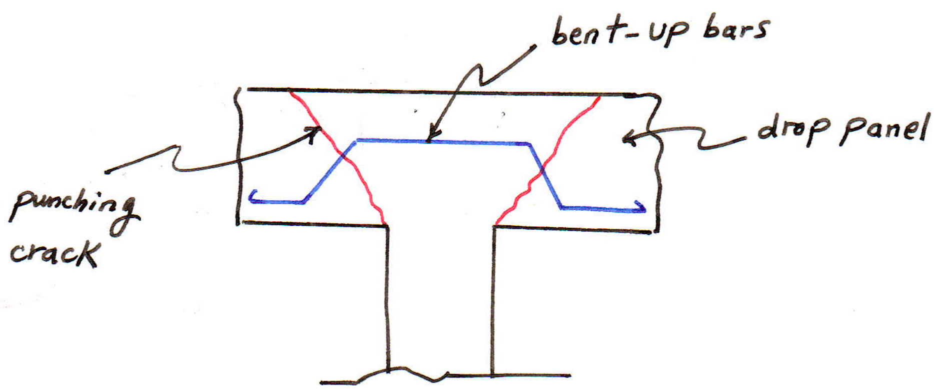
$$v_c = \frac{\phi}{3}\sqrt{f_c'} = \frac{0.85}{3}\sqrt{27.5} = 1.486 \ MPa .$$

Note:-
here punching is occurs??
يحصل الانبعاج ؟
تشكل تقعر في
الكونكريت ؟

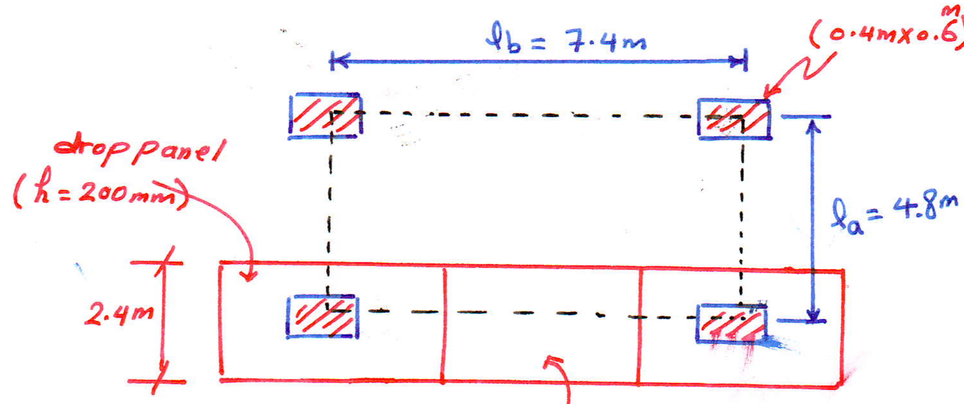
since $v_c < v_u$, then shear reinforcement is needed.

في مثل هذه الحالة نستخدم (L-bars) كتسليح فوق الأعمدة ، وكما مبين ، لتجنب فشل القص؟؟





في حالة حساب M^- نأخذ (سمك السقف + الزيادة d.P.) (slab + drop panel) ؟؟ بينما في
حسابات M^+ نأخذ سمك السقف فقط (slab) ؟



total moment

total load

$$\boxed{M_o = 0.1\, W_u \cdot l_b \cdot F \cdot \left(1 - \frac{2 C_b}{3 l_b}\right)^2}$$

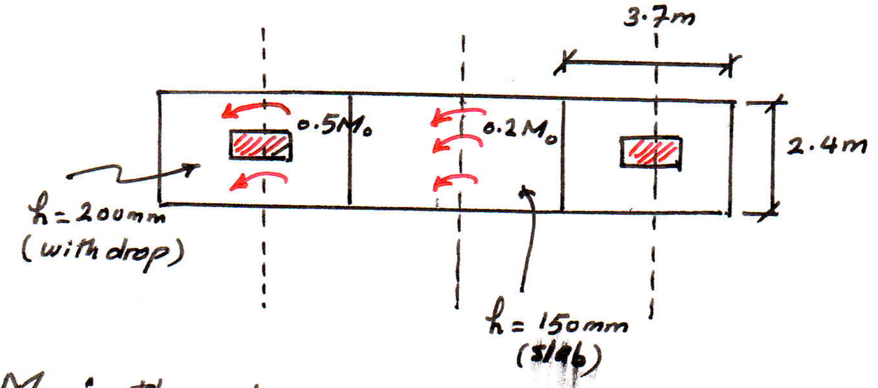
Here $W_u = 20.5734 * 4.8 * 7.4$
$= 730.767$ KN

$$F = 1.15 - \frac{C_b}{l_b} = 1.15 - \frac{0.6}{7.4} = 1.068 > 1.0$$

$$M_o = 0.1 * 730.767 * 7.4 * 1.068 * \left(1 - \frac{2 * 0.6}{3 * 7.4}\right)^2$$
$$= 516.790 \text{ KN.m}$$

Apportion This moment :

لاحظ الجدول (توزيع العزم) ؟



The negative B.M. in the column strip is :

$$M^- = 0.50 M_o = 0.50 * 516.790$$
$$= 258.395 \text{ KN.m}/(2.4m \text{ width})$$

The effective depth is :

$$d = \left(200 - 20 - \frac{12}{2}\right) = 174 \text{mm}$$

Use $R = \dfrac{M_u}{\phi f'_c\, b d^2} = \dfrac{258.395 * 10^6}{0.9 * 27.5 * 2400 * (174)^2}$

From Table :

$$\omega = \begin{cases} 0.158 & \text{when } R = 0.1433 \\ 0.159 & \text{when } R = 0.1441 \end{cases}$$

use, $\omega = 0.1585$ for $R = 0.14368$

The steel ratio ρ is,

$$\rho = \omega f'_c / f_y$$
$$= 0.1585 * 27.5/350$$
$$= 0.012453$$

النسبة عالية؟ اهنا؟
why?

Thus $A_s = \rho b d$

$$= 0.01245 * 2400 * 174$$
$$= 5200\ mm^2/(2.4m\ width)$$

No. of ϕ 12mm bars $= \dfrac{5200}{113} = 46$ bars

spacing $S = \dfrac{2400}{46} = 52mm$

Use $S = 50mm$

Note:- It is better to Use ϕ 16mm in this case, The spacing will be about 100 – 90 mm.

يتيسر المجال بين القضبان وكذلك يوزع التسليح بشكل منتظم أكثر ؟؟

For M^+:

$$M^+ = 0.20\, M_o$$
$$= 0.20 * 516.790$$
$$= 103.358\ kN.m/(2.4m\ width)$$

The effective depth d here is

* slab thick. only

$$d = 150 - 20 - \frac{12}{2} = 124mm$$

Find ; $R = \dfrac{103.358 * 10^6}{0.9 * 27.5 * 2400 * (124)^2} = 0.113165$

very large value ??

From the tables :

$$\omega = \begin{cases} 0.121 & \text{for } R = 0.1124 \\ 0.122 & \text{for } R = 0.1133 \end{cases}$$

use $\omega = 0.1218$

The steel ratio $\rho = 0.1218 * 27.5 / 350$

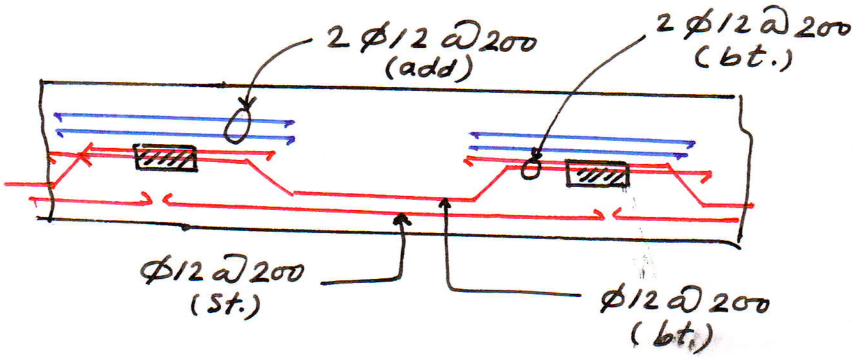
$= 0.009570$

Steel area ;

$As = 0.009570 * 2400 * 124 = 2848 \text{ mm}^2$

No. of $\phi 12$mm bars $= \dfrac{2848}{113} = 25$ bars

spacing $S = \dfrac{2400}{25} = 95.2 \text{ mm} \simeq 100 \text{ mm}$



3. (Analysis of flat slabs)

An interior flat slab has spans 5.4m & 6.6m & is supported on circular columns 0.4m in diameter. The slab is without drop panels & has total thickness 150mm. The slab is reinforced by $\phi 12$mm @ 125mm in the positive bending moment of the column strip in the long span & by $\phi 12$mm @ 62.5mm in the negative bending moment in the same column strip. The surfacing load is 0.75 kN/m², for Concrete: $f'_c = 25$ MPa, $\gamma_c = 24.5$ kN/m³, For steel $f_y = 310$ MPa, clear cover is 20mm. Find the live load capacity of this slab.

Hint:- Find w_u for 4 conditions:

1. beam shear 2. punching shear 3. M^+ in column strip (C.S.) 4. M^- in (C.S.).

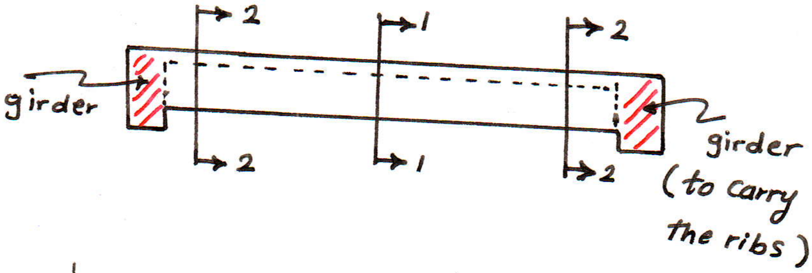
* (live load = w_u − d.L.)

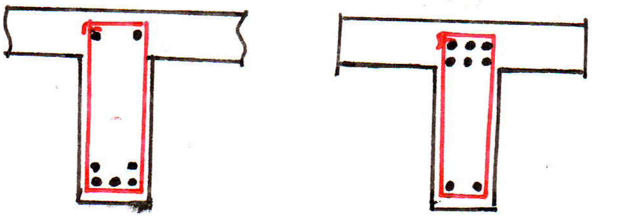
2. The web. This is usually rectangular or trapezoidal.

The web must be designed to take the shearing forces. No shear reinforcement is needed. In Iraq, minimum ϕ 8mm or ϕ 10mm stirrups at $d/2$ near the ends & at d in the middle are used.

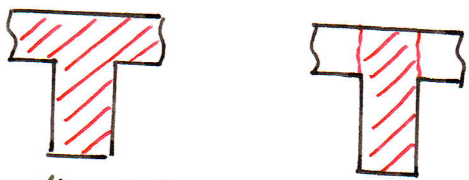
Thus, $$\frac{V_u}{bd} \leqslant v_c \qquad \text{where } v_c = \frac{\phi}{6}\sqrt{f'_c}$$

The flexural (or bending) steel in the web is calculated at two positions.





steel for M^+ | steel for M^-



section 1-1 (T-section under positive B.M.) | section 2-2 (rectangular web under negative B.M.)

b_{web} = 200 - 250 mm

b_f = 800 - 400 mm

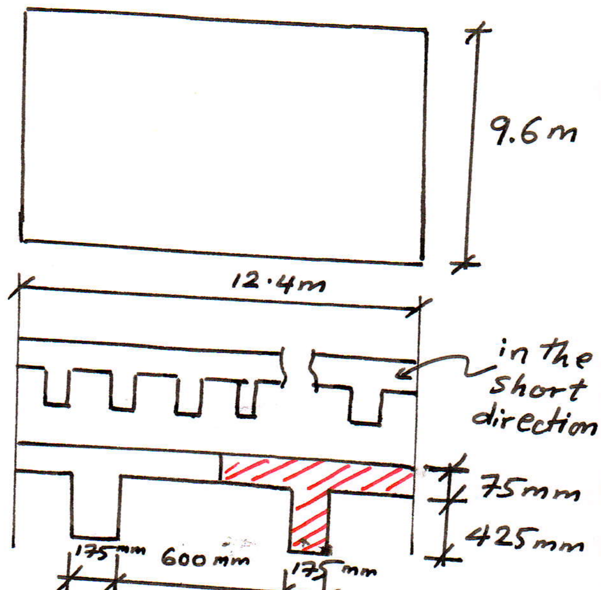
Example (design of 1-way ribbed slab):

A rectangular area with clear spans (9.6m * 12.4m) is to be covered by a 1-way ribbed slab. The ribs are given in the figure. The imposed live load is 5 KN/m² & the surfacing load 1.0 KN/m². For concrete; $f'_c = 27.5\ MPa$

$\gamma_c = 24.5\ KN/m^3$

For steel; $f_y = 310\ MPa$

Use ϕ16mm for flexural steel in the ribs & ϕ8mm in the flange. Check the shear in the ribs & calculate the flexural steel? Cover = 40mm ?



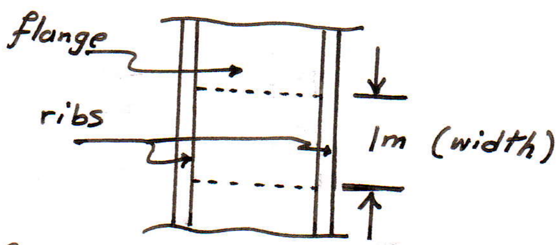
Solution:-

First calculate the ultimate load.

* لاحظ عرضها 600mm ، وطولها 9600mm ؟

Take the flange (1-way slab)

$$w_u = 1.4(0.075 * 1 * 24.5 + 1.0) + 1.7(5.0 * 1.0)$$
$$= 12.4725\ KN/m/m\ (width)$$



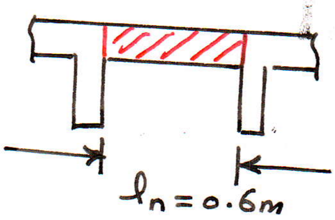
For simplicity take;

الفرق قليل عن الواقع (ليؤخذ المعامل ١٠)

$$M^+ = M^- = \frac{w_u \cdot l_n^2}{10}$$

Thus

$$M^+ = M^- = \frac{12.4725 * 0.6^2}{10}$$
$$= 0.449\ KN.m/m\ (width)$$



Use effective depth, $d = \frac{75}{2}$

* الحديد في المنتصف

$= 37.5\ mm$

Use $R = \dfrac{M_u}{\phi f'_c\, b d^2}$

$$= \frac{0.449 * 10^6}{0.9 * 27.5 * 1000 * 37.5^2} = 0.0129$$

This gives,

$$\omega = 0.013$$

The steel ratio $\rho = \omega f'_c / f_y$

$$= 0.013 * 27.5 / 310$$

$$= 0.001153 < \rho_{min}$$

So use, $\rho_{min} = 0.002$ (for distribution steel)?

, but check? ∴ دائماً نستخدم (ρ_{min}) لأن الناتج قليل لكن لتدقيق الحل ؟

Thus $A_s = 0.002 * 37.5 * 1000$

$$= 75\ mm^2/m \text{ (width)}$$

Use ϕ 8mm with area $50.2 mm^2$. The number of bars;

$$\frac{75}{50} = 1.5 \text{ bars}/m$$

spacing $s = \dfrac{1000}{1.5} = 666.6\ mm$

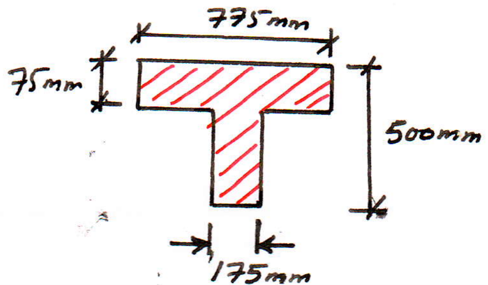
Use $s = 600$ mm (in both directions).

Next take the rib with the flange as T-section :-

Calculate the ultimate load:-

$$w_u = 1.4\left[(0.175 * 0.425 + 0.075 * 0.775) * 24.5 + 0.775 * 1.0\right] + 1.7 * (0.775 * 5)$$

$$= 12.217\ KN/m$$



Then checking shear

$$V_u = w_u \left[\frac{\ell_n}{2} - d \right]$$

$$d = (425 + 75) - 40$$
$$= 460\,mm$$

$$V_u = 12.217 \left[\frac{9.6}{2} - 0.46 \right]$$
$$= 53.02178\ kN$$

$$v_u = \frac{53.02178}{0.175 * 0.46} = 658.655\ kN/m^2$$

b_w d

$$= 0.6586\ MPa$$

$$v_c = \frac{\phi}{6}\sqrt{f_c'}$$
$$= \frac{0.85}{6}\sqrt{27.5}$$
$$= 0.7429\ MPa$$

$v_c > v_u$ o.K.

Then, design of ribs :-

assume the section is T- sec,

$a < h_f$ (rectangular)

$$M = \frac{W_u \ell_n^2}{10} = \frac{12.217 * (9.6)^2}{10} = 112.6\ kN.m / m\ (width)$$

$$\text{Then find, } R = \frac{112.6 * 10^6}{0.9 * 27.5 * \cancel{1000} * (460)^2} = 0.02149$$

775 0.027742

$\rightarrow \omega = 0.0222$

& so , $\rho = \omega f_c' / f_y$ check $\omega = ?$

$= 0.0222 * 27.5 / 310$ & Then ρ

$= 0.0019693$

$$\rho_{min} = \frac{1.4}{f_y} * \frac{b_w}{b_f} \qquad \text{for T-sec.}$$

$$\rho_{min} = \frac{1.4}{310} * \frac{175}{775} = 0.00101977$$

, but $\rho > \rho_{min}$, use $\rho = 0.00101977$

Then, $A_s = \rho b d$

~~= 0.0010019~~

$= 0.001969 * 775 * 460$

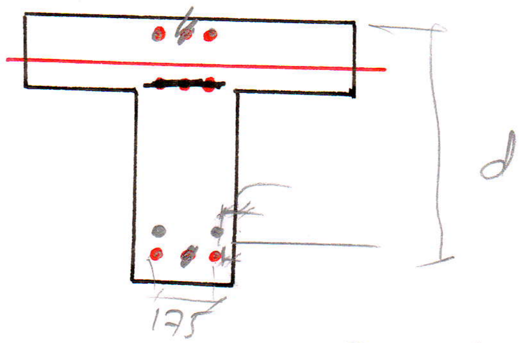
$= 701.948 \simeq 702 \ mm^2/m$ (width) 0.775 m

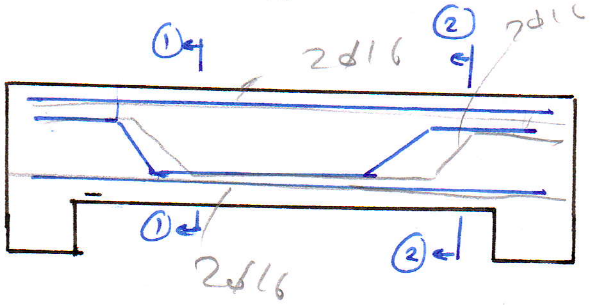
Use ϕ16mm with area 201 mm^2.

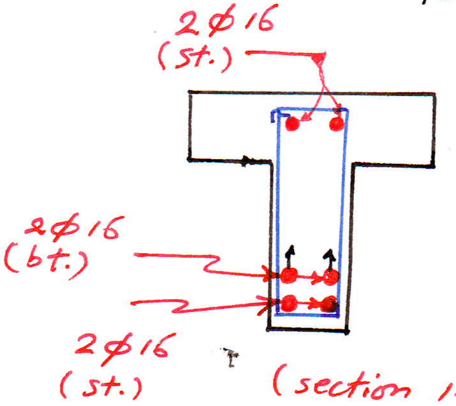
The number of bars

$$\frac{702}{201} = 3.5 \text{ bars/m}$$

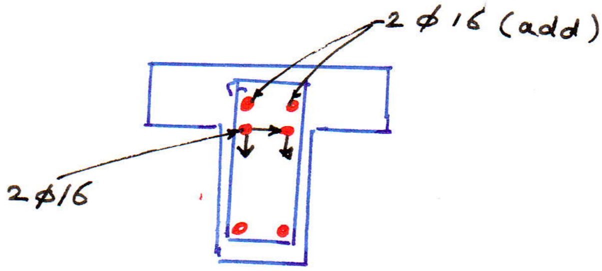
Then use $4\phi16$ for M^- & M^+.







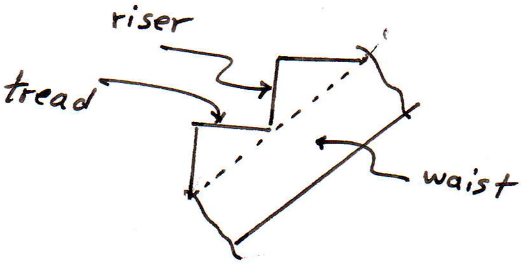
(section 1-1)
at mid span
M^+

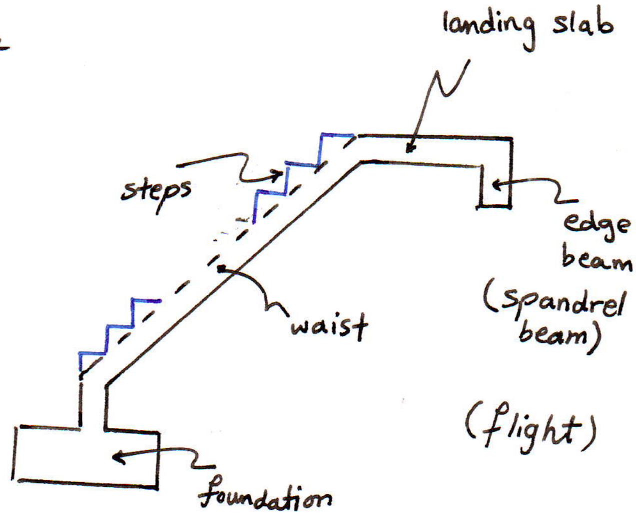


(section 2-2)
(at edge)
M^-

R.C. STAIR CASES

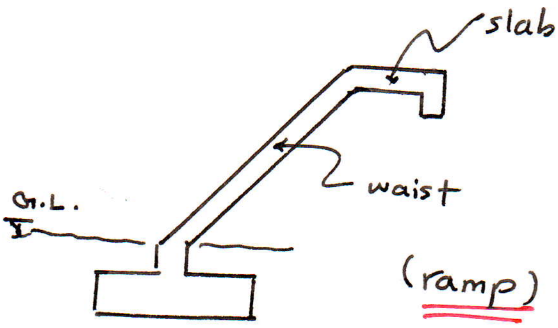
A staircase is a structure with steps (or stairs) used to provide movement between different levels.



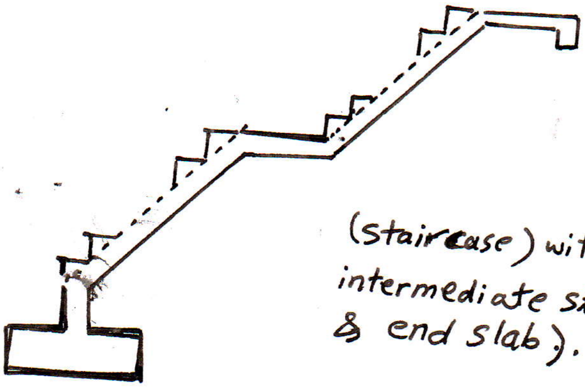


(flight)

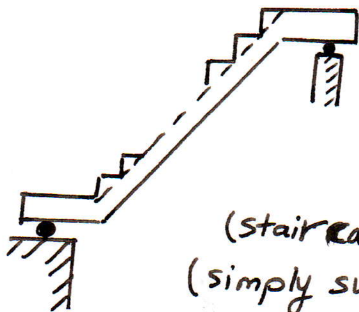
A ramp provides movement between different levels without steps (or stairs). It has only a waist.



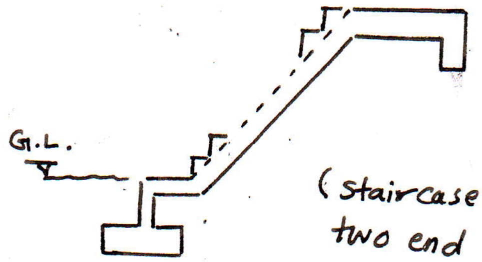
(ramp)



(staircase) with intermediate slab & end slab).



(staircase) (simply supported at both ends).



(staircase with two end slabs.)

(Here, stair cases with single flights will be studied).

Design of Stair Cases (with steps) or ramps (without steps).

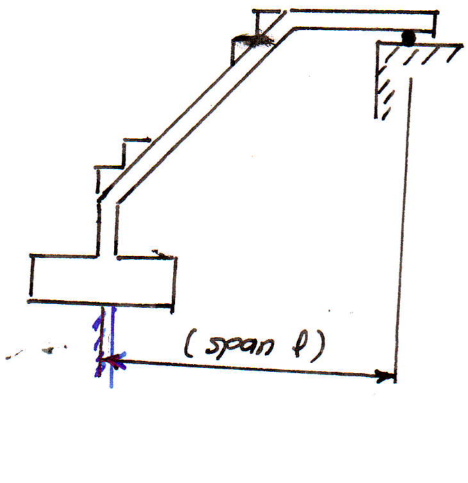
There are different Codes. Here a simplified method will be used.

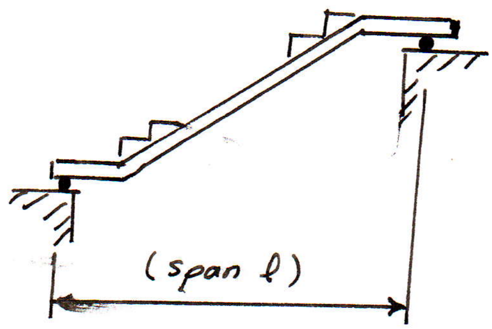
1) The inclined staircase is assumed to span horizontally.

2) The staircase is assumed simply supported at points of contraflexure (or zero bending moments).

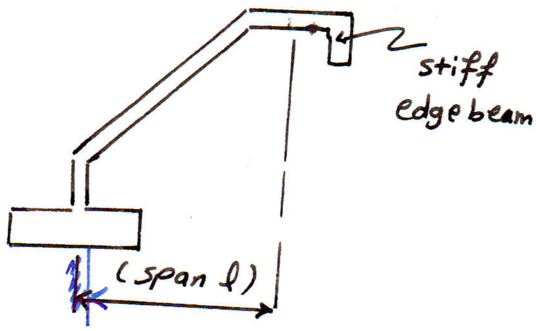
3) The effective span is the horizontal distances between the two points of contraflexure. (zero M.)?

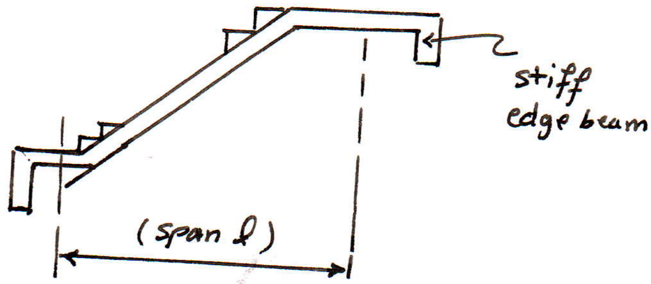
i) If the ends are simply supported, then the effective span is the horizontal distance between these ends.





ii) If the staircase has landing slab at one or both ends, the point of contraflexure is taken at middle of these slab when these slabs are fixed to stiff edge beams.





Thus : A staircase is designed as a simply supported beam with horizontal span. عتبة بسيطة أفقية

Notes :-

1. The dead loads especially the selfweight should be calculated for the whole inclined staircase. This self-weight includes the waist and the steps (for the horizontal span only).

2. The surfacing loads are usually horizontal (on treads of steps).

3. The live load is usually high (above $5 KN/m^2$) and acts vertically (on horizontal span).
دائماً الحمل الحي عالي جداً، لتلافي الفشل عند الازدحام في المدارس والسينما مثلاً؟ يعني لمقدار كبير؟؟

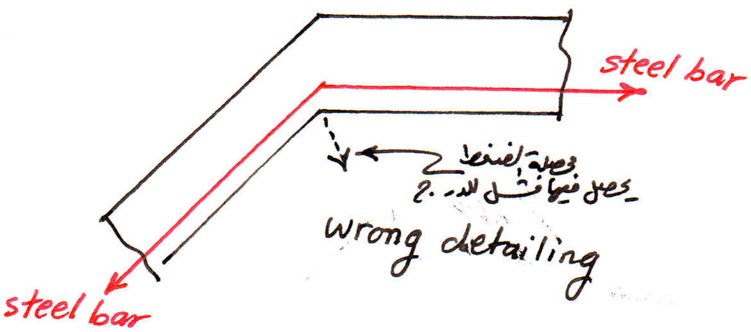
4. If W_u is the total ultimate load, the maximum positive bending moment is $\boxed{M^+ = \frac{W_u \cdot l}{8}}$

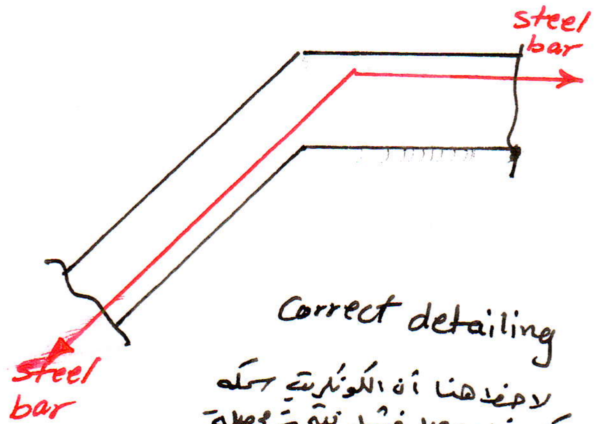
هذا العزم المؤثر المركز (لكل الفضاء)؟ لم يؤخذ $\frac{w_u \cdot l^2}{8}$ ؟
لماذا؟ لأنه هنا أي هذا المقدار (per unit length) ؟

Important note on reinforcement detailing:- ملاحظة مهمة لتفصيل الحديد

The staircase usually has bends (كسرات). The reinforcement at the bends must not follow the bend otherwise the small concrete cover at the bend will spall off (تتفتت أو يقشر).

التسليح لا يتبع الكسرات لأنه يقشر!



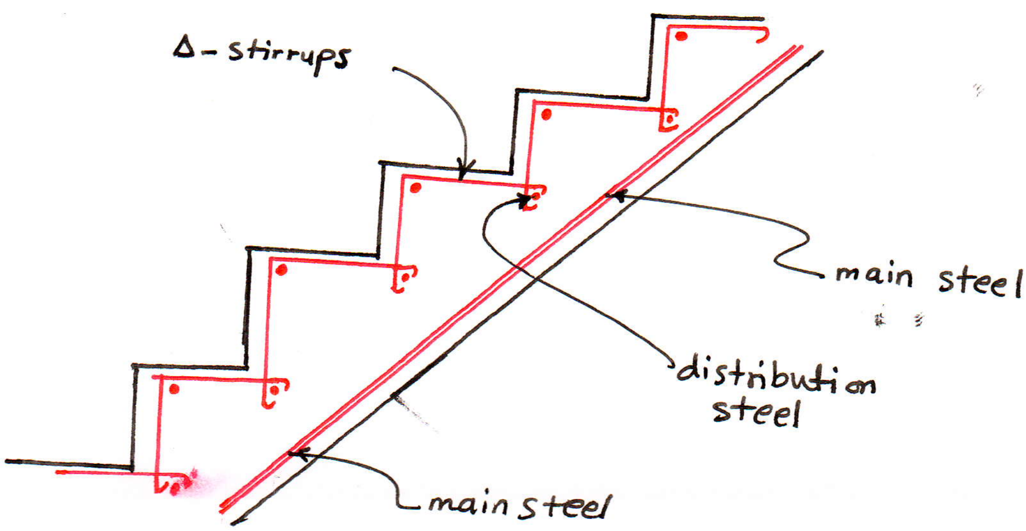


لاحظ هنا أن الكونكريت حمله كبير فلا يحصل فشل نتيجة محصلة الشد بالاتجاهين (من حديد التسليح)؟

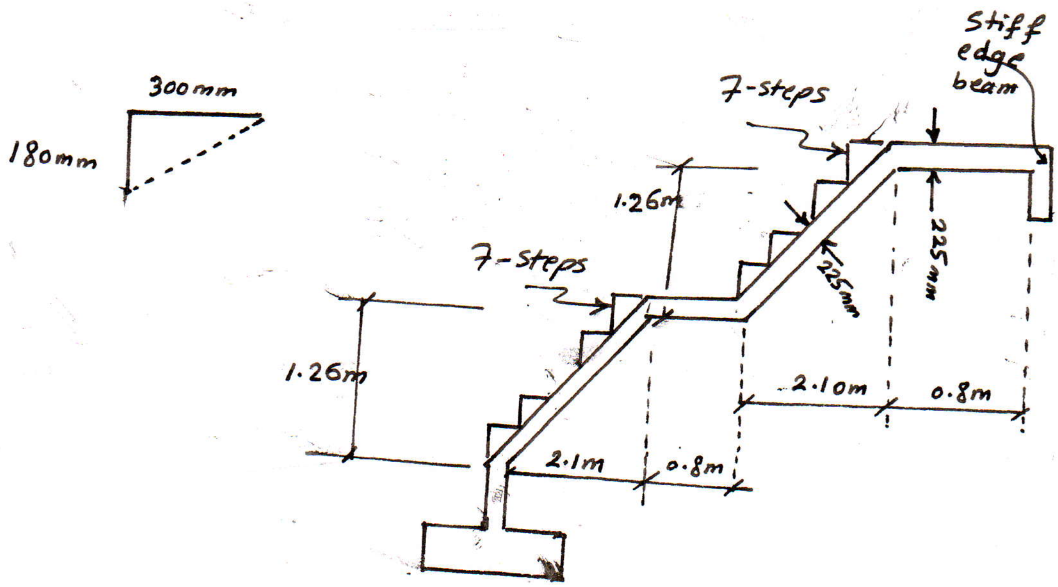
ملاحظة :- في حالة الاضطرار بأن الحديد يتبع الكسرة الـ (bend) يربط هذا الحديد بأتاري (Stirrups) أو بحديد عرضي وبواسطة الـ (welding) اللحام، كل ذلك من أجل عدم الضغط على الكونكريت الذي تحت حديد التسليح (مقدار سمكه قليل ؟؟) ، حيث يدفع الحديد السفلي للأعلى بواسطة الحديد العرضي ؟؟

The steps theoretically need no reinforcement. They are dead loads. In Iraq, nominal steel is used.

* لا تسلح لأنها حمل ميتة وغالباً ما تسلح أحياناً (حديد رمزي) بسيط؟



Example :- (design of staircase)



width of staircase $b = 1.10$ m , concrete strength $f'_c = 25$ MPa , concrete weight , $\gamma_c = 24.5$ kN/m^3 , steel yield $f_y = 345$ MPa. Tiling load 0.98 kN/m^2 (horizontally), live load 5.0 kN/m^2 (horizontally), use or take waist سمك العتبة عدا الباية ! thickness 225mm . Use clear cover 20 mm. Use ϕ16mm for التسليح الطولي main Steel & ϕ10mm بقية التفاصيل for others.

Solution :-

First the total selfweight must be calculated. length of each going is

$$\sqrt{1.26^2 + 2.10^2} = 2.449 \text{ m}$$ طول كل صعدة

Total selfweight of waist (only):

السمك العرض الصعدة 1 الصحن الأول الصعدة 2 نصف الصحن الأخير

$$W_g = 0.225 * 1.10 * \left(2.449 + 0.8 + 2.449 + \frac{0.8}{2}\right) * 24.5$$

$$= 36.977 \text{ KN}$$

Total load of steps:

$$w_s = \left(\frac{1}{2} * 0.180 * 0.300 * 1.1\right) * 24.5 * 14 \leftarrow \text{steps No.}$$

$$= 10.187 \text{ kN}$$

Tiling load

$$w_t = 1.1 * \left(2.1 + 0.8 + 2.1 + \frac{0.8}{2}\right) * \underbrace{0.98}_{\text{tiling load kN/m}^2}$$

$$= 5.821 \text{ kN}$$

Total dead loads

$$w_d = w_g + w_s + w_t$$

$$= 52.985 \text{ kN}$$

Total live load $w_\ell = 1.1 * \left(2.1 + 0.8 + 2.1 + \frac{0.8}{2}\right) * 5.0$

$$= 29.7 \text{ kN}$$

Total ultimate load $W_u = 1.4 w_d + 1.7 w_\ell$

$$= 1.4\,(52.985) + 1.7\,(29.7)$$

$$= 124.669 \text{ kN}$$

The effective span is $\ell = 2.1 + 0.8 + 2.1 + \frac{0.8}{2}$

$$= 5.4 \text{ m}$$

Thus the max. positive B.M. (at center) is:-

$$M^+ = \frac{W_u \cdot \ell}{8} = \frac{(124.669) * (5.4)}{8}$$

$$= 84.152 \text{ kN.m}$$

The effective depth is

$$d = 225 - 20 - \frac{16}{2}$$

$$= 197 \text{ mm}$$

Find ρ (by any method):

$$\rho = 0.006716$$

check this with $\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{345} = 0.004058$

use $\rho = 0.006716$

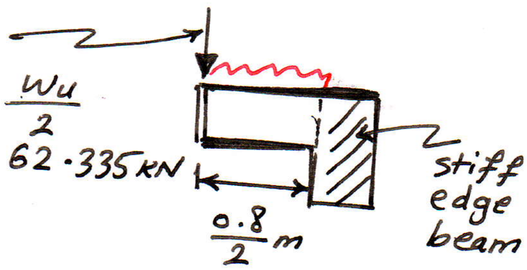
Then

$$A_s = 0.006716 * 1100 * 197$$
$$= 1455.4 \, mm^2$$

use $8\phi 16 \rightarrow A_s = 1609 \, mm^2$ (o.k.)

Next consider the cantilever part of the landing slab :

$P_u = \frac{W_u}{2}$
$= 62.335 \, kN$



The concentrated load (at free edge of slab) is :

$$P_u = \frac{W_u}{2} = \frac{124.669}{2} = 62.335 \, kN$$

self weight $\quad W_g = 0.225 * 1.10 * \frac{0.8}{2} * 24.5$
$= 2.426 \, kN$

weight of tiling $\quad W_t = 1.10 * \frac{0.8}{2} * 0.98$
$= 0.431 \, kN$

Total distributed dead load $\quad W_d = W_g + W_t$
$= 2.857 \, kN$

Total distributed live load

$W_l = 1.10 * 0.4 * 5.0$
$= 2.2 \, kN$

Total ultimate load distributed :

$W_u = 1.4 W_d + 1.7 W_l$
$= 7.740 \, kN$

Max. negitive B.M. (at the fixed end) is:

هذه هي نقطة تأثير max B.M. في الموقع

$$\bar{M} = P_u \cdot l + \frac{W_u \cdot l}{2}$$

$$= 62.335 * 0.4 + \frac{7.740 * 0.4}{2}$$

$$= 26.482 \text{ KN.m}$$

As befor $d = 197$ mm

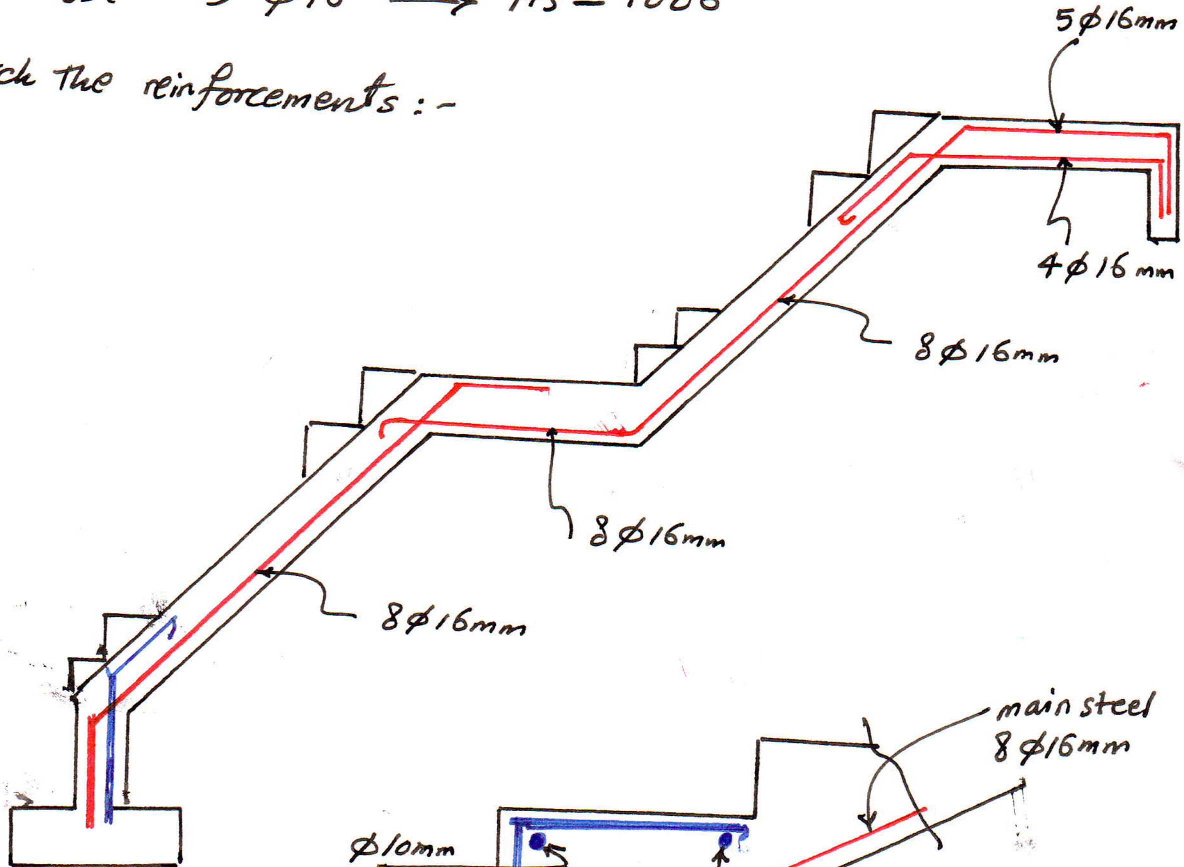
find $\rho = 0.002032$

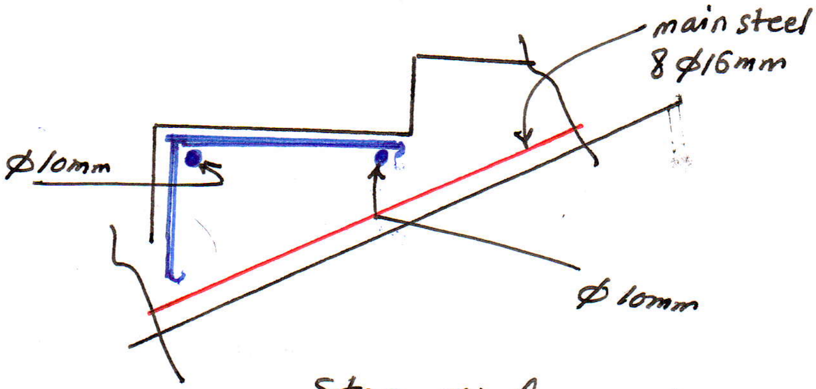
This $\rho < \rho_{min}$, so use $\rho = \rho_{min} = 0.004058$

$As = 0.004 * 1100 * 197 = 866.8 \text{ mm}^2$

use $5 \phi 16 \longrightarrow As = 1006$

Sketch the reinforcements :-

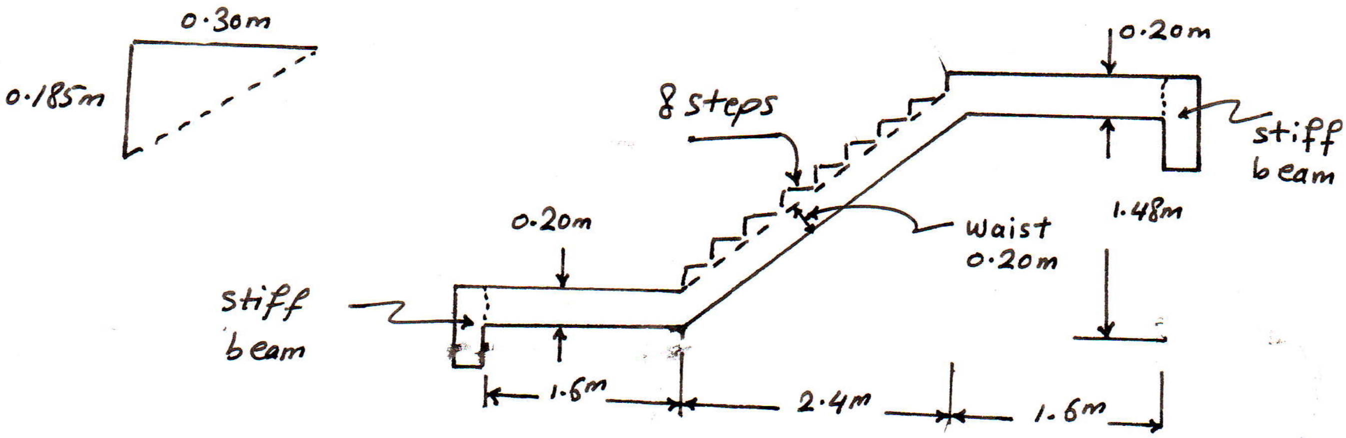




Step-reinforcement

($\phi 10^{mm}$ - Δ - stirrups)

Design of R.C. staircase



Design and sketch accurately the steel reinforcement for the staircase shown in the figure above.

For concrete $f'_c = 27.5$ MPa $\qquad \gamma_c = 24.5\ kN/m^3$

For steel $f_y = 345$ MPa $\qquad \phi 12$ mm bars

surfacing load $0.92\ kN/m^2$ (horizontally).

Live load $5.80\ kN/m^2$ (horizontally).

stair width $b = 1.20$m. clear cover 20 mm.

Hint: weight of waist 25.990 KN. weight of steps 6.527 KN. surfacing load 4.416 KN.

$W_d = 36.933$ KN, $W_\ell = 27.84$ KN, $W_u = 99.034$ KN

span $\ell = 4.0$m, $M_u = \dfrac{W_u \cdot \ell}{8} = 49.517$ KN.m,

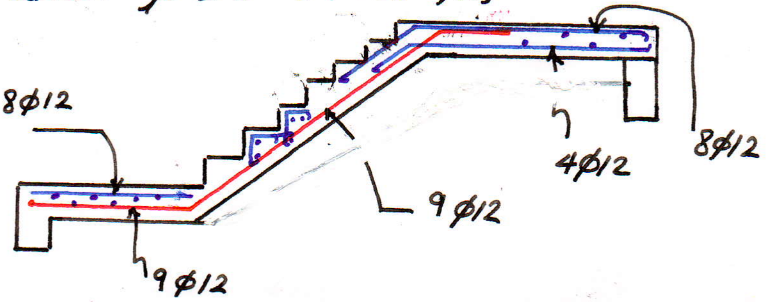
$d = 174$ mm, $\rho = 0.004544$, $A_s = 939.6\ mm^2$,

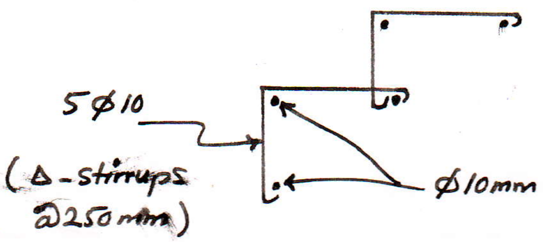
use $9\phi 12$mm $\rightarrow A_s = 1018 mm^2$. For landing slabs

$M_u = 49.517 * 0.8 + \frac{1}{2} * 21.710 * 0.8^2$

$= 46.561$ KN.m $\rho = 0.004255$, $A_s = 887.4\ mm^2$

use $8\phi 12$.

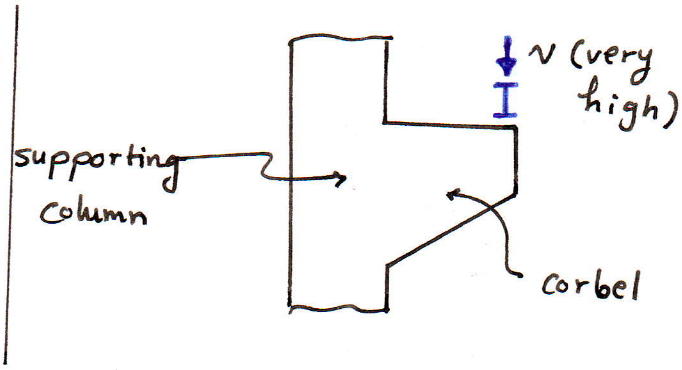


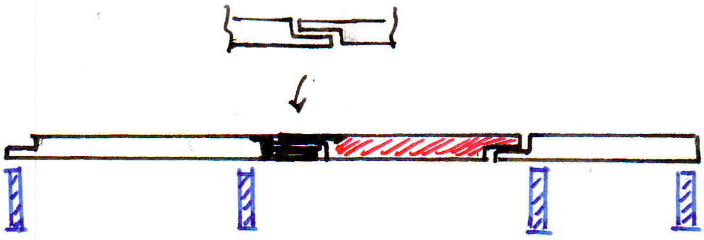


CORBELS (OR BRACKETS)

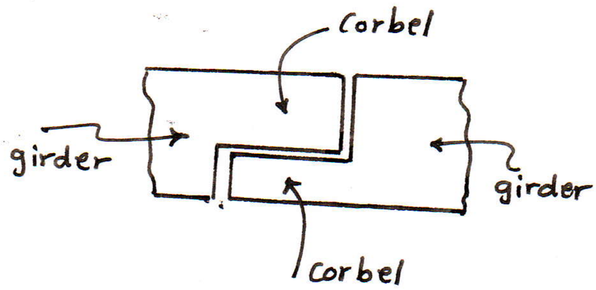
A corbel (or bracket) is a very deep and short cantilever carrying high loads.

(In industrial buildings and warehouses to carry travelling cranes)

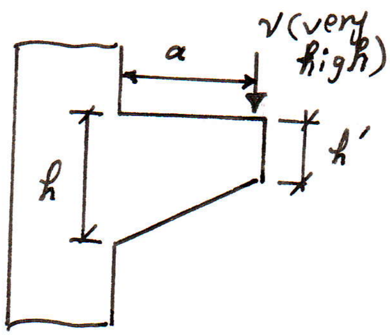




(central beam on corbels)



Behaviour of corbels under high concentrated loads:

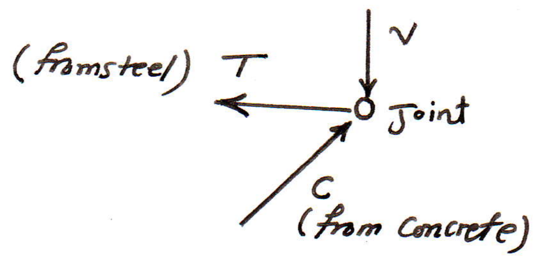


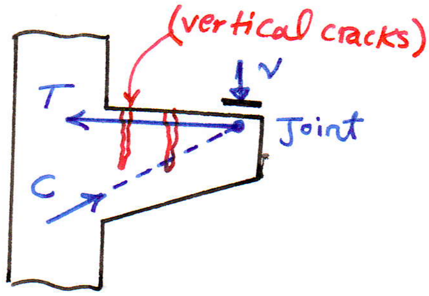
The following notations are used.

a is the shear span.
h is the depth at end.
h' is the depth at free end. (usually $h' \geq \frac{1}{2} h$)
b width of the corbel.

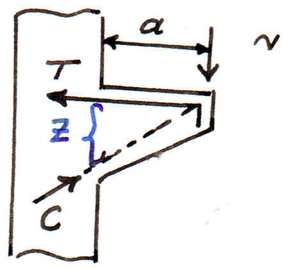
The top of the corbel is under tensile stresses (from bending). The cross section is also under high shearing stress. At failure no diagonal cracks occurs because $a < h$. At failure, vertical cracks will occur.

The behaviour is like a simple truss (truss analogy). The main flexural steel (at top in tension) is the tension bar. Concrete from top at the load to the bottom end near the column will act like a bar in compression.





The lever arm z at the end is usually $z = 0.80h$ (approximately).



Design of Corbels:

هنا نضع stimps
أفقية لأن التشققات
عمودية؟

Here the shear span (a) and the load (V) are given. Also the width (b) is usually given. Find the depths h and h' and calculate the flexural steel A_s (for bending) and the shear steel A_h for shear. Horizontal stirrups are used here because the cracks are vertical.

Procedures:

Here a simple method for design of Corbels is given.

1. Specify the depth h and then h'. Try $h = \frac{a}{0.6}$, $h' \geq \frac{1}{2} h$

2. Specify the effective depth (d). Take total cover about 75 mm. Then

$$d = h - 75$$

3. Specify the lever arm Z as:

$$Z = 0.85\,d$$

(approximately)

4. Find the flexural steel As from moments at bottom of the end:

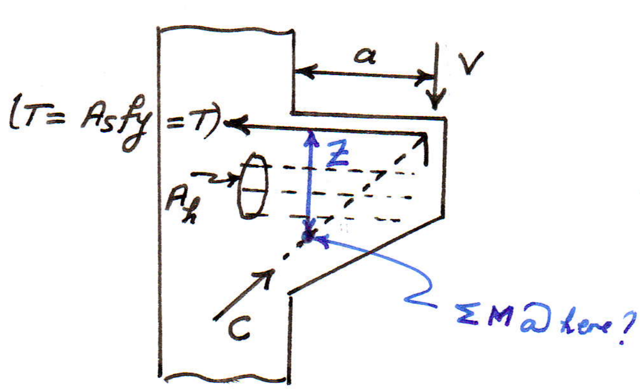
$$T \cdot Z = V \cdot a \quad \text{, Thus} \quad A_s = \frac{V a}{\phi f_y Z}$$

where $\phi = 0.90$ for bending

Check $\dfrac{A_s}{bd} > 0.04 \dfrac{f'_c}{f_y}$

If this not checked, then decrease (h) & so (d) to increase A_s.

لاحظ المعادلة



5. Find the horizontal shear reinforcement A_h.

Use $v_u = \dfrac{V}{\phi\, b d}$, ($\phi = 0.85$ for shear)

shear force

actual reinf. shearing stress.

x-sec. area

This is the actual ultimate shearing stress in the Concrete section (at end).

Also $v_c = 0.54\left(1 - \dfrac{a}{2d}\right)(1 + 64\rho_v)\sqrt{f'_c}$

for the shear strength of concrete (in MPa). Here

$$\rho_v = \frac{A_s + A_h}{bd}$$

put $v_u = v_c$ & find ρ_v.

check $\rho_v < 0.20 \dfrac{f_c'}{f_y}$

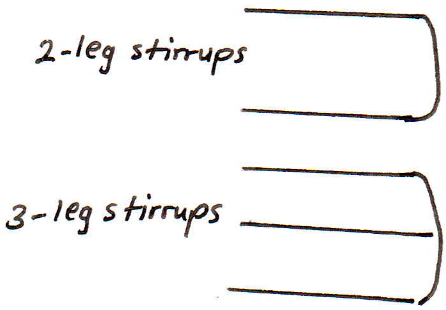
If this is not checked, then increase h & so d to decrease ρ_v.

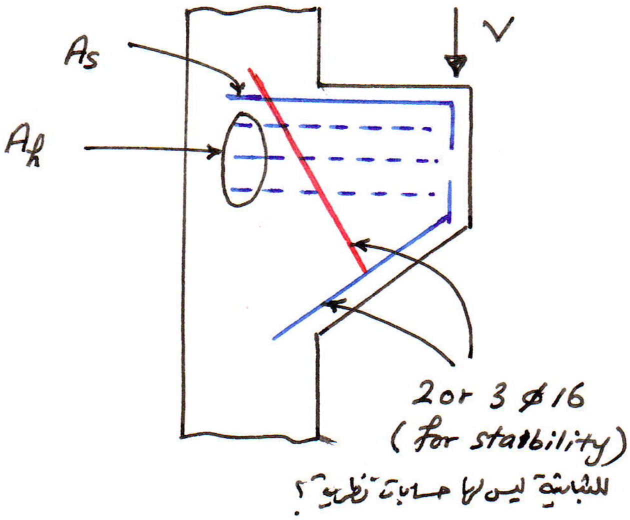
After that find horizontal shear steel

$A_h = \rho_v \cdot bd - A_s$, check $\frac{1}{2} A_s < A_h < A_s$

put the horizontal stirrups A_h in layers at $\frac{2}{3} d$ from top.

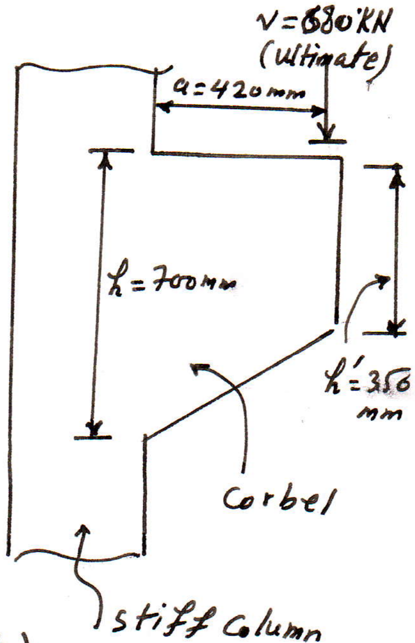
6. Draw the details of reinforcement.





Example (design of Corbels):

Design and sketch accurately the steel reinforcement for the Corbel shown in the figure.



For Concrete $f'_c = 27.5$ MPa

For steel $f_y = 345$ MPa

(Use $\phi 16$ mm bars)

clear cover 75 mm

width of corbel $b = 400$ mm

Solution:

Shear span $a = 420$ mm

load $V = 680$ KN (ultimate)

width $b = 400$ mm

Depth $h = 700$ mm (at end)

$f'_c = 27.5$ MPa

$f_y = 345$

Cover 75 mm

effective depth $(d) = 700 - 75 - \frac{16}{2} = 617$ mm

Find: i) Flexural reinforcement A_s

ii) Horizontal shear reinforcement A_h.

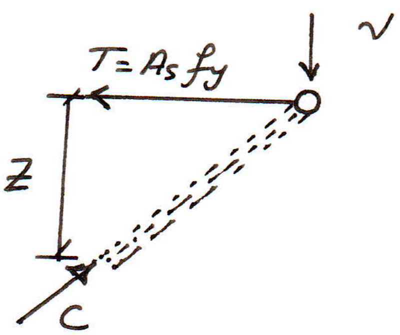
First find A_s. Assume a truss analogy.

Then the lever arm z is estimated as:

$z = 0.85 d$

$= 0.85 * 617$

$= 524.450$ mm



By moments, then $A_s = \dfrac{V a}{\phi f_y z}$

$$A_s = \frac{680 * 10^3 * 420}{\underset{\phi}{\underline{\underline{0.90}}} * 345 * 524.450} = 1753.850 \ mm^2$$

Use $9 \phi 16 \ mm$ bars $\rightarrow A_s = 1810 \ mm^2$

check $\rho_s = \frac{A_s}{bd} > 0.04 \frac{f'_c}{f_y}$

الحد الفاصل لايعمل
(عاطل)؟

Here $\rho_s = \frac{1753.850}{400 * 617} = 0.007106$

$$0.04 \frac{f'_c}{f_y} = 0.003188$$

Thus here $\rho_s > 0.04 \frac{f'_c}{f_y}$ (o.k.)

Notice that if $A_s = 1810 \ mm^2$ is used then

$$\rho_s = \frac{1810}{400 * 617} = 0.007293 \quad \text{(o.k.)}$$

Next find A_h ?

The actual ultimate shearing stress in the concrete section is :

لاحظ أن المساحة مضروبة
بـ(ϕ) لزيادة الضمان ؟

$$v_u = \frac{V}{\phi b d}$$

$$= \frac{680 * 10^3}{0.85 * 400 * 617} = 3.241 \ N/mm^2 \text{ (or MPa)}$$

The shear strength of concrete

$$v_c = 0.54 \left(1 - \frac{a}{2d}\right)(1 + 64 \rho_v) \sqrt{f'_c}$$

Let $v_u = v_c$, then , $3.241 = 0.54 \left(1 - \frac{420}{2 * 617}\right) * (1 + 64 \rho_v) \sqrt{27.5}$

This gives,

$$\rho_v = 0.011485$$

check $\rho_v < 0.2 \frac{f'_c}{f_y}$

Here $\quad 0.20 * \dfrac{27.5}{345} = 0.015942 \quad$ (o.k.)

But $\quad \rho_v = \dfrac{A_s + A_h}{bd} \quad$, then $\quad A_h = \rho_v \cdot bd - A_s$

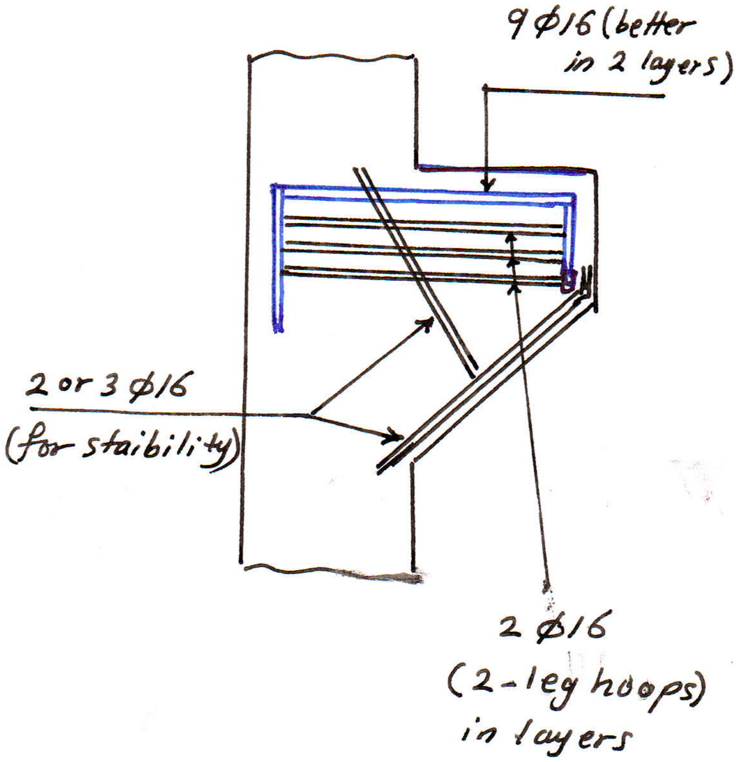
or $\quad A_h = 0.011485 * 400 * 617 - 1810$

$\quad\quad = 1024.498 \text{ mm}^2$

Use $\quad 6\phi16 \rightarrow A_h = 1207 \text{ mm}^2$

check $\quad \frac{1}{2}A_s < A_h < A_s \quad$ Here o.k.

Use $\quad$ 3 layers for A_h:



Example: (design of corbels)

width of corbel 400mm

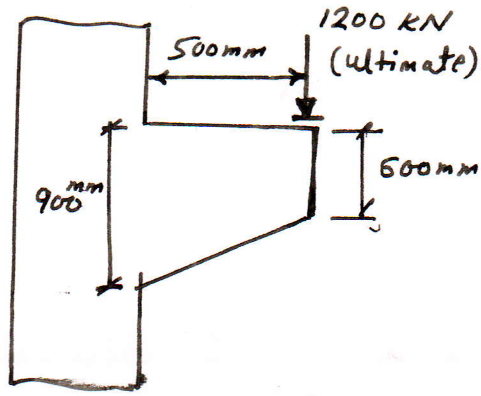
For concrete $\quad f'_c = 25 \text{ MPa}$

For steel $\quad f_y = 350 \text{ MPa}$

use $\phi18$mm for bending steel.

$\quad \phi12$mm for shear steel.

clear cover 75mm ?

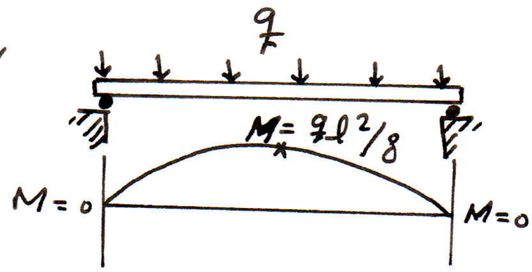


PLASTIC HINGES IN BEAMS AND YIELD (OR CRACK) LINES IN SLABS

The Concept of Collapse in beams:

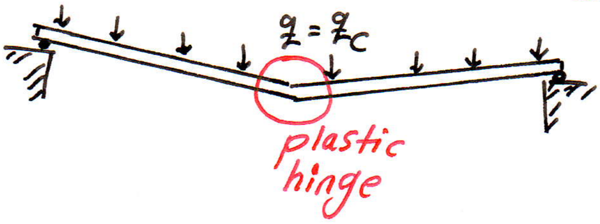
1. Consider a simply supported beam under uniform load q (per unit length).

Let the beam have a reinforced Concrete section $(b*d)$. The Ultimate moment is:



$$M_u = \phi \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) b.d^2$$

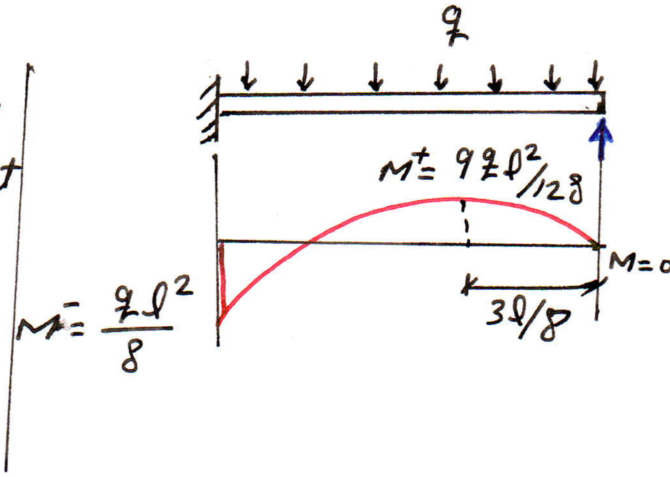
Now assume the load q to increase gradually. The maximum bending moment (at mid-span) will also increase. when the load q reaches q_c (collaps load), the bending moment $M = \frac{q_c \cdot l^2}{8}$ will be equal to the ultimate moment of the sec. M_u. A plastic hinge will be formed in the mid-span and the beam will collapse. The beam will be a mechanism,



The collapse load q_c is obtained from:

$$\frac{q_c . l^2}{8} = M_u \Rightarrow \boxed{q_c = \frac{8 M_u}{l^2}}$$

2. Consider a beam with one simply supported & one fixed end. Let q (per unit length) be the load.

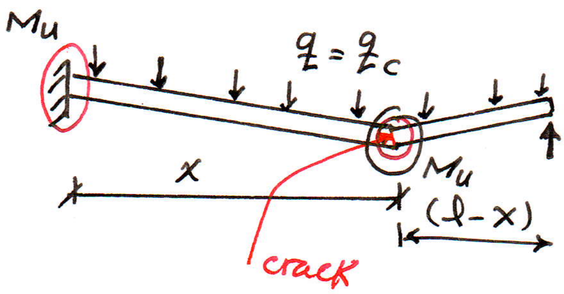


Here the negative bending moment is $M^- = q\ell^2/8$ at the fixed & the positive bending moment is:

$M^+ = 9\,q\ell^2/128$,

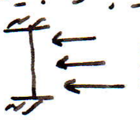
near the mid-span. Let the beam have a reinforced concrete section with $M_u = \phi \rho f_y (1 - 0.59 \frac{\rho f_y}{f_c'}) . bd^2$ at all sections.

Let q increases, then a plastic hinge is first formed at the fixed end when $M_u = M^- = q\ell^2/8$. The beam will not collapse but it will behave as a simply supported beam for further increase in q. The collapse load $q = q_c$ will occur when a second plastic ~~collapse load~~ hinge is formed at or near the mid-span. How to find this collapse load q_c !!



- يجب أن يفشل في موقعين كي يسقط؟ حيث يتحول إلى (mechanism) حيث يبدأ الحمل بالزيادة تدريجياً؟

q ضغط كبير جداً؛ ليس مجرد حمل؟ مثلاً تأثير ضغط عالٍ على عتبات في بناية؟ يحدث قرب الانفجار

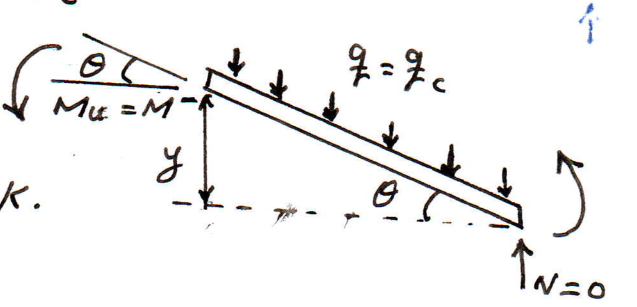


q (very high pressure) ?

Assume the second plastic hinge is at distance x from the fixed end. Also assume (or in fact) the shearing force $V=0$ at the second (or interior) hinge. Use the free body diagram of the left portions:

There are two methods to find q_c:

1) The method of equilibrium.
2) The method of virtual work.



By equilibrium, take moments about the fixed end:

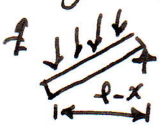
$$M_u + M_u - q_c \cdot x \cdot \frac{x}{2} = 0$$

Thus $\quad q_c = \dfrac{4M_u}{x^2}$; In this case $x = \dfrac{l}{2}$ at collapse, thus:

$$q_c = \frac{4M_u}{(l/2)^2} = \frac{16M_u}{l^2}$$

Note:

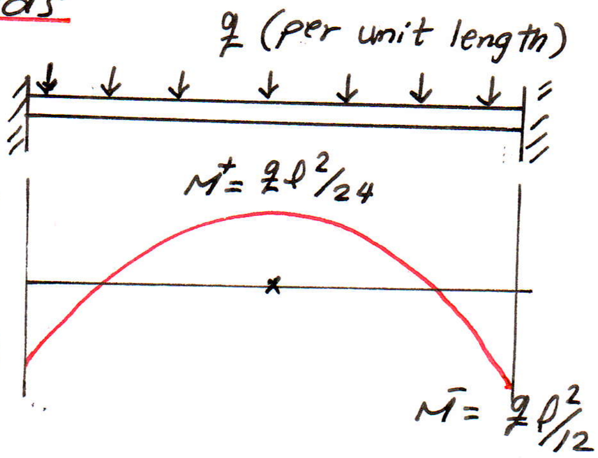
x can be found exactly by taking the equilibrium of right-side portion.



$$M_u - q_c \cdot (l-x) \cdot \frac{(l-x)}{2} = 0$$

3. A beam fixed at both ends

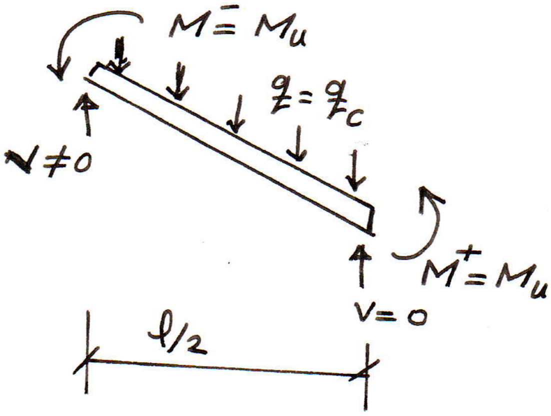
Assume M_u constant for all cross sections. At collapse ($q=q_c$), three plastic hinges are formed.



By equilibrium :

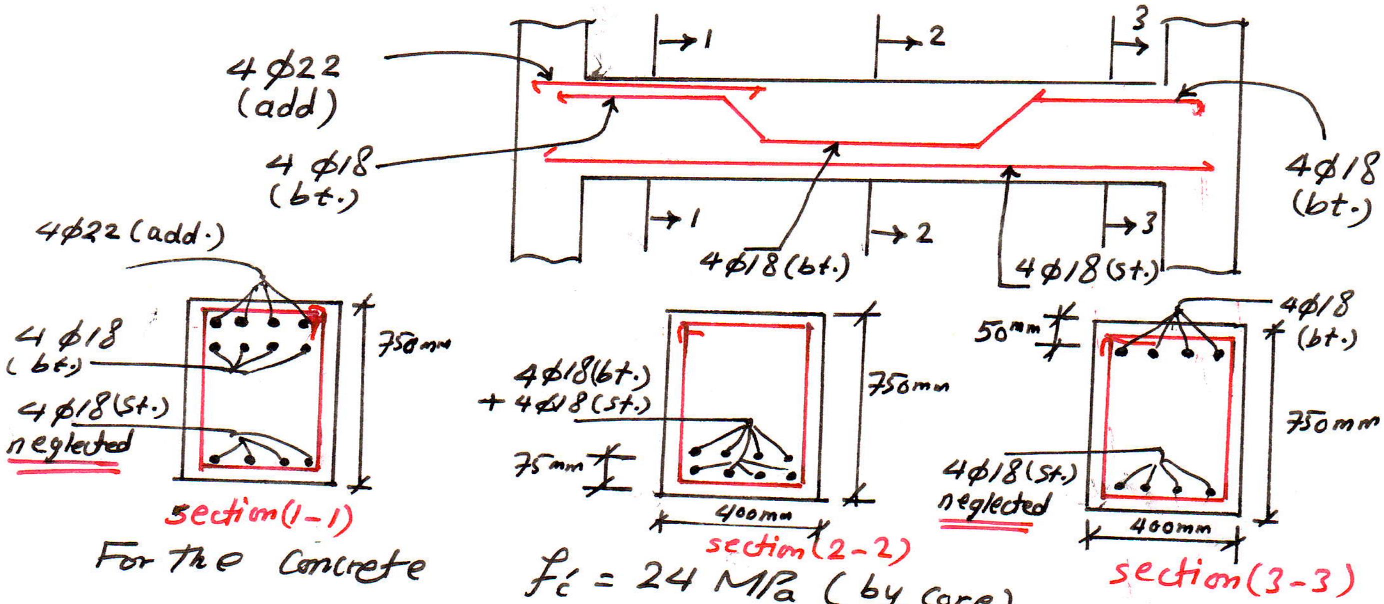
$$M_u + M_u - q_c \cdot \frac{l}{2} \cdot \frac{l}{4} = 0$$

This gives $\quad q_c = \dfrac{16 M_u}{l^2}$



Example (plastic hinges in R.C. beams)

A beam of clear span 9.6m is connected to columns at both ends. The sections and reinforcements are shown in the figure:



For the Concrete $\quad f'_c = 24$ MPa (by core)

$\gamma_c = 25$ kN/m3

For steel $\quad f_y = 310$ MPa

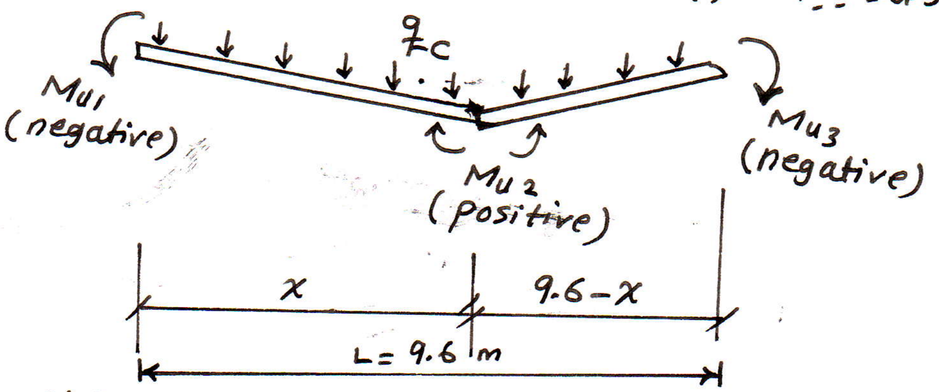
Calculate the total Collapse load q_c & then the imposed Collapse load q_i ?

Hint: First calculate the ultimate moments (M_{u1}, M_{u2} and M_{u3}) at sections 1-1, 2-2 and 3-3.

بتستخدم أي طريقة من الطرق ؟

Next assume a mechanism of collapse:

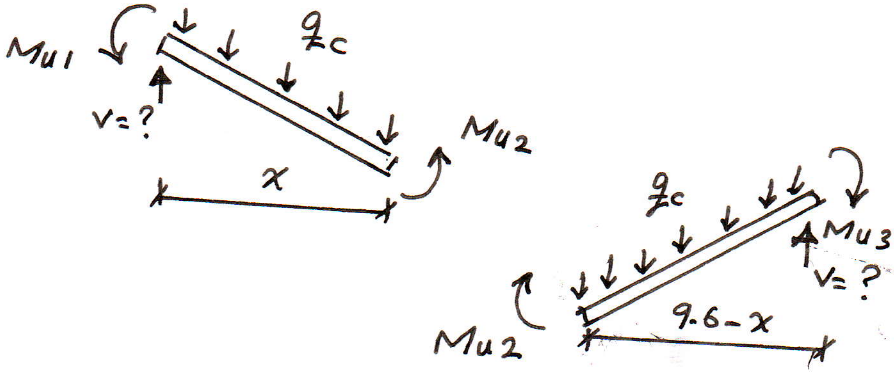
افترض ميكانيكية الانكسار ؟



Then use equilibrium (or virtual work) to calculate the two unknowns (x & q_c).

توجد مجهولان ، q_c & x ؟ لذا نحتاج معادلتين للتوازن ؟ لكل جزء ؟ على حده ؟

(assume the shearing force V at interior hinge (2) is zero).



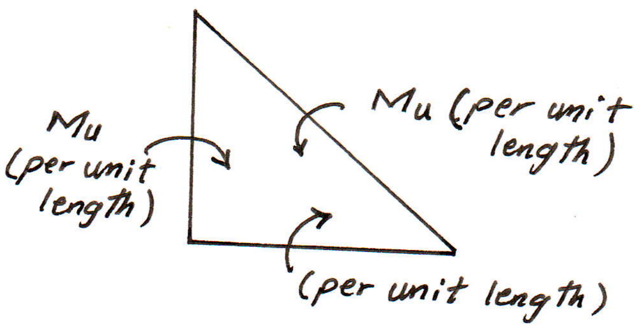
The Coneept of Yield Lines in Slabs

Assume the load q (per unit area) on the slab increases gradually. The positive bending moments (at interior regions) and negative bending moments (at edges) will also increase gradually.

Cracks will appear when the moments from the applied loads become equal to the ultimate moments of the sections. The slab will not collapse and can take more and more loads. New sections will reach their ultimate moments and the cracks will spread تنتشر.

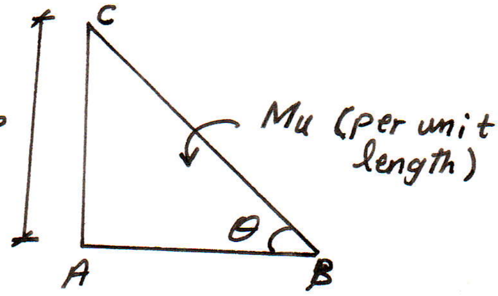
The process will continue until all cracks will join together and the slab will collapse into pieces.

Here only slabs with isotropic reinforcement is considered, therefore Mu is the same in the short and in the long directions and in fact in all directions (like hydrostatic pressure).



Assumptions:

1. The reinforcement is isotropic. Mu is the sam in all directions.
2. The yield lines are straight (in rectangular and polygonal slabs).
3. shearing forces are zero at interior yield lines.
4. Interior yield lines cut the free edges (or other edges) at right angles.
5. Ultimate moments Mu (per unit length) are multiplied by their length to obtain the total ultimate moments. These are vectors (like forces).



From figure: moment on

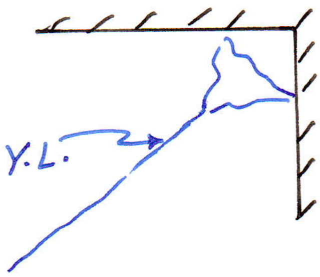
$$\overline{BC} = M_u \cdot \overline{BC}$$ العزم الكلي

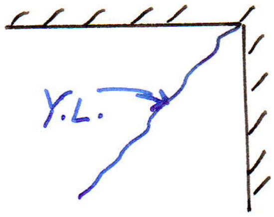
$$\text{moment on } \overline{AB} = (M_u \cdot \overline{BC}) \cdot \cos\theta$$
$$= M_u \cdot (\overline{BC} \cdot \cos\theta)$$
$$= M_u \cdot \overline{AB}$$

$$\text{moment on } \overline{AC} = (M_u \cdot \overline{BC}) \cdot \sin\theta$$
$$= M_u \cdot (\overline{BC} \cdot \sin\theta)$$
$$= M_u \cdot \overline{AC}$$

مهم جداً : Here right-hand rule is used for total moments (as vectors). كمية اتجاهية

Note: Usually at corners of slabs, the yield lines will have branches. To simplify calculations, these branches are neglected.





The collapse mechanism is correct if and only if each piece of slab can rotate as a free body about a fixed axis.

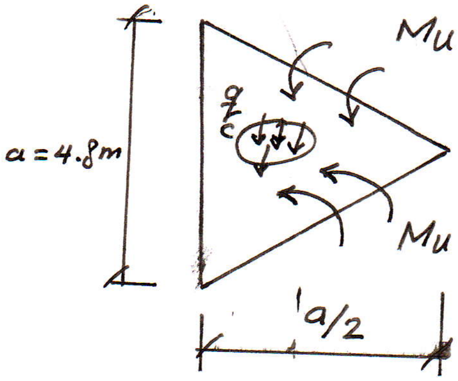
Example (Collapse of R.C. Slabs):

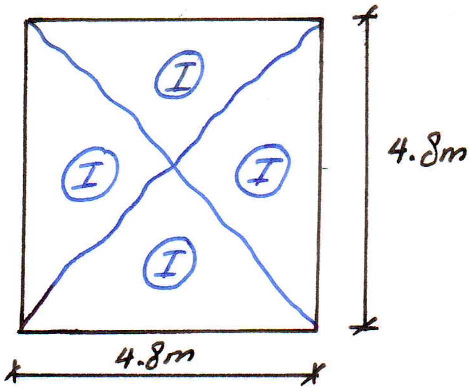
1. A square slab of clear spans 4.8 m * 4.8 m is simply supported over the edges. The slab is reinforced by $\phi 10$ mm @ 200 mm in both directions. The cover is 25 mm. The slab thickness is 125 mm. For concrete $f'_c = 21.7$ MPa.
$\gamma_c = 24.5$ kN/m^3.
For steel $f_y = 275$ MPa.

Calculate the imposed collapse load? حمل السقوط المفروض المطلوب

Solution:-

Due to symmetry in both directions, the yield (or cracks) lines are diagonal. The slab will crack into 4 equal pieces. Take one piece and let q_c (per unit area) be the collapse load and M_u the ultimate moment of a section inside the slab (in any direction due to isotropy).





(Yield line Pattern)

Take moments about the edge :-

مقطع العزم

مساحة المثلث

مركز الحمل

$$M_u \cdot a - q_c \left(\frac{1}{2} a \cdot \frac{a}{2}\right) \cdot \frac{a}{6} = 0$$

Then $q_c = \dfrac{24 M_u}{a^2}$

Find M_u : $A_s = 393\ mm^2/m$; $\rho = \dfrac{393}{1000 * 100} = 0.003930$

use ; $M_u = \phi \rho f_y \left(1 - 0.59 \dfrac{\rho f_y}{f_c'}\right) b.d^2$

$$= 0.9 * 0.003930 * 275 \left(1 - 0.59 * \frac{0.003930 * 275}{21.7}\right) * (1000 * 100^2).$$

$$= 9.441\ KN.m/m$$

substitute , then $q_c = \dfrac{24 * 9.441}{4.8^2} = 9.834\ KN/m^2$

The imposed collapse load;

* الوزن الإضافي لوزن السقف
لاحداث الانهيار (الفشل)
الحمل الاضافي اللازم ؟

$$q_i = 9.834 - 0.125 * 24.5$$
$$= 6.772\ KN/m^2$$

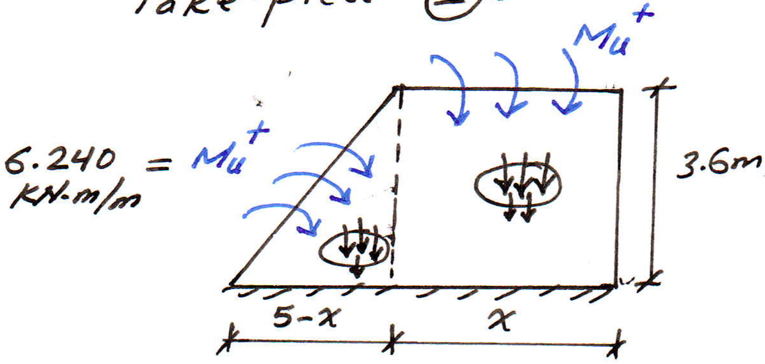
2. simply supported:

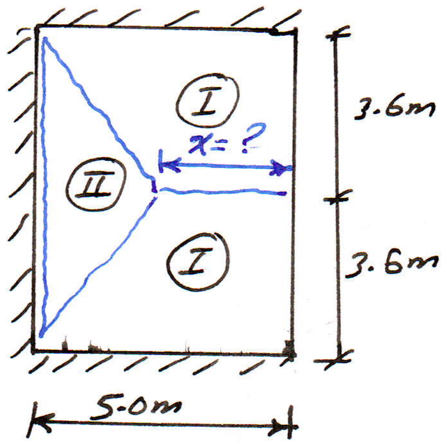
A rectangular slab with isotropic reinforcement has spans and edges as shown in the figure. The ultimate positive moment is 6.240 KN. m/m (inside the slab) and the ultimate negative moment is 1.25 * 6.240 = 7.80 KN.m/m (at the fixed edge). For the Yield pattern shown here in, find the collapse load.

Solution:

There are 2 unknowns (x & q_c). Two equations are needed.

Take piece (I):





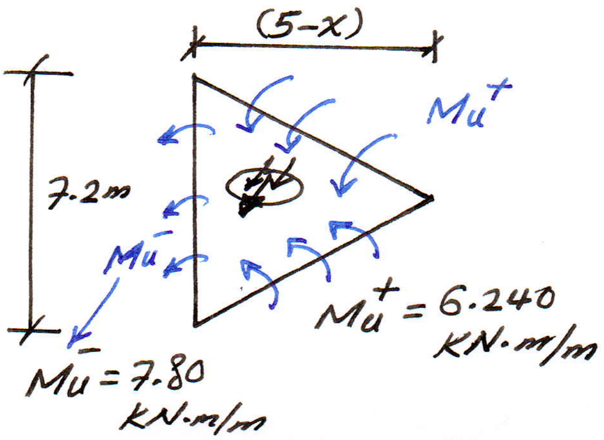
$$M_u * 5.0 - q_c \cdot \left\{ \left[\frac{1}{2} (5-x) * 3.6 * \frac{3.6}{3} \right] + \left[(3.6x) * \frac{3.6}{2} \right] \right\} = 0$$

---- (1)

Take piece (II):

$$M_u^+ * 7.2 + M_u^- * 7.2 - q_c \cdot \left\{ \frac{1}{2} * 7.2 * (5-x) * \frac{(5-x)}{3} \right\} = 0$$

---- (2)



Solve ① & ②

$x = \text{----}\ m$

$q_{fc} = \text{----}\ KN/m^2$

check $0 < x < 5.0$

3. Here $M_u^- = 1.20\, M_u^+$,

$q_{fc} = 12.4\ KN/m^2$ (given)

$f'_c = 27.6\ MPa$

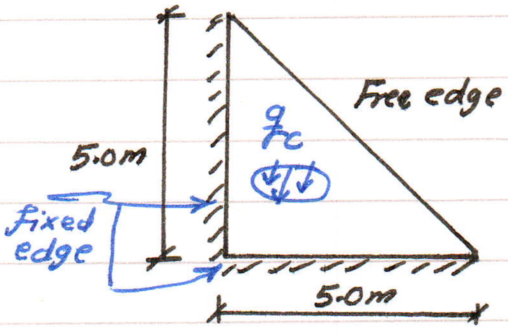
$f_y = 345\ MPa$

$h = 175\ mm$ (Slab thickness)

clear cover 25 mm (in both directions)

use $\phi 12$ mm steel bars.

Find the spacing ?



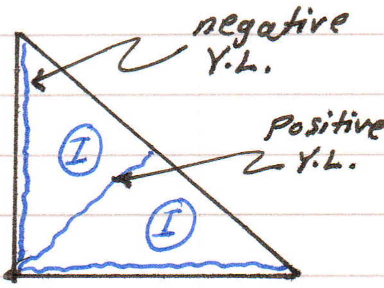
Note: This is an example of design of slabs by yield lines).

دائماً في سقوف الملاجئ نصمم باستخدام (yield lines)

Solution :-

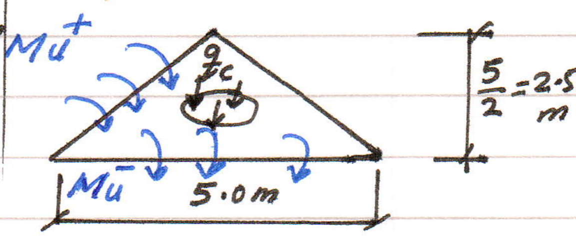
The yeild line (or crack line) pattern is shown:

There are equal pieces. Take one piece,



$$M_u^- * 5.0 + M_u^+ * \frac{5.0}{2} - q_{fc} * \left(\frac{1}{2} * 5.0 * \frac{5.0}{2} * \frac{1}{3} * \frac{5.0}{2}\right) = 0$$

substitute, then $M_u = 23.6\ kN.m/m$



To find As , use

$$R = \frac{M_u}{\phi f_c' b d^2} = \frac{23.6 * 10^6}{0.90 * 27.6 * 1000 * 150^2}$$

$= 0.0419$, this gives $\omega = 0.043$ (nearly)

So $\rho = \omega f_c' / f_y$

$= 0.043 * 27.6 / 345$

$= 0.003440 > \rho_{min}$

So $A_s = \rho b d$

$= 0.00344 * 1000 * 150$

$= 516 \ mm^2/m$ (width)

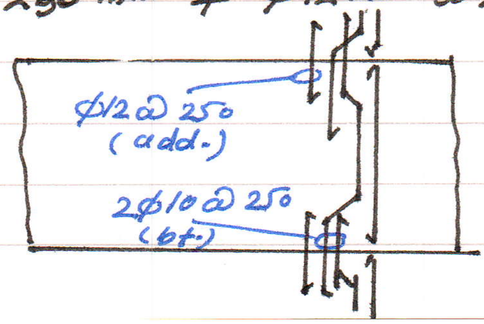
use $\phi 12$ mm @ 200 mm $\rightarrow A_s = 595 \ mm^2/m$ (width)

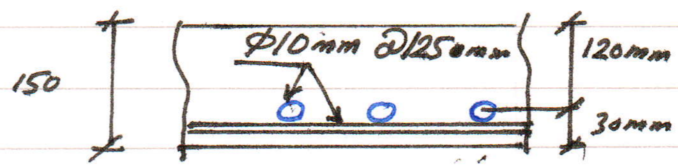
o.k.

Example (Yield line in slabs):

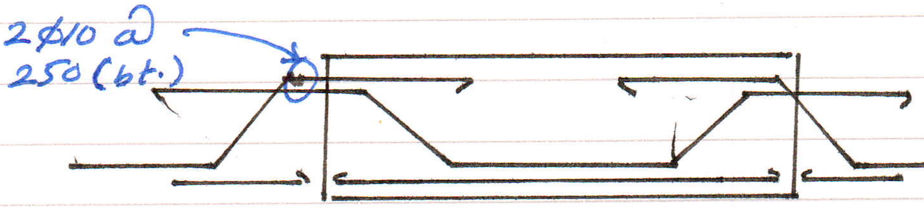
An interior slab (with continuous edges) has clear spans 5.1 m in the short direction & 7.5 m in the long direction &. The slab thickness is 150 mm. The slab has isotropic reinforcement in the interior regions (regions of positive bending moments), $\phi 10$ mm @ 125 mm (both ways) & effective cover 30 mm ?

The reinforcement in the long edge for the negative bending moment in the short direction is $2\phi 10$ mm @ 250 mm + $\phi 12$ mm @ 250 mm.



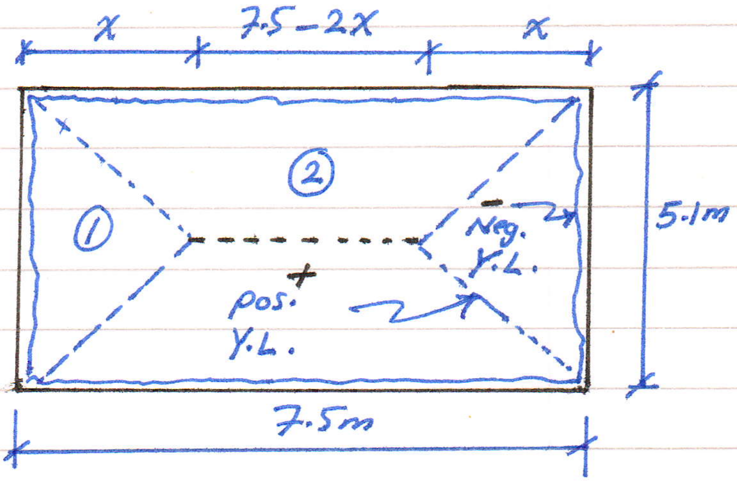


The reinforcement in the short edge (for the negative bending moment in the long direction) is $2\phi 10$ mm @ 250 mm.



* لاحظ أنه (two-way slab) لذا فانه يحتاج للتسليح السالب في long d. أيضاً لكنه أقل من (short) حيث أن الاجهاد هو البسيط

For the yield line pattern (given in the figure), determine the collapse load q_c. For concrete $f'_c = 25$ MPa, steel $f_y = 275$ MPa.

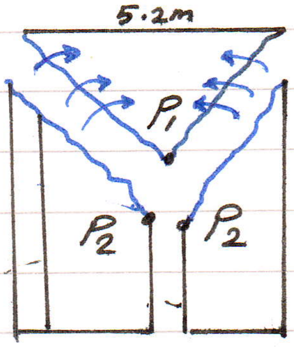


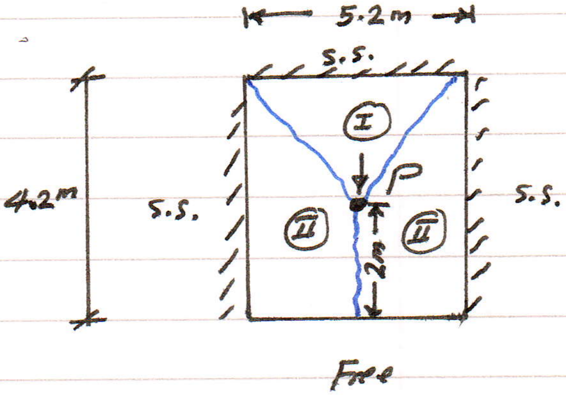
* لاحظ الشكل بدقة حيث أن كل beam يأخذ ثقل المثلث أو شبه المنحرف الذي من جهته وذلك يفيد في تصميم الجسور والعتبات إلا حد كبير لمعرفة الثقل المسلط على العتبة في حالة التصميم.

* في حالة (1-way) نقسم الـ (slab) إلى مستطيلين لأن المثلث سوف يكون صغيراً جداً نسبة لمساحة شبه المنحرف المتكون ؟

Ex. Calculate the collapse load $P_c = ?$
3 edge s.s., 1 edge free?

$M_u^{+} = 30 \text{ kN}\cdot\text{m/m}$





Take segment I :

$$(30)(5.2) = P_1(2.2)$$

$$\Rightarrow P_1 = \frac{30 * 5.2}{2.2} = 70.909 \text{ kN}$$

segment II :

$$30(4.2) = P_2(2.6)$$

$$\Rightarrow P_2 = 48.461 \text{ kN}$$

$\therefore \quad P_{collapse} = P_1 + 2P_2 = 167.831 \text{ kN} ?$

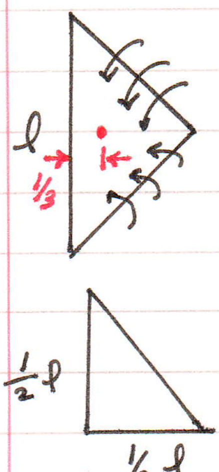
Yield Lines By Virtual Work

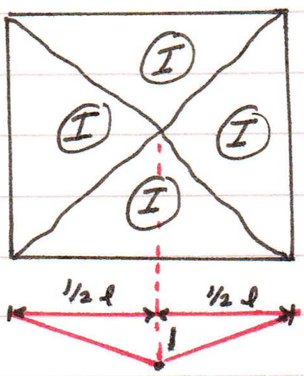
The main idea is

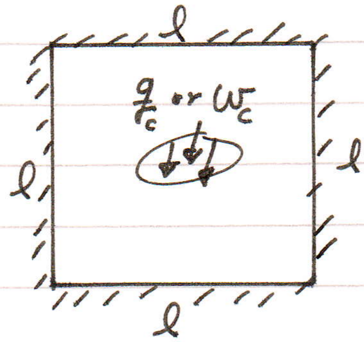
$$\underset{\text{External}}{\sum W.\delta} = \underset{\text{Internal}}{\sum M L \theta}$$

1. Example:

Calculate the collapse load (w_c or q_c) for the simply square slab.







$$\text{External work} = [q_c * \tfrac{1}{2} * \ell * \tfrac{1}{2}\ell] * \overset{\text{4 Parts}}{4} * \frac{1}{3}$$

$$= \frac{q_c \ell^2}{3}$$

$$\text{Internal work} = M * \ell * \underbrace{\frac{1}{\tfrac{1}{2}\ell}}_{\theta} * \underbrace{4}_{\text{4 parts}}$$

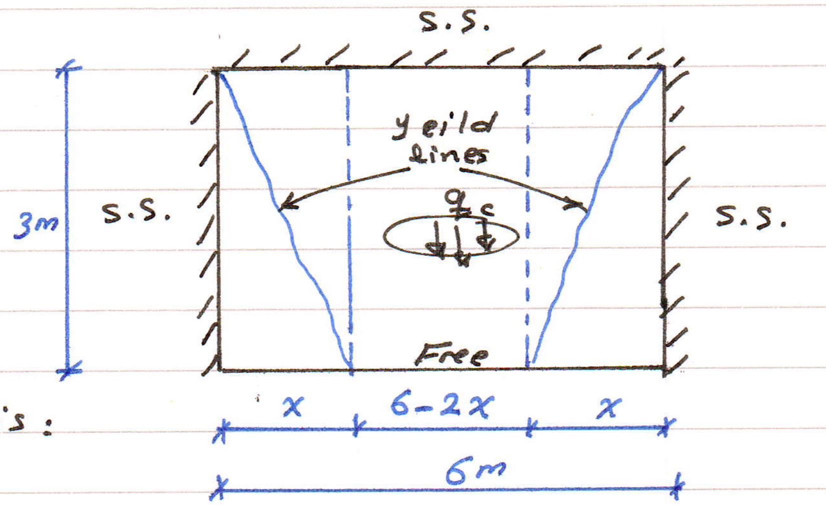
$$= 8M$$

Then

$$\frac{q_c \ell^2}{3} = 8M \Rightarrow \boxed{q_c = \frac{24M}{\ell^2}}$$

2. Example :

calculate the collapse load q_c ?



The main equation is :

$$\Sigma W\delta = \Sigma M L \theta$$

area (R) location No. of parts

first $\Sigma W\delta = q_c\left[\underbrace{\tfrac{1}{2} * x * 3}_{\text{area}} * \underbrace{\tfrac{1}{3}}_{\text{(R) location}} * \underbrace{2}_{\text{No. of parts}}\right]$

$$+ q_c\left[\tfrac{1}{2} * x * 3 * \tfrac{1}{3} * 2\right]$$

$$+ q_c\left[(6-2x) * 3 * \tfrac{1}{2}\right]$$

$$= q_c\,(9-x)$$

Then find

$$\Sigma M L \theta = M\left[3 * \tfrac{1}{x}\right] * 2$$

$$+ M\left[x * \tfrac{1}{3}\right] * 2$$

$$+ M\left[(6-2x) * \tfrac{1}{3}\right]$$

$$= M\left[\tfrac{6}{x} + 2\right]$$

External work = Internal work

$q_c\,(9-x) = M\left(\frac{6}{x} + 2\right)$, use $\frac{\partial q_c}{\partial x} = 0 \Rightarrow$ then find x ?

3. Example:

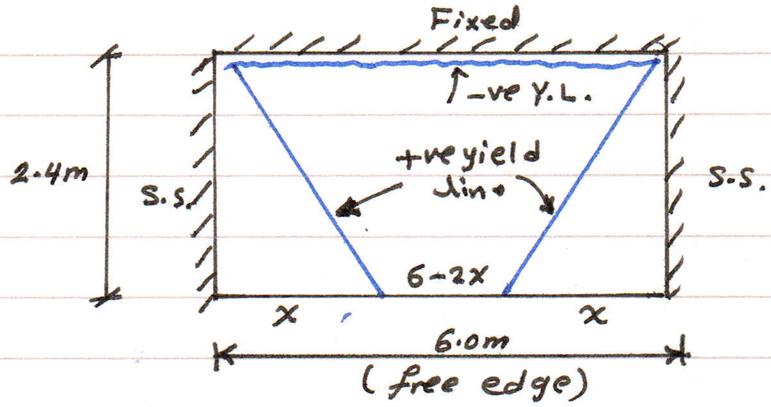
A rectangular concrete slab (2.4m * 6m). The slab thickness is 150mm, the average cover is 30 mm, For concrete $f'_c = 25$ MPa

For steel $f_y = 350$ MPa

$A_s^+ = \phi 12 @ 200$

$A_s^- = \phi 12 @ 200 + \phi 12 @ 400$ (add)

$$M_u = \phi \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right) b d^2 \quad ?$$



solution:

For M_u^+, $A_s^+ = \frac{(12)^2 \pi}{4} * 5 = 565 \text{ mm}^2$

$d = 150 - 30 = 120$ mm, $\rho = \frac{A_s}{bd} = \frac{565}{1000 * 120} = 0.0047$

$$M_u^+ = 0.9 * 0.0047 * 350 \left(1 - 0.59 * \frac{0.0047 * 350}{25}\right) * 1000 * (120)^2$$

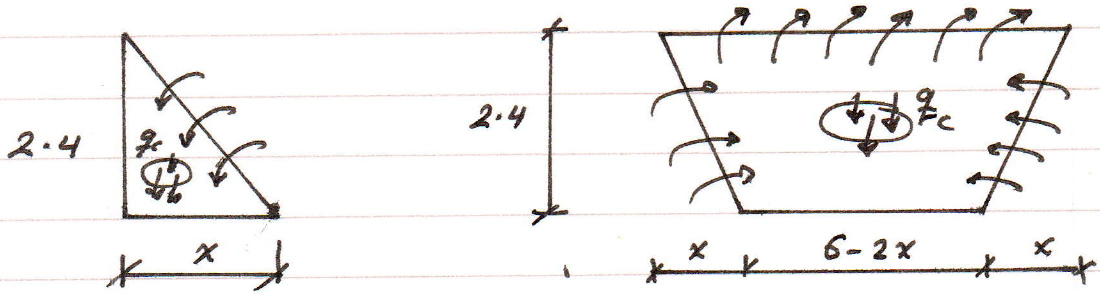
$M_u^+ = 20.5$ KN.m/m

For M_u^-, $A_s^- = 847.5 \text{ mm}^2 \rightarrow \rho = 0.00706$

Then,

$$M_u^- = 0.9 * 0.00706 * 350 \left(1 - 0.59 * \frac{0.00706 * 350}{25}\right) * 1000 * 120^2$$

$= 30$ KN.m/m



External work $= q_c\left[\left(\frac{1}{2} * x * 2.4 * (2) * \left(\frac{1}{3}\right)\right)\right.$

$+\left(\frac{1}{2} * x * 2.4 * (2) * \left(\frac{1}{3}\right)\right)$

$\left.+\left((2.4)(6-2x)\left(\frac{1}{2}\right)\right)\right]$

$= q_c\,[0.8x + 0.8x + 1.2(6-2x)]$

$= q_c\,[1.6x + 7.2 - 2.4x]$

$= q_c\,(7.2 - 0.8x)$

Internal work $= \Sigma\, M L \theta$

$= (2)\ \overset{M^+}{(20.5)}\ \overset{l}{(2.4)}\ \overset{\theta}{\left(\frac{1}{x}\right)}$

$+\ \overset{M^+}{(20.5)} * \overset{l}{(x)} * (2) * \left(\frac{1\theta}{2.4}\right)$

$+\ \overset{M^-}{(30)}\ \overset{l}{(6)}\ \left(\frac{1\theta}{2.4}\right)$

$= \frac{98.4}{x} + 17.08x + 75$

External work = Internal work

$q_c\,(7.2 - 0.8x) = \left(\frac{98.4}{x} + 17.08x + 75\right)$

$q_c = \frac{\left(\frac{98.4}{x} + 17.08x + 75\right)}{[7.2 - 0.8x]} = \frac{98.4 + 75x + 17.08x^2}{x\,[7.2 - 0.8x]}$

solve by try & error or by Newton-Raphson method:

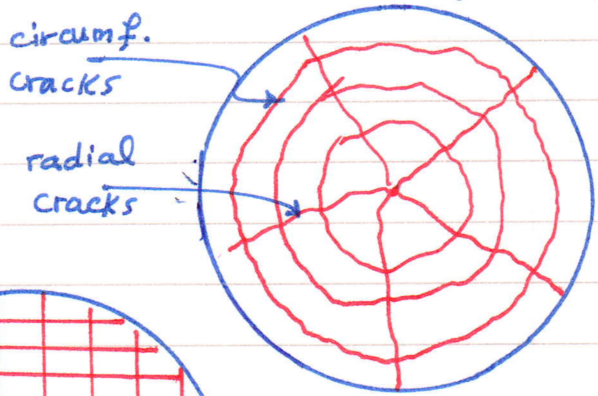
x	q_c
2	–
2.2	–

CIRCULAR SLABS

Circular slabs are generally used as base slabs for columns or cover slabs for circular storage tanks.

These slabs when loaded will deflect in the form of a dish. The slab at failure (or collapse) will develop circumferential and radial cracks.

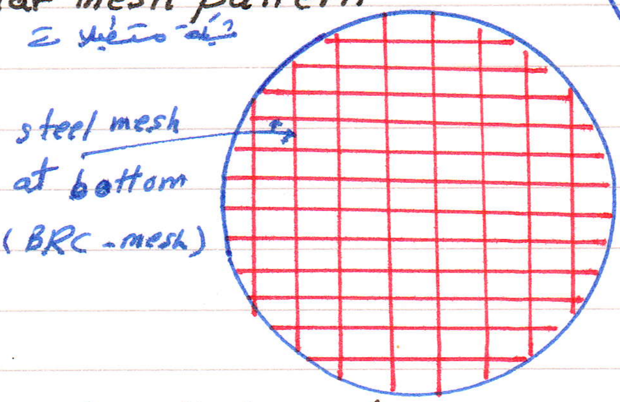
Therefore, reinforcement are needed (at right angles to these cracks).



Reinforcement detailing:-

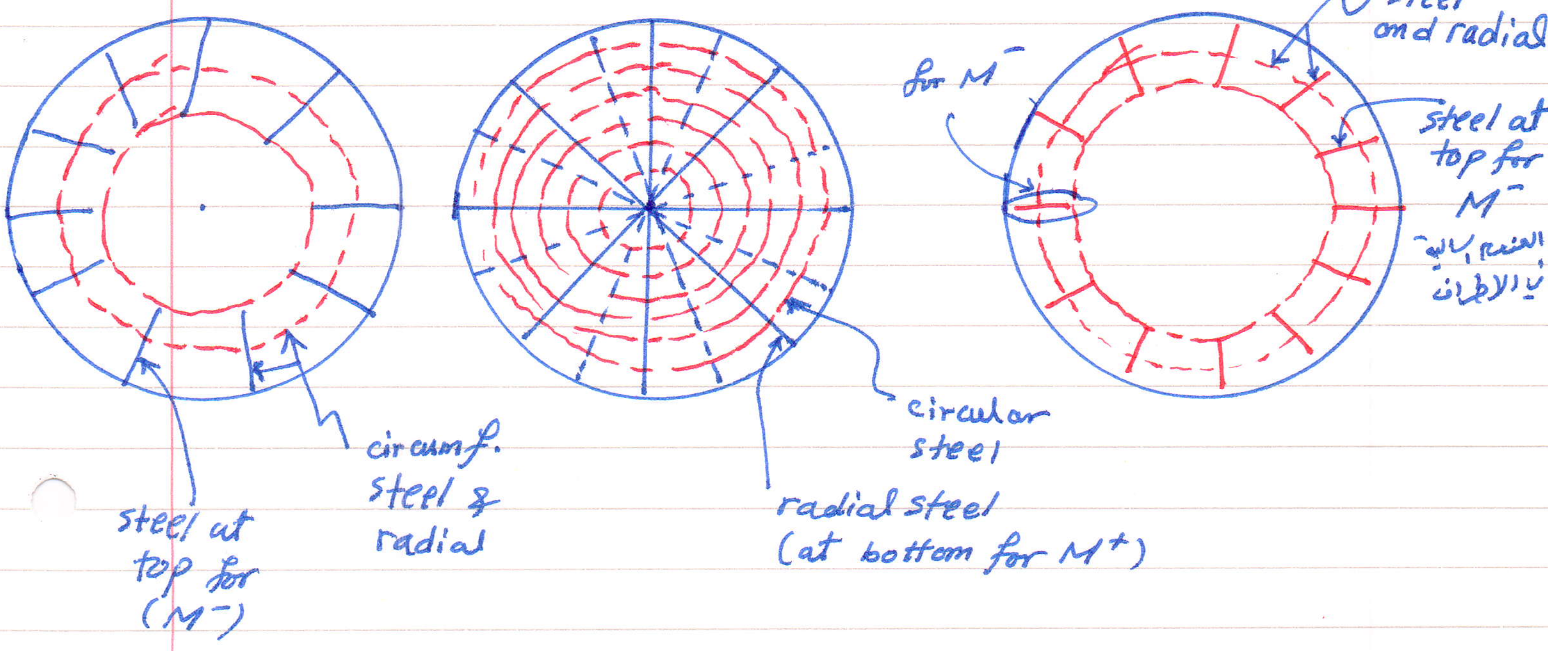
1. Rectangular mesh pattern

شبكة متعامدة =



من مزايا هذه الطريقة أنها سهلة التطبيق

2. Circular and radial mesh

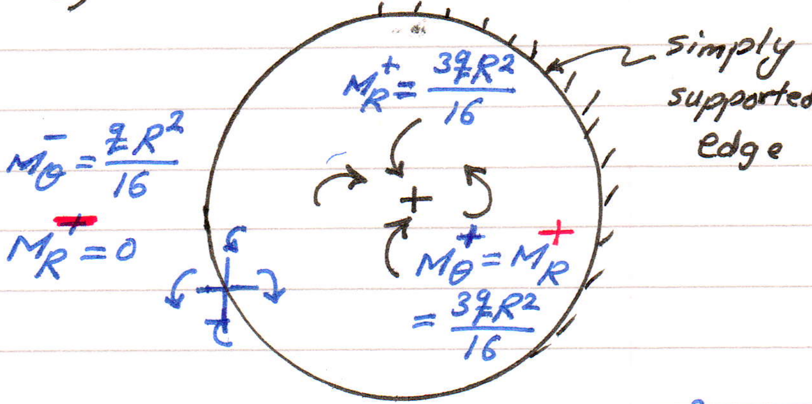


Bending Moment in Circular Slabs:-

Let q be the load per unit area (or w_u the ultimate load per unit area). Let also R be the radius of the slab. There are 2 cases:

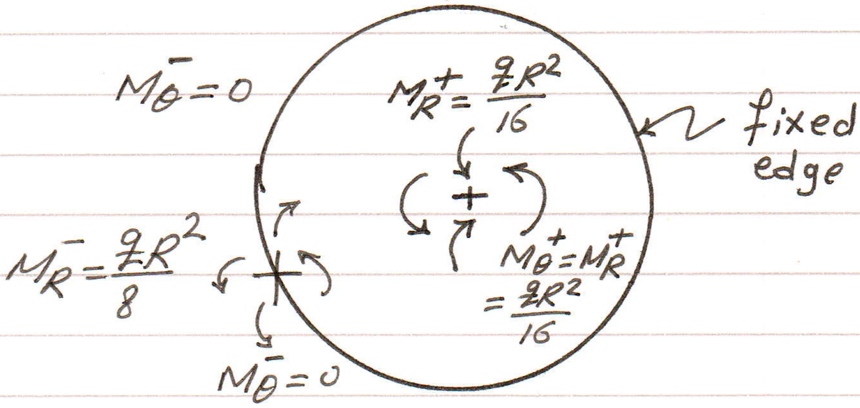
1. circular slab simply supported at the outer edge.

(Here M_R is in the radial plane while M_θ in the circumferential plane).



*(but use $M_\theta^- = \frac{1}{3} M_R^+ = \frac{qR^2}{16} = M_\theta^-$)

2. Circular slab with outer edge fixed.



(but use $M_\theta^- = \frac{1}{2} M_\theta^+ = \frac{qR^2}{32}$)

Examples:

1.

A circular slab is to be used as a cover for a circular area of radius 3.0m. The slab is assumed to have a simply supported edge. Take a slab thickness $h = \frac{2\pi R}{180}$

$= \frac{2\pi * 3000}{180} = 105\ mm$; for more safety use $h = 125\ mm$.

نزيد بسمك هنا لأن ال (deflection) عالي لأنه بسقف (s.s.)، ونفرضنا السمك، الـسقف (2-way) ؟

The imposed live load is $3 KN/m^2$. calculate the necessary reinforcements. For Concrete $f'_c = 25\ MPa$, $\gamma_c = 24.5\ KN/m^3$, for steel $f_y = 275\ MPa$. Use $\phi 10mm$ bars with clear cover 20mm.

Solution :-

effective cover $= 20 + 10$
$= 30\ mm$

effective depth $= 125 - 30$
$= 95\ mm$

The dead load

$w_d = 0.125 * 24.5$
$= 3.0625\ KN/m^2$

The live load $w_l = 3.000\ KN/m^2$ (given)

The ultimate load

$$w_u = 1.4 * 3.0625 + 1.7 * 3.000$$
$$= 9.3875\ KN/m^2$$

At center of slab, the positive bending moments are

$$M_R^+ = M_\theta^+ = \frac{3 w_u R^2}{16} = \frac{3 * 9.3875 * 3^2}{16} = 15.845\ KN.m/m \text{ (width)}$$

To find steel reinforcement, use $R = \frac{M_u}{\phi f'_c b d^2}$

$$R = \frac{15.841 * 10^6}{0.90 * 25 * 1000 * 95^2} = 0.0780$$

This gives $\omega = 0.082$

Thus

$$\rho = \omega f'_c / f_y = 0.082 * 25 / 275 = 0.007454$$

Steel area, $A_s = 0.007454 * 1000 * 95 = 708 \text{ mm}^2/\text{m (width)}$

Use $\phi 10\text{mm}$ @ 100 mm $\rightarrow A_s = 785 \text{ mm}^2/\text{m (width)}$

بالاتجاهين وبشكل مشبك مستطيل لتحقيق التسليح المطلوب

The negative bending moments at the outer edge:

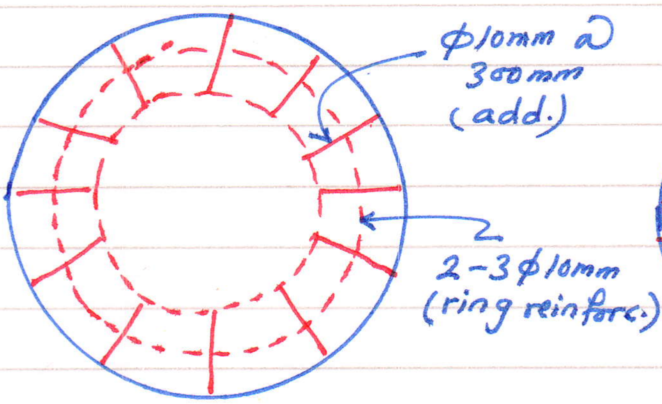
$$M_\theta^- \neq M_R^- = \frac{w_u R^2}{16} \qquad \left(M_\theta^- = \frac{w_u R^2}{16}\right)$$

0

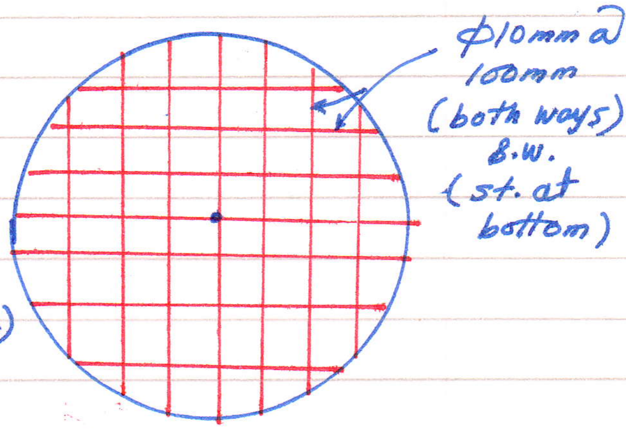
$$= \frac{9.3875 * 3^2}{16} = 5.280 \text{ KN.m/m (width)}$$

لاحظ أن قيمة العزم السالب هي: $(\frac{1}{3} M^+_{R,\theta})$ لذلك:

By proportions, use $\phi 10$ mm @ 300 mm ?



Top reinforcement (for M_R^- and M_θ^-)



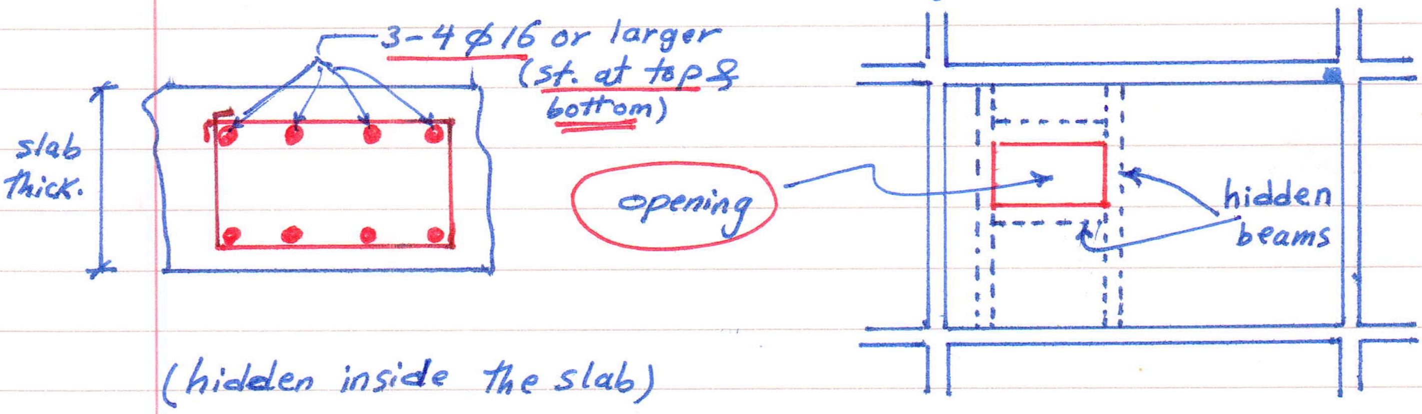
Bottom reinforcement (for M_R^+ & M_θ^+)

OPENINGS IN SLABS

1. Large openings:

Large openings are dangerous because these openings weaken the slab (especially openings at max. pos. B.M. at center or max. neg. B.M. at center of edges.

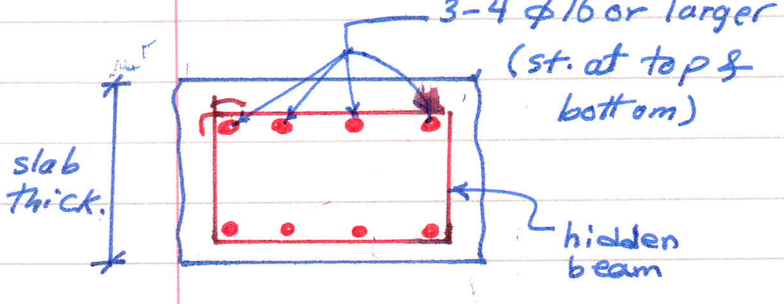
R.C. beams are needed at the edges of these openings to carry the loads and strengthened the slab. It is advised to extend these beams to the edges of the slab. (especially in short direction).

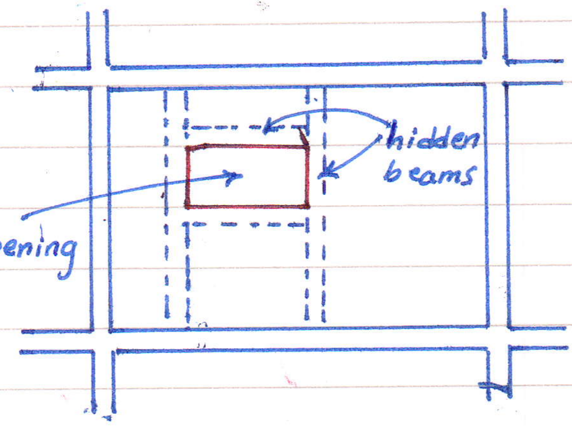


(hidden inside the slab)

2. Medium-size openings:

The slab is strengthened by hidden beams at the edge of the openings. Also it is advised to extend these hidden beams to the edges of the slab. (especially in the short direction). A hidden beam usually has 3 or 4 bars of size 16mm or 18mm (at top & bottom) & sometimes larger. stirrups are needed to hold these bars.





(hidden beam inside the slab)

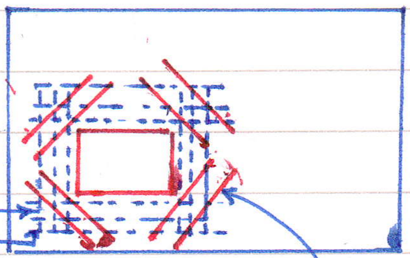
* ($t_{beam} = t_{slab}$)

3. Small openings:

In this case, the slab is strengthened by additional reinforcements (equal to the cut or intercepted bars). More bars may be needed normal to the diagonals at corners to prevent cracking at these corners.

حديد التعويض يوضع بنفس مكان الحديد المقطوع؟ عند الانشاء؟

Compensating reinforcement (at same position of intercepted bars)



Corner reinfor. (2 or 3 bars at top & bottom) (more bars)

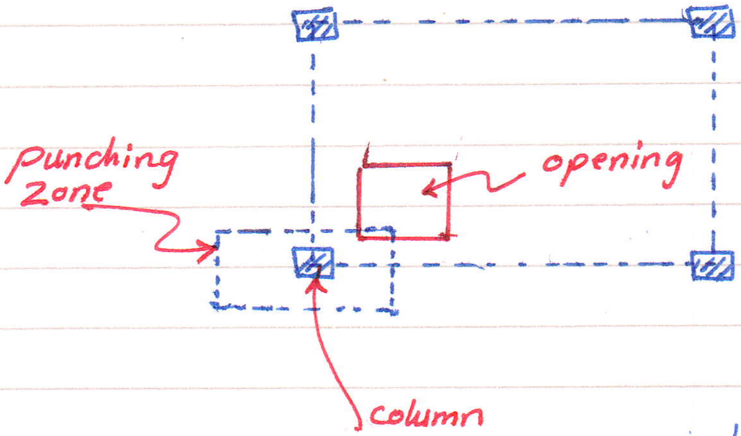
* التسليح عمودي على (diagonal) لتفادي التشققات التي تبدأ من الزوايا عادة ؟

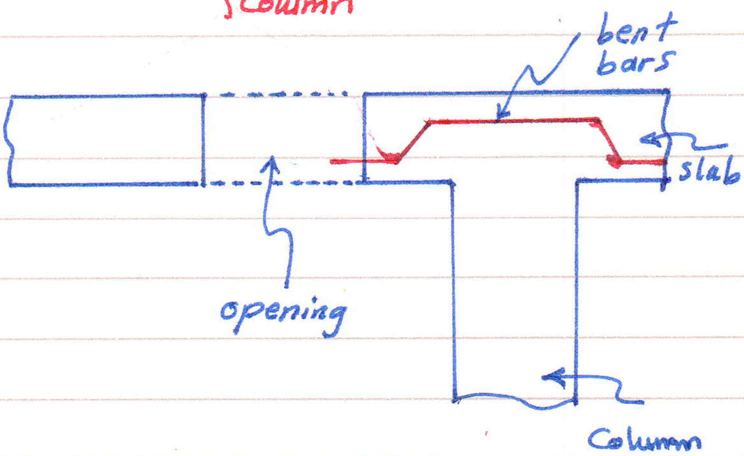
4. openings near columns in flat slabs:

Here, there is a danger of weaking the slab in punching shear (when the opening is in the punching zone).

For this case usually shear reinforcements over the columns are provided. These shear reinforcements are in the form of cross bars or bent bars.

* نستخدم حديد زاوية
أو حديد معوج للحصول على
تقوية للعمود لمنع إنفصال
لقرب الفتحة منه ؟



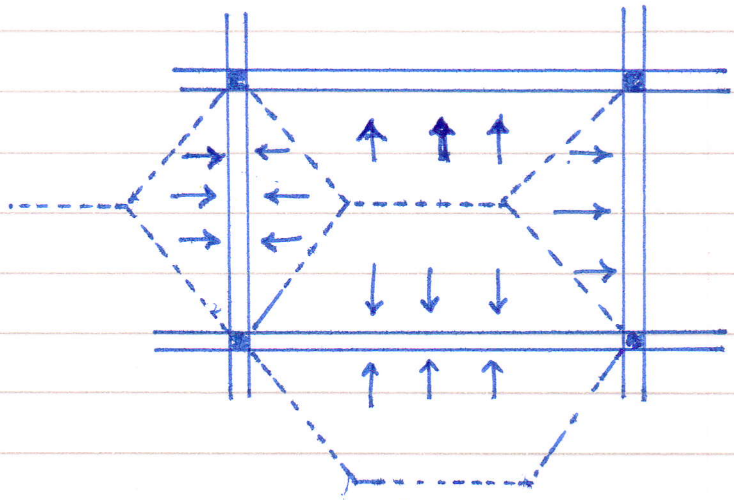


TRANSFER OF LOADS IN BUILDING FRAMES

A R.C. building is usually made from slabs, beams and columns. The main live loads and also dead loads are on slabs. The loads from the slabs are transferred to the beams and then from beams to columns. Finally the loads are transferred from columns to the foundations.

1. Transfere of loads from slabs to beams:-

The concept of yield lines is used. At collapse, the slab breaks into pieces and each piece will be carried by an edge beam. For rectangular slabs, the yield lines make 45° at corners. Thus the rectangular slab is divided into two triangles and two trapezoidals. The total load on ~~the~~ each beam is divided by the span (or length) of the beam to obtain a distributed load per unit length.



Example:

A building frame has rectangular slabs 4.8m * 7.2m. The slabs are supported on beams. The imposed live load is $6 KN/m^2$ and the estimated dead load $4.8 KN/m^2$. Calculate the transferred loads on the beams.

solution:

Take the beam of 7.2m span:

total live load is , $W_l = 2\left[6 * \frac{1}{2}(2.4 + 7.2) * 2.4\right]$

$= 138.24 KN$

The distributed live load on this beam is,

$$w_l = \frac{138.24}{7.2} = 19.2 \ KN/m \ (\text{length})$$

Total dead load on this beam is , $W_d = 2\left[4.8 * \frac{1}{2}(2.4 + 7.2) * 2.4\right]$

$= 110.592 KN$

The distributed dead load on this beam is

$$w_d = \frac{110.592}{7.2} = 15.36 \ KN/m \ (\text{length})$$

This beam has additional dead loads from its weight or weights of walls on it.

Next take the beam of span 4.8m, total live load on this beam is :

$$W_l = 2\left[6 * \frac{1}{2} * 4.8 * 2.4\right]$$

$= 69.12 KN$

The distributed live load on this beam is , $w_l = \frac{69.12}{4.8} = 14.4$ KN/m (length)

Total dead load from the slab to this beam is:

$$W_d = 2\left[4.8 * \left(\frac{1}{2} * 4.8 * 2.4\right)\right] = 55.298 \text{ kN}$$

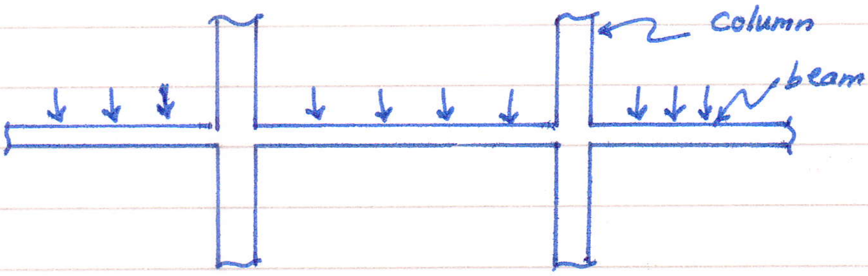
The distributed dead load on this beam is

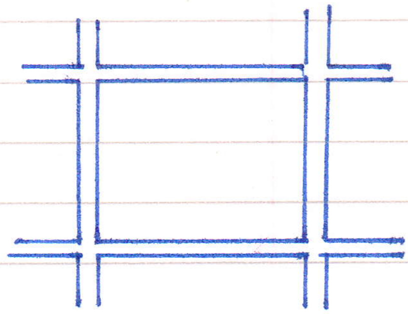
$$W_d = \frac{55.298}{4.8} = 11.52 \text{ kN/m (length)}$$

This beam has additional dead loads from its weight & weights of walls on it.

2. Transfer of loads from beams to Columns:-

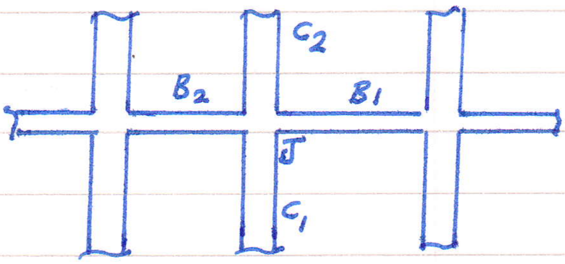
Usually the column will have axial load & bending moments. For the total axial load, the half loads from each beam is transferred to the column. Remember the column will take the loads from upper stories (or floors).





Bending moments on Columns:

The bending moments comes from the dead loads from the two beams on both sides and from the live loads from one beam on one side.



Dead load bending moments:

1. Calculate the fixed-end moments in beam on both sides of the column (due to the dead load w_d)

For beam B_1 $$M^F_{B_1} = \frac{w_d \cdot l_{B_1}^2}{12}$$

$$M^F_{B_2} = \frac{w_d \cdot l_{B_2}^2}{12}$$

The difference $(M^F_{B_1} - M^F_{B_2})$, is distributed at joint J according to $\frac{EI}{l}$ of the member.

* يوزع هذا الفرق في العزم بموجب $(\frac{EI}{l})$ والذي يحدث بسبب الميلان ؟

Thus for column C_1 (at top):-

$$M_{C_1} = \frac{(\frac{EI}{l})_{C_1}}{(\frac{EI}{l})_{C_1} + (\frac{EI}{l})_{C_2} + (\frac{EI}{l})_{B_1} + (\frac{EI}{l})_{B_2}} * (M^F_{B_1} - M^F_{B_2})$$

Similarly for column C_2 (at bottom):-

$$M_{C_2} = \frac{(\frac{EI}{l})_{C_2}}{(\frac{EI}{l})_{C_1} + (\frac{EI}{l})_{C_2} + (\frac{EI}{l})_{B_1} + (\frac{EI}{l})_{B_2}} * (M^F_{B_1} - M^F_{B_2})$$

Live load bending moment :-

العتبة الطويلة 1. put the live load on one beam (usually the larger beam).

2. Calculate the fixed-end moment at ends of this beam,

$$M_{B1}^{F} = \frac{w_{l} \cdot l_{B1}^{2}}{12}$$

3. Distribute this moment at joint J according to $\left(\frac{EI}{l}\right)$ of each member.

~~For column C_2 at bottom,~~ or take C_1 at top first,

$$M_{C_1} = \frac{\left(\frac{EI}{l}\right)_{C_1}}{\left(\frac{EI}{l}\right)_{C_1} + \left(\frac{EI}{l}\right)_{C_2} + \left(\frac{EI}{l}\right)_{B1} + \left(\frac{EI}{l}\right)_{B2}} * M_{B1}^{F}$$

For column C_2 at bottom,

$$M_{C_2} = \frac{\left(\frac{EI}{l}\right)_{C_2}}{\left(\frac{EI}{l}\right)_{C_1} + \left(\frac{EI}{l}\right)_{C_2} + \left(\frac{EI}{l}\right)_{B1} + \left(\frac{EI}{l}\right)_{B2}} * M_{B2}^{F}$$

The total moment on a column is the sum from the dead load and live load.

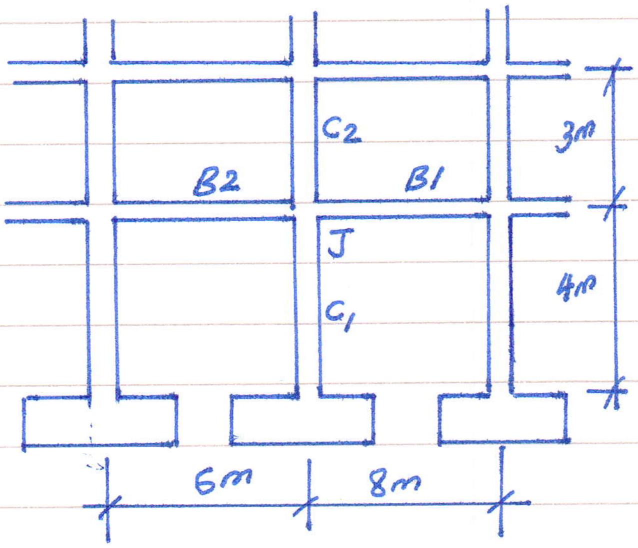
or we can write as , the difference is:

$$\Delta M_D = \left(\frac{w_D l^2}{12}\right)_{BR} - \left(\frac{w_D l^2}{12}\right)_{BL}$$ (from dead load)

and , $\frac{K_{member}}{\Sigma K_{Joint}}$, $E = E_c = 4700\sqrt{f'_c}$, $I = I_g = \frac{bh^3}{12}$

Example: (moments on column):

Calculate the total moments at top of column C_1 & bottom of column C_2. Here dead loads on beam $w_d = 12$ KN/m, live loads on beam $w_l = 15$ KN/m, column sections are 300 x 400 mm. Beam sections are 300 * 600 mm?



$$I_{column} = \frac{1}{12}(300)(400)^3 \quad *$$

$$I_{beam} = \frac{1}{12}(300)(600)^3$$

- نضع (load حمل) بإتجاه العقب الكبيرة للحصول على أكبر عزم
- لاحظ حساب قيمة (I) للحصول على أكبر (I) ممكن.

Example:

Dead loads on beams (32 KN/m), and live load (26 KN/m), calculate the ~~beam~~ moments on the interior column. Try exterior column. Find axial load?

solution:

F.E.M from dead load is,

$$(M_D)_{\substack{right \\ (R)(beam)}} = \frac{32(8)^2}{12} = 170.667 \text{ KN.m}$$

$$(M_D)_{BL} = \frac{32(6)^2}{12} = 96 \text{ KN.m}$$

$\therefore$ The difference between moments,

$$\Delta M_D = 170.667 - 96 = 74.667 \text{ KN.m}$$

F.E.M from W_l :

longer beam

$$(M_L)_{BR} = \frac{26(8)^2}{12} = \underline{138.667}\ KN.m$$

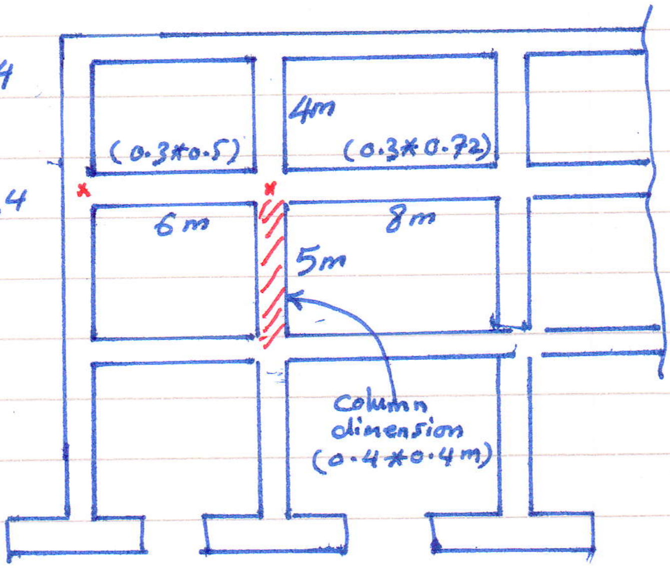
take the longer beam

$$I_{BR} = \frac{0.3(0.72)^3}{12} = 0.009331 m^4$$

$$I_{BL} = \frac{0.3(0.5)^3}{12} = 0.003125\ m^4$$

$$I_{C_B} = I_{C_T} = \frac{0.4(0.4)^3}{12}$$

$$= 0.002133 m^4$$



Then, moment on column C_B from W_D,

ΔM_D

$$(M_D)_{CB} = \frac{\frac{0.002133}{5}}{\left(\frac{0.002133}{5}\right)+\left(\frac{0.002133}{4}\right)+\left(\frac{0.009331}{8}\right)+\left(\frac{0.003125}{6}\right)} * (74.667)$$

$$= 12.033\ KN.m$$

$\therefore$ The moment on column C_B from W_l :

column (5m) — $(M_L)_{BR}$ (longer beam)

$$(M_L)_{CB} = \frac{K_{member}}{\Sigma K_{Joint}} * \underline{138.667} = 22.356\ KN.m$$

$$\therefore\ M_{C_B} = \underbrace{12.033}_{\text{from dead load}} + \underbrace{22.348}_{\text{from live load}} = 34.381\ KN.m$$

Exterior column :

$$\text{F.E.M from } W_D,\ (M_D)_{BR} = \frac{32(6)^2}{12} = 96\ KN.m$$

$$(M_D)_{CB} = \frac{\left(\frac{0.002133}{5}\right)}{\left(\frac{0.002133}{5}\right)+\left(\frac{0.002133}{4}\right)+\left(\frac{0.003125}{6}\right)} * 96 = 27.659\ KN.m$$

F.E.M. from w_ℓ :

$$(M_L)_{BR} = \frac{26(6)^2}{12} = 78 \ KN.m$$

$$(M_L)_{CB} = \frac{K_{member}}{\sum K_{Joint}} (78) = 22.498 \ KN.m$$

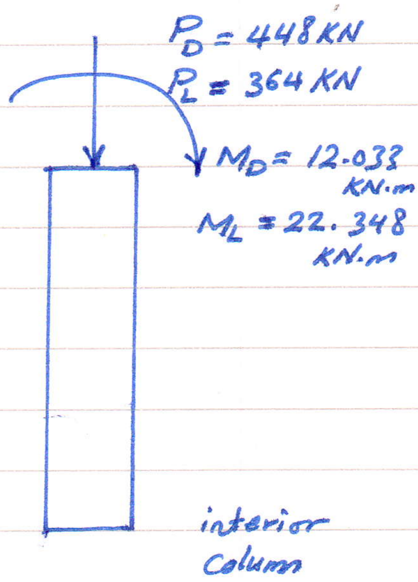
$$M_{CB} = 27.659 + 22.473$$
$$= 50.132 \ KN.m$$

axial load :-

from W_D :

$$P_D = 32 * \left(\frac{6}{2}\right) + 32 * \left(\frac{8}{2}\right)$$
$$= 224 \ KN$$
$$\therefore P_D = 2 * (224) = 448 \ KN$$

from W_ℓ : $P_L = 26 * \left(\frac{6}{2}\right) + 26 * \left(\frac{8}{2}\right)$
$$= 182 \ KN$$
$$\therefore P_L = 2 * (182)$$
$$= 364 \ KN$$

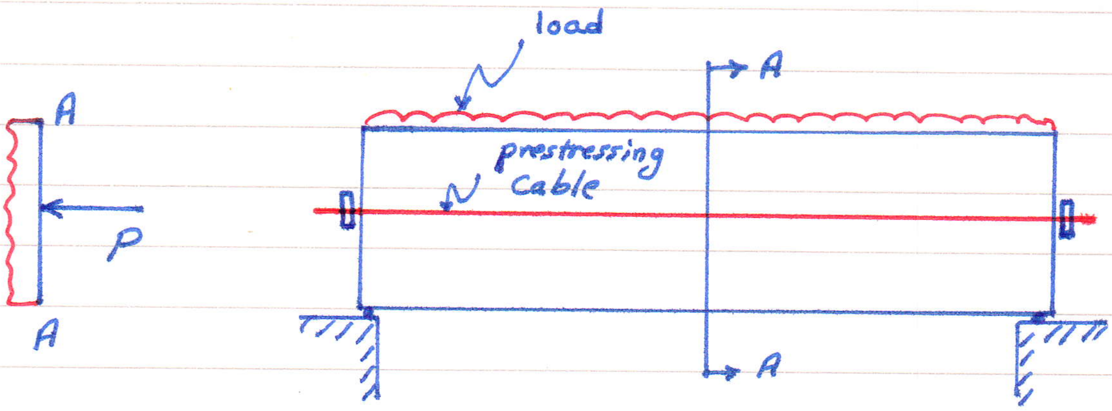


PRESTRESSED CONCRETE

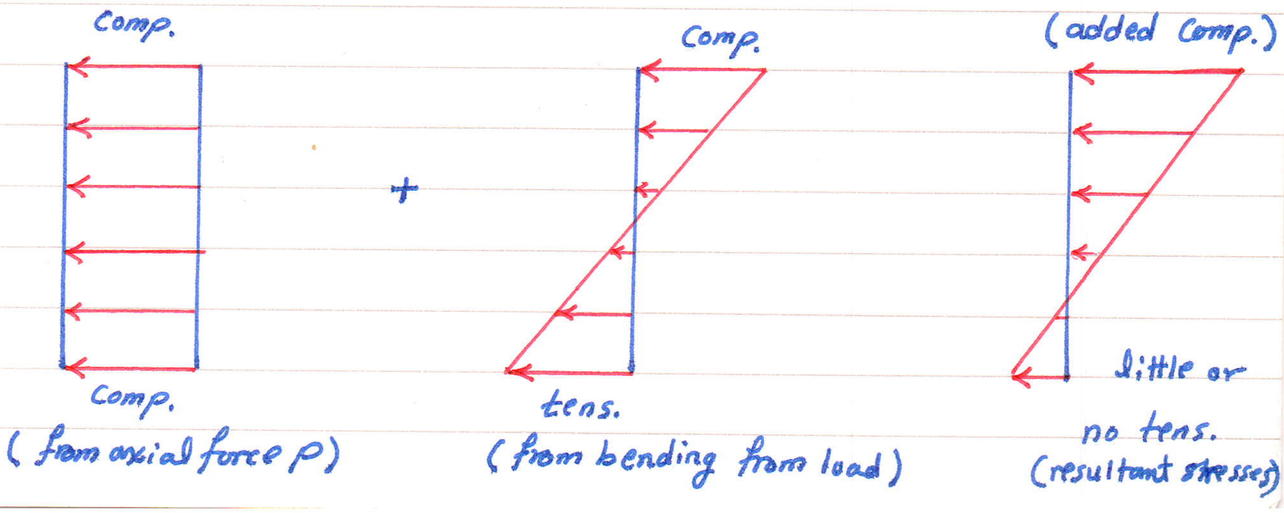
The concept of prestressing of concrete:

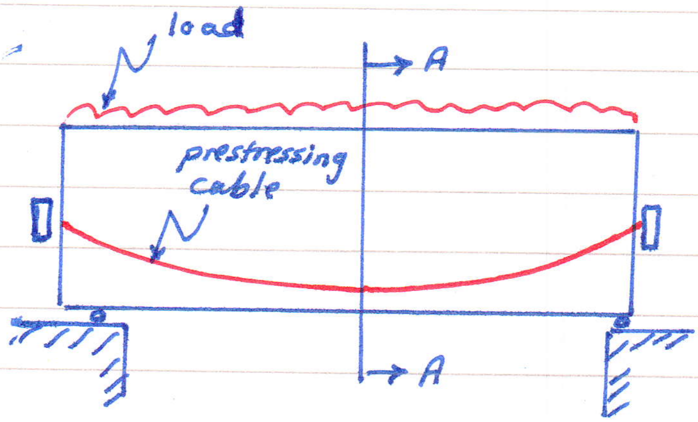
Concrete is strong in compression but weak in tension. Therefore steel reinforcements are used to take the tension in concrete structures. This is the concept of reinforced concrete.

It is possible to apply stresses to the concrete opposite or almost opposite to the stresses from loads. The resulting stresses will have little or no tension. This is the concept of prestressed concrete.

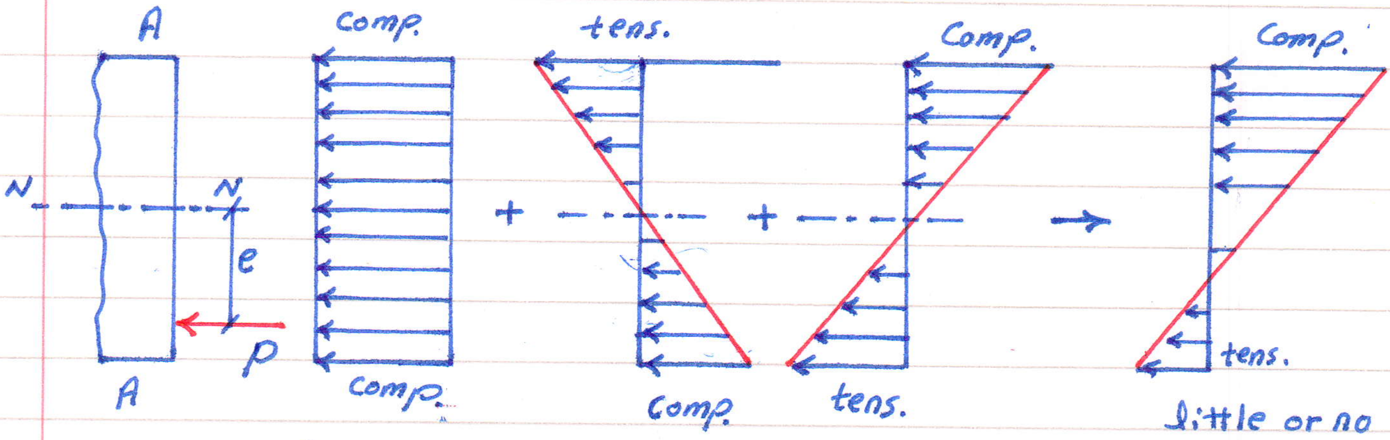


(simply supported beam with central prestressing cable)





(simply supported beam with eccentric prestressing cable)



(from axial force P) (from bending moment - P.e) (from bending - from loads) little or no tens. (resultant stresses)

Note:- Prestressed concrete requires, high tensile steel (HTS) with ultimate strength $f_{pu} > 1000\,MPa$.

This steel has no clear yield stress. Also high strength concrete (HSC) with $f'_c > 40\,MPa$ is used.

Advantages :-

1. The full concrete section is effective in resisting the external loads. Therefore, smaller sections (and so lighter beams) can be used.

2. The beams are stronger than identical beams with ordinary reinforcement.

3. The beams will have smaller deflections than ordinary R.C. beams. This is because prestressing produces opposite deflections to the loads.

4. Most precast beams are prestressed.

Dangers :

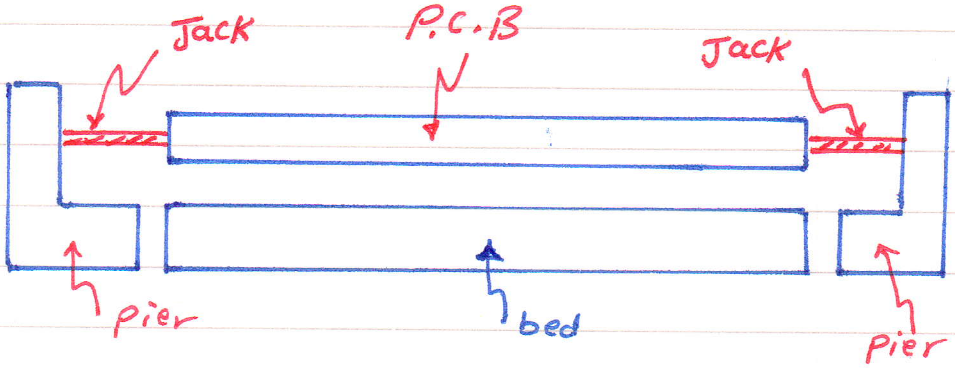
1. Lack of control.

2. Corrosion of prestressing steel (by chlorides).

تردّي 3. Deterioration of concrete (by sulphates)

Methods of Prestressing:-

First attempt was by the use of jacks for a applying compressive forces to simply supported beams (used in 1930 in France)



Nowadays, prestressing is made in factories under good control.

Generally, there are two methods of prestressing.

1. Pretensioning:

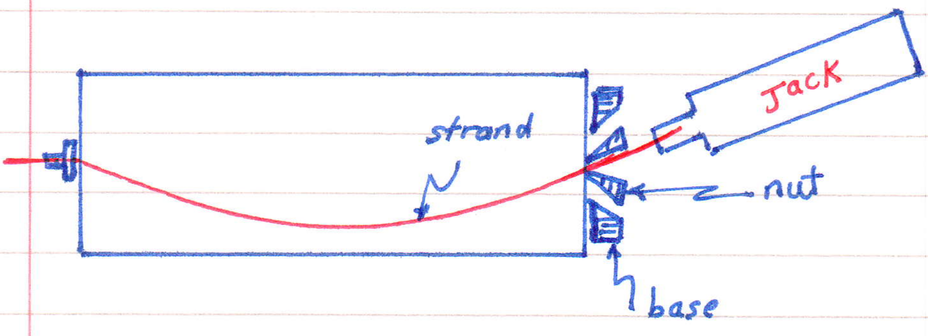
Here the prestressing steel wires are tensioned while in the mould before concrete is cast. The wires remain tensioned until the concrete is hardened. Then the tension is released and goes to compression in the concrete.

This method is usually applied to small prestressed concrete beams such as lintels (over openings in walls) & نائمة تحت سكة الحديد sleepers (under rail tracks).

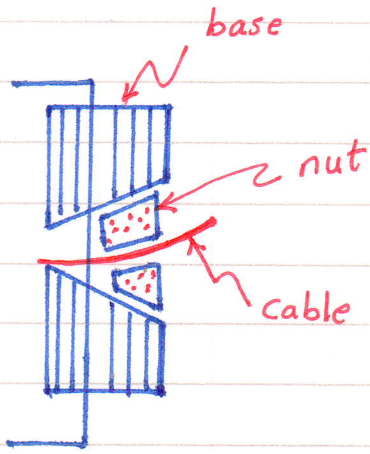
2. Post-tensioning:

Here the concrete is cast and allowed to become hardened. Ducts are provided inside the beam for inserting the prestressing cables (or strands).

The prestressing is applied to the concrete beam by these Cables. The prestressing Cable is fixed by proper anchorage at one end, jacks are used at the end to apply tension forces. Then the jacks are released and the tension in the cables will go to compression to the concrete. This method is used for large prestressed Concrete beams (such as girders).



(Anchorage)



(Anchorage)

Losses of Prestressing forces :-

There are several factors which cause losses or reduction in the prestressing forces.

1. slip at the transfer: (انزلاق الانتقال من Jack ← إلى الكونكريت)

The prestressing forces is applied by jacks to the Cables. when these cables are released from jacks, Certain slips (although very small) will occure. This slip will reduce the prestressing force. هنا البسيطة مهمة "جداً" لمعرفة القوة الحقيقية المنتقلة إلى الكونكريت

2. Elastic shortening of concrete:

when the cables are released from Jacks, the concrete will be in compression (especially round the cables). Thue this concrete will have compressive strains and becomes shorter. The steel also becomes

shorter. Thus some prestressing force is lost.

3. Shrinkage in concrete:

Concrete (like clay) will shrink when loosing moisture (or water). There are two types of shrinkage in Concrete:-

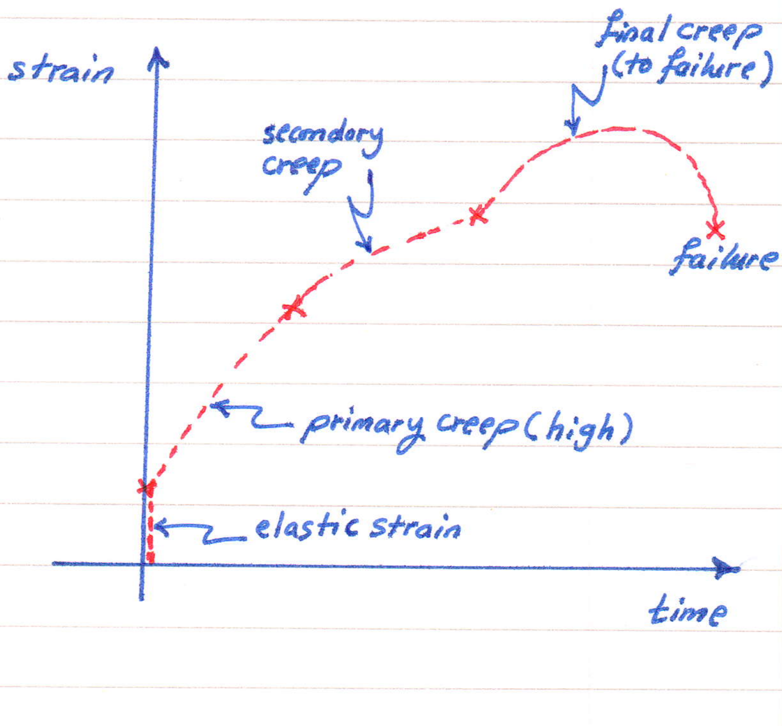
i) plastic shrinkage (when the concrete is still plastic in mould).

ii) Drying shrinkage (when the concrete is hardened & out of the mould.

In prestressed concrete, drying shrinkage is, important. This shrinkage will make the concrete shorter & also the steel cables. Thue more or some prestressing force is lost.

4. Creep of concrete:

The creep of a material is the increase of strain with time when the material is under constant stress.



The concrete under compressive prestressing force will have creep. This creep will make the concrete shorter & shorter with time. The steel also becomes shorter, thus some prestressing force is lost with time.

5. Relaxation in the prestressing steel:

Relaxation in a material is the decrease in stress with time when the material is kept under constant strain.

The high prestressing force in steel cables will become smaller after some time (by relaxation).

Note: All these losses can be calculated properly. However, they are 10 – 20% of the prestressing force from jacks.

Analysis of Prestressed Concrete Beams

Only simply supported beams are considered. Most prestressed concrete beams are simply supported. Analysis means the calculations of stresses in prestressed concrete beams (in different situations). These are 4 different situations or stages.

Notes:

Here the full section is used in the analysis because the concrete is not cracked & also the full section takes the stresses. The area of steel is usually small. Therefore the gross section is used for the area A & 2nd moment of the area I, (Not the transformed section used), The neutral axis passes through the centroid of the section. Also here:

tensile stresses are positive.
Compressive stresses are negative.

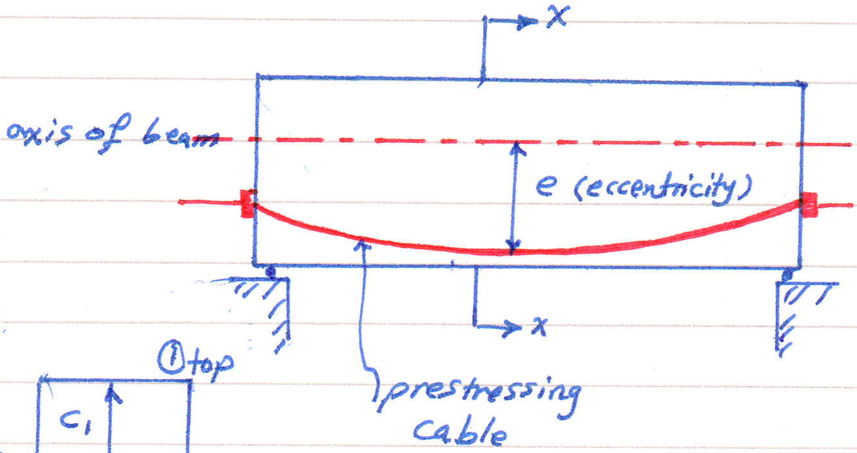
For the strength of concrete use:

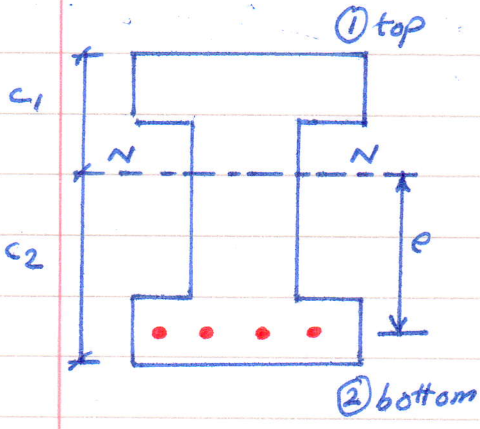
f'_{ci} initial strength (befor 28 days)

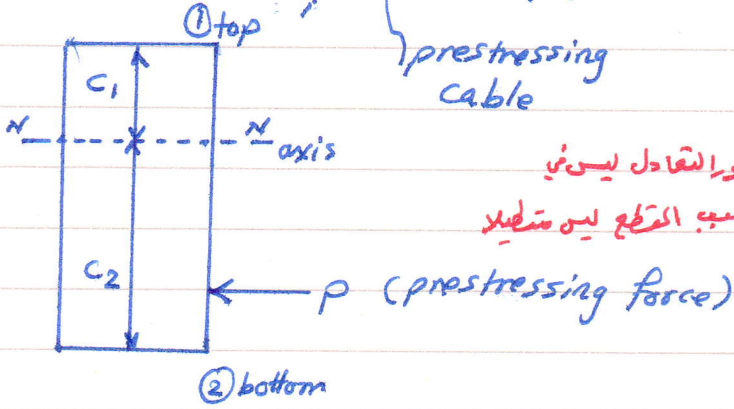
f'_c final " (after 28 days)

usually $f'_{ci} < f'_c$

لاحظ امتداد الكيبل عكس B.M.
الناتج عن الأحمال







* لاحظ محور التعادل ليس في
المنتصف بسبب القطع ليس متماثلا
$c_1 \neq c_2$

* العتبات المجهدة مسبقاً الإجهاد
(sec-(I)) لأن الكونكريت يكون غير فعال
في الوسط؟

The Stages:

1. Transfere of prestressing force from jacks to concrete. Here the initial prestressing force P_i is used ($P_i < P_J$ from jacks). بسبب الانزلاق

Also f'_{ci} for concrete is used ($f'_{ci} < f'_c$).
At top face 1 (in a simply supported beam):

$$f_{1P_i} = \underbrace{-\frac{P_i}{A}}_{\text{axial}} + \underbrace{\frac{\overset{(M)}{P_i \cdot e}}{I} \cdot c_1}_{\text{bending}}$$

f = stress
1 = top
P = prestressed
i = initial

$$f_{2P_i} = \underbrace{-\frac{P_i}{A}}_{\text{axial}} - \underbrace{\frac{P_i \cdot e}{I} \cdot c_2}_{\text{bending}}$$

− comp., + tens.

Check by ACI code:

$$f_{1p_i} < 0.25\sqrt{f'_{ci}} \quad \text{(tensile)}$$

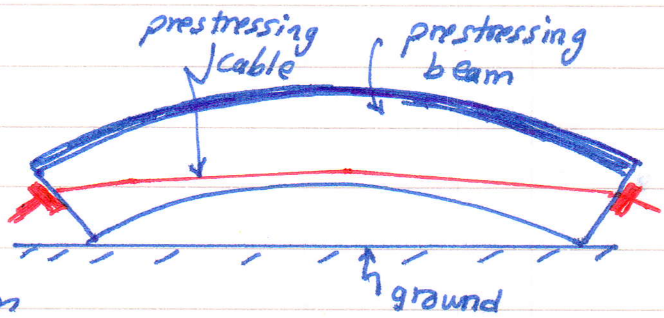
* مرحلة مهمة لغرض تدقيق الإجهادات ومقارنتها مع المواصفات الأمريكية

$$f_{2p_i} < 0.6\, f'_{ci} \quad \text{(compressive)}$$

2. Initial prestressing force & the selfweight of beam:

يستريح بكل طوله على الارض

Originally the beam rests fully on the ground. The prestressing force will produce negative bending moments in the beam. So, the beam will bend upwards & rest only at ends.



Let

وزن العتبة لوحدة الطول

$$w_g = \gamma_c \cdot A$$

وزن العتبة لوحدة الحجم — مساحة — وزن لوحدة الطول

be the selfweight of beam (per unit length). The maximum bending moment (at mid-span) is:-

$$M_g = \frac{w_g \cdot l^2}{8}$$

This bending moment will produce additional stresses.

At top face 1:

$$f_{1g_i} = f_{1p_i} - \frac{M_g}{I} \cdot c_1$$

At bottom face 2:

$$f_{2g_i} = f_{2p_i} + \frac{M_g}{I} \cdot c_2$$

* هذه المرحلة تقلل الإجهادات (الخطورة فيها قليلة)

3. Effective prestressing force & the selfweight:

After some time, losses in the prestressing force will occur. Thus the effective prestressing force (P_e) must be used ($P_e < P_i$). Also concrete will have f'_c (where $f'_c > f'_{ci}$).

At top face 1:

$$f_{1ge} = [-\underbrace{\frac{P_e}{A}}_{\text{axial from } P_e} + \underbrace{\frac{P_e \cdot e}{I} \cdot c_1}_{\text{bending from } P_e}] - \underbrace{\frac{M_g}{I} \cdot c_1}_{\text{bending from } w_g}$$

At bottom face 2:

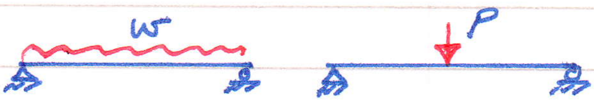
$$f_{2ge} = [-\underbrace{\frac{P_e}{A}}_{\text{axial from } P_e} - \underbrace{\frac{P_e \cdot e}{I} \cdot c_2}_{\text{bending from } P_e}] + \underbrace{\frac{M_g}{I} \cdot c_2}_{\text{bending from } w_g}$$

* هذه المرحلة - أيضاً غير خطرة ؟

4. Effective prestressing force & the total service loads:

Here, the beam will be in service. The total service load is:

$$w_s = \underbrace{w_d}_{\text{dead}} + \underbrace{w_l}_{\text{live}}$$



The maximum bending moment (at mid-span in a simply supported beam) is:

$$M_s = \frac{w_s \cdot l^2}{8}$$

This produces stresses:

لاحظ الاشارات متعاكسة ؟

At top face 1:

$$f_{1se} = \left[-\frac{P_e}{A} + \frac{P_e \cdot e}{I} \cdot c_1\right] - \frac{M_s}{I} \cdot c_1$$

At bottom face 2:

$$f_{2se} = \left[-\frac{P_e}{A} - \frac{P_e \cdot e}{I} \cdot c_2\right] + \frac{M_s}{I} \cdot c_2$$

Check by ACI code:

$f_{1se} < 0.45 f'_c$ (compressive)

* مرحلة خطرة ؟

$f_{2se} < 0.5\sqrt{f'_c}$ (tensile)

Examples:-

1. A pretensioned cantilever of length 1.8m & rectangular section 350mm * 120mm is used for projecting stair steps. The initial prestressing force is $P_i = 108$ kN & the effective prestressing force is $P_e = 92$ kN (after all losses). The added service load is 3.15 kN/m (not including the self weight) $f'_{ci} = 30$ MPa, $f'_c = 41$ MPa, check the stresses in all stages?

Solution:-

Take the fixed end. Use 1 for the bottom face & 2 for the top face.

$A = 42 * 10^3\ mm^2 \quad (A = bh)$

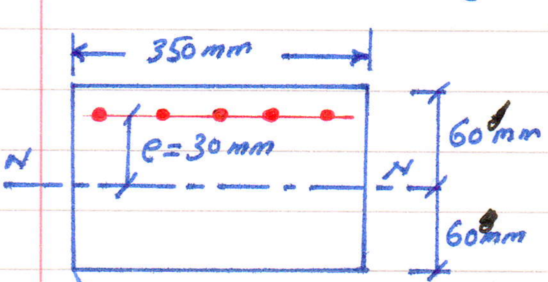
$I = 50.4 * 10^6\ mm^4 \quad (I = \frac{1}{12} b h^3)$

$c_1 = c_2 = \frac{h}{2} = 60mm$

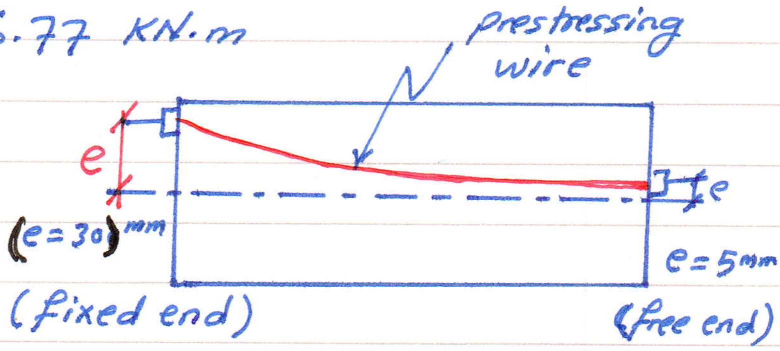
$w_g = 1.029$ kN/m

$M_g = \frac{1}{2} w_g \ell^2 = 1.667 \text{ kN.m}$

$M_s = \frac{1}{2}(w_a + w_g)\ell^2 = 6.77 \text{ kN.m}$



(x-section at the fixed end)



stage 1: $f_{1p_i} = 1.286$ MPa (tens.)

$f_{2p_i} = -6.429$ MPa (comp.)

Check: $0.25\sqrt{f'_{ci}} = 1.369$ MPa (tens.)

$0.6 f'_{ci} = 18$ MPa (comp.)

Both o.K.

stage 2: $f_{1g_i} = -0.699$ MPa (comp.)

$f_{2g_i} = -4.444$ MPa (comp.)

stage 3: $f_{1ge} = -0.889$ MPa (comp.)

$f_{2ge} = -3.491$ MPa (comp.)

stage 4: $f_{1se} = -6.964$ MPa (comp.)

$f_{2se} = 2.584$ MPa (tens.)

Check:

$0.45 f'_c = 18.45$ MPa (comp.)

$0.5\sqrt{f'_c} = 3.202$ MPa (tens.)

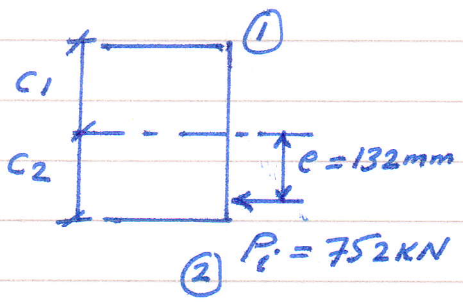
(Both o.K.)

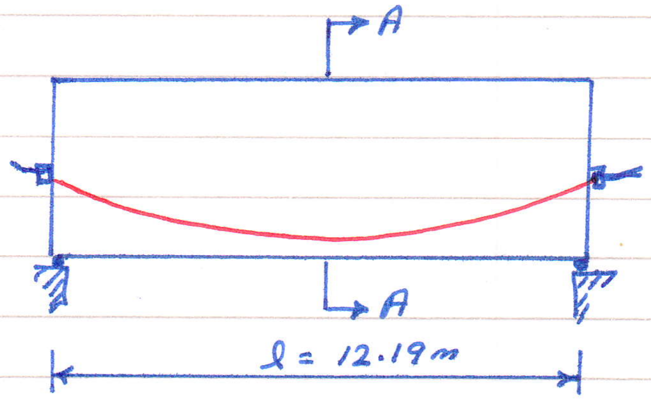
2. Ex: (Prestressed Concrete beam).

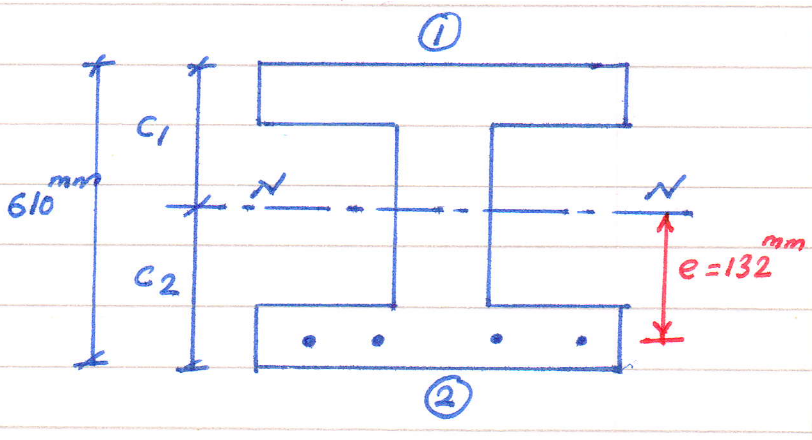
A simply supported prestressed concrete beam of span $l = 12.19\,m$ & I-section of area $A = 114 * 10^3\,mm^2$ & 2nd moment of area $I = 4.99 * 10^9\,mm^4$, the I-section is symmetric, $c_1 = c_2 = \frac{h}{2} = 305\,mm$, self weight of beam $w_g = 2.67\,KN/m$.

self weight of beam $w_g = 2.67\,KN/m$
Added service load $= 8.02\,KN/m$
strength of concrete $f'_{ci} = 32.5\,MPa$
$f'_c = 42\,MPa$
prestressing force $P_i = 752\,KN$
$P_e = 639\,KN$

Eccentricity at mid span $e = 132\,mm$
check the stresses at all four stages
This example is from the book by Winter.







solution: stage 1: (Transfer of prestressing force from jack to concrete)

At top : $f_{1P_i} = -\frac{P_i}{A} + \frac{P_i \cdot e}{I} \cdot c_1$

$= -\frac{752*10^3}{114*10^3} + \frac{752*10^3*132}{4.99*10^9} * 305$

-0.529

$= -0.572$ MPa (comp.)

At bottom : $f_{2P_i} = -\frac{P_i}{A} - \frac{P_i \cdot e}{I} \cdot c_2$

$= \frac{-752*10^3}{114*10^3} - \frac{752*10^3*132}{4.99*10^9} * 305$

$= -12.664$ MPa (comp.)

check by ACI code:

$f_{1P_i} < 0.25\sqrt{f'_{ci}}$ (tensile)

$0.25\sqrt{f'_c} = 0.25\sqrt{32.5}$

$= 1.425$ MPa (ten.)

$f_{2P_i} < 0.6 f'_{ci}$ (comp.)

$0.6 * 32.5 = 19.5$ MPa (comp.)

Both o.k.

Stage 2 (initial prestressing force + selfweight)

Here the beam bends upwards and will be simply supported at ends.

At mid-span : $M_g = \frac{w_g \cdot l^2}{8} = \frac{2.67*(12.19)^2}{8} = 49.594$ kN.m

$= 49.594*10^6$ N.mm

At top : f_{1g}

$= f_{1P_i} - \frac{M_g}{I} \cdot c_1$

$= -0.572 - \frac{49.594*10^6}{4.99*10^9} * 305$

$= -3.603$ MPa (comp.)

At bottom:

$$f_{2gi} = f_{2pi} + \frac{M_g}{I} . c_2$$

$$= -12.666 + \frac{49.594*10^6}{4.99*10^9} * 305$$

$$= -9.635 \text{ MPa (comp.)}$$

Stage 3: (effective prestressing force + self weight)

At top : $$f_{1ge} = \left[-\frac{P_e}{A} + \frac{(P_e . e)}{I} . c_1 \right] - \frac{M_g}{I} . c_1$$

$$= -\frac{639*10^3}{114*10^3} + \frac{639*10^3*132}{4.99*10^9} *305 - \frac{49.594*10^6}{4.99*10^9} * 305$$

$$= -3.481 \text{ MPa (comp.)}$$

At bottom : $$f_{2ge} = \left[-\frac{P_e}{A} - \frac{P_e . e}{I} . c_2 \right] + \frac{M_g}{I} . c_2$$

$$= \frac{-639*10^3}{114*10^3} - \frac{639*10^3*132}{4.99*10^9} * 305 + \frac{49.594*10^6}{4.99*10^9} * 305$$

$$= -7.730 \text{ MPa (comp.)}$$

Stage 4: (effective prestressing force + total service load).

total service load is,

$$w_s = \underbrace{2.67}_{\text{beam}} + \underbrace{8.02}_{\text{added}}$$

$$= 10.69 \text{ KN/m}$$

At mid-span,

$$M_s = \frac{w_s . l^2}{8} = \frac{10.69*(12.19)^2}{8} = 198.561 \text{ KN.m}$$

At top:

$$f_{1se} = \left[-\frac{P_e}{A} + \frac{P_e \cdot e}{I} \cdot c_1\right] - \frac{M_s}{I} \cdot c_1$$

$$= \frac{639*10^3}{114*10^3} + \frac{639*10^3*132}{4.99*10^9}*305 - \frac{198.651*10^6}{4.99*10^9}*305$$

$$= -12.586 \text{ MPa (comp.)}$$

At bottom:

$$f_{2se} = \left[-\frac{P_e}{A} - \frac{P_e \cdot e}{I} \cdot c_2\right] + \frac{M_s}{I} \cdot c_2$$

$$= \frac{-639*10^3}{114*10^3} - \frac{639*10^3*132}{4.99*10^9}*305 + \frac{198.561*10^6}{4.99*10^9}*305$$

$$= 1.376 \text{ MPa (tens.)}$$

Check by ACI code : $f_{1se} < 0.45 f'_c$ (comp.)

$$0.45 f'_c = 0.45*42$$
$$= 18.90 \text{ MPa} \quad \text{(o.K.)}$$

$$f_{2se} < 0.5\sqrt{f'_c} \quad \text{(tens.)}$$

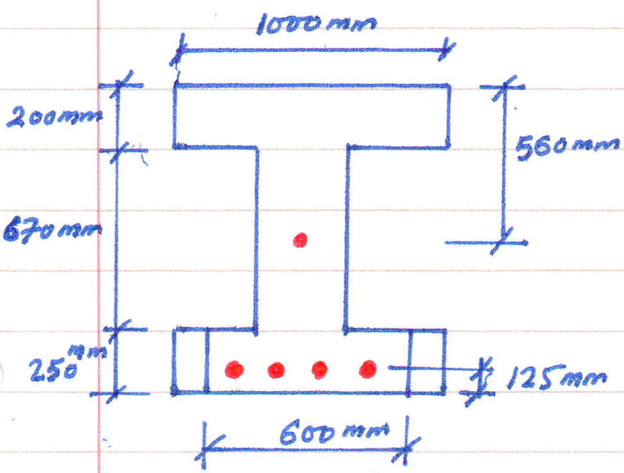
$$0.5\sqrt{f'_c} = 0.5\sqrt{42} = 3.24 \text{ MPa}$$

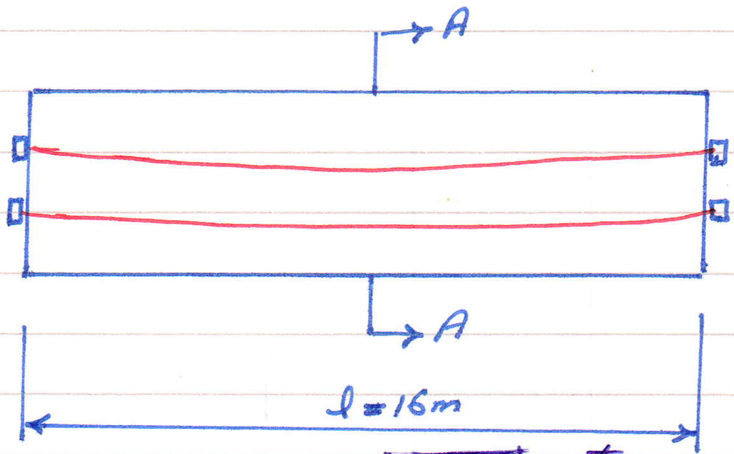
(tens.)

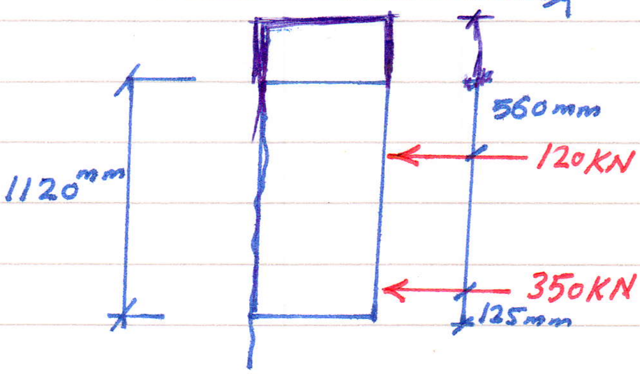
(o.K.)

3. Example:

$f'_c = 38\,MPa$, $\gamma_c = 24.5\,kN/m^3$.
added service loads = 20 kN/m'.
check the stresses at final stage.







القوة الأولى والثانية هي بعد حصول الضائعات
(after losses).

service moments in prestressed Concrete beams:

Sometimes the maximum service moment of an existing prestressed concrete beam is required. For this purpose, use,

$$f_{1se} = -\frac{P_e}{A} + \frac{P_e \cdot e}{I} \cdot c_1 - \frac{M_s}{I} \cdot c_1$$

put $f_{1se} = -0.45 f'_c$ (comp.)

& find M_s

Also, use

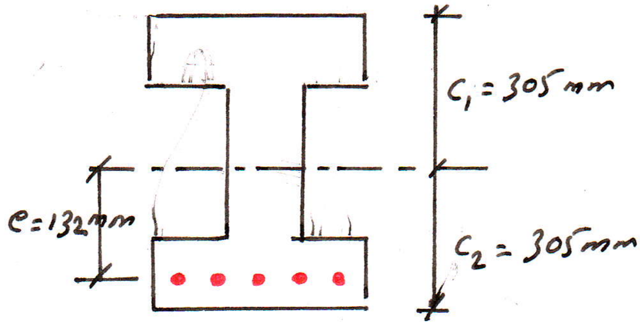
$$f_{2se} = -\frac{P_e}{A} - \frac{(P_e \cdot e)}{I} \cdot c_2 + \frac{M_s}{I} \cdot c_2$$

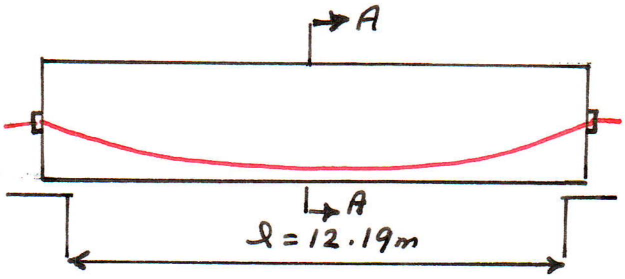
put $f_{2se} = +0.5\sqrt{f'_c}$ & find M_s.

Take the smaller M_s. when M_s is known, then the added service loads can be calculated.

$$M_s = \frac{(w_g + w_{added}) \cdot l^2}{8}$$

Example : (calculation of service moment):-





self weight — $w_g = 2.67\ KN/m$; $A = 114 * 10^3\ mm^2$

Concrete strength — $f'_c = 42 MPa$; 2nd moment of area

Eff. pres. force — $P_e = 639\ KN$; $I = 4.99 * 10^9\ mm^4$

Eccentricity — $e = 132\ mm$;

calculated the added service load.

Solution : Use, (for top face):

$$-0.45 f'_c = -\frac{P_e}{A} + \frac{P_e.(e)}{I}.C_1 - \frac{M_s}{I}.C_1$$

$$-0.45 * (42) = -\frac{639*10^3}{114*10^3} + \frac{639*10^3*132}{4.99*10^9}.305 - \frac{M_s}{4.99*10^9}*305$$

solve, $-18.9 = -5.605 + 5.155 - 61.12 * 10^{-9} M_s$

$$M_s = \frac{18.9 - 5.605 + 5.155}{61.12} * 10^9 = 301.865 * 10^6\ N.mm$$

$$= 301.865\ KN.m$$

At bottom face, Use,

$$+0.5\sqrt{f'_c} = -\frac{P_e}{A} - \frac{(P_e.e)}{A}.C_2 + \frac{M_s}{I}.C_2$$

$$0.5\sqrt{42} = -\frac{639*10^3}{114*10^3} - \frac{639*10^3*132}{4.999*10^9}*305 + \frac{M_s}{4.999*10^9}*305$$

solve,

$$M_s = \frac{3.240 + 5.605 + 5.155}{61.12 * 10^{-9}} = 229.058 * 10^6 \ N.mm$$

$$= \underline{229.058} \ KN.m$$

Use $M_s = 229.058 \ KN.m$

The added service load is obtained from:

$$M_s = \frac{(W_g + W_{added}).l^2}{8} \ ; \quad 229.058 = \frac{(2.67 + W_{added}) * (12.19)^2}{8}$$

$$W_{added} = \underline{\underline{9.662}} \ KN/m$$

Cracking moment in prestressed concrete beams

For a simply supported beam, use the tensile stress at bottom face;

$$f_{2se} = -\frac{P_e}{A} - \frac{(P_e \cdot e)}{I} \cdot c_2 + \frac{M_s}{I} \cdot c_2$$

The concrete section is uncracked. Therefore full (or gross) area A & 2nd moment of area I are used. But if the loads increase above the service loads, the section will crack. The section will crack when $f_{2se} \geqslant f_r$.

when f_r is the modulus of rupture ACI-Code gives,

$$f_r = 0.7\sqrt{f'_c} \quad \text{(in MPa)}$$

Thus substitute :-

$$0.7\sqrt{f'_c} = -\frac{P_e}{A} - \frac{(P_e \cdot e)}{I} \cdot c_2 + \frac{M_{cr}}{I} \cdot c_2$$

So the cracking moment M_{cr} is calculated.

Notice $\quad M_{cr} > M_s$

Example: Calculation of M_{cr} ?

$A = 114 * 10^3\ mm^2$

$I = 4.99 * 10^9\ mm^4$

$P_e = 6.39\ KN$ (Prestressed force (eff.))

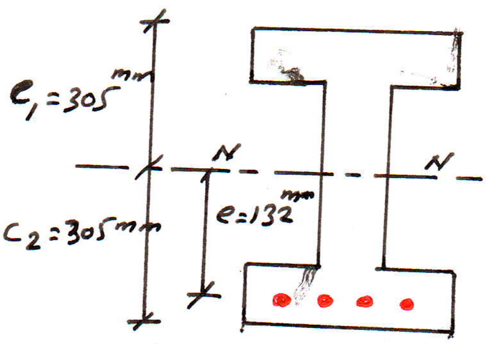
self weight $w_g = 2.67\ KN/m$

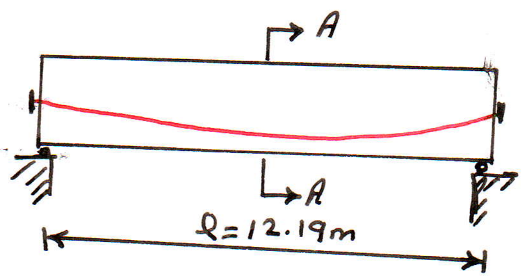
Eccentricity $e = 132\ mm$

$c_1 = c_2 = h/2 = 305\ mm$

- calculate the cracking moment (M_{cr}) & the added load ?

- $M_{cr} > M_s$ & The imposed (live & dead) load to cause cracking ?





solution:- first find f_r,

$$f_r = 0.7\sqrt{f'_c}$$

$$= 0.7\sqrt{42} = 4.536\ MPa$$

Use,

$$4.536 = -\frac{639*10^3}{114*10^3} - \frac{639*10^3*132}{4.99*10^9}*305 + \frac{M_{cr}}{4.99*10^9}*305$$

This gives, $M_{cr} = 250.265\ KN.m > M_s = 229.058\ KN.m$

For the simply supported beam, use $M_{cr} = \dfrac{q_r \cdot \ell^2}{8}$

Thus

$$q_r = \frac{8 * 250.265}{(12.19)^2} \quad , \text{ so } \quad q_r = 13.474 \text{ KN/m}$$

The imposed load to cause cracking

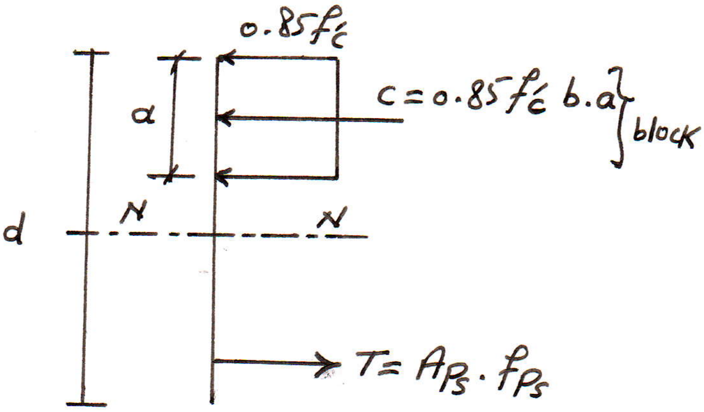
$$q = q_r - w_g$$
$$= 13.474 - 2.670$$
$$= 10.804 \text{ KN/m}$$

Ultimate moment in prestressed Concrete sections:-

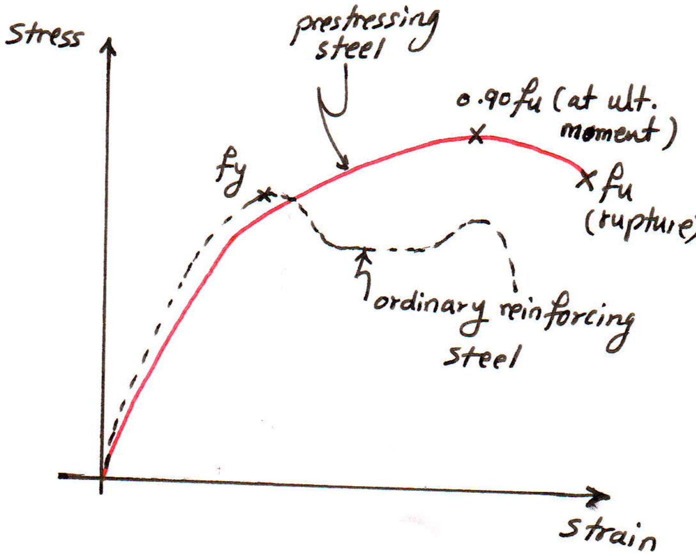
The prestressed concrete has no cracks at service moments. Above the service moment, the concrete will have cracks (at cracking moment). when the section is cracked, the concrete will not take tensile stresses. All tensile stresses in the concrete will go to the prestressing steel. The section will behave exactly like an ordinary reinforced concrete.

At ultimate moment the concrete will ~~take~~ make a rectangular block of compressive stresses (equal to $0.85 f'_c$) & depth $a = 0.85c$, where c is the depth of the neutral axis. The steel in tension will have high stresses (near to the ultimate strength f_u). Remember the prestressing has no abvious yield stress (no f_y in this steel).

For approximate analysis, the stress in steel at ultimate moment is taken $0.90 f_u$.



Stresses in P.S.C. (section at ultimate moment)



(Stress – strain relation for high tensile steel)

Thus at ultimate, consider a rectangular section (b = width & d = effective depth). At ultimate moments,

$$C = T$$

$$0.85 f'_c \, b a = A_{ps} f_{ps}$$

This gives,

$$a = \frac{A_{ps} f_{ps}}{0.85 f'_c b} = 1.176 \rho \frac{f_{ps}}{f'_c} \cdot d$$

where $\rho = A_{ps}/bd$ (steel ratio)

The ultimate moment is,

$$M_u = T \cdot \left(d - \frac{a}{2}\right)$$

$$= A_{ps} \cdot f_{ps} \cdot \left(d - \frac{1.176}{2} \rho \frac{f_{ps}}{f'_c} \cdot d\right)$$

$$= \rho b d \cdot f_{ps} \left(1 - 0.59 \rho \frac{f_{ps}}{f'_c}\right) \cdot d$$

$$= \rho f_{ps} \left(1 - 0.59 \rho \frac{f_{ps}}{f'_c}\right) b \cdot d^2$$

use a reduction factor $\phi = 0.90$, then:

$$\boxed{M_u = 0.90 \rho f_{ps} \left(1 - 0.59 \rho \frac{f_{ps}}{f'_c}\right) b \cdot d^2}$$

This is exactly the formula for ordinary reinforced section but f_{ps} is used in place of f_y.

write $\omega = \rho f_{ps} / f'_c$, then

$$M_u = 0.9 \omega f'_c (1 - 0.59 \omega) \cdot bd^2$$

or

$$\frac{M_u = ?}{0.9 f'_c bd^2} = \omega (1 - 0.59 \omega)$$

or $\boxed{R = \omega (1 - 0.59 \omega)}$

Thus table of $R \rightarrow \omega$ can be used.

Example : For concrete $f'_c = 36 MPa$

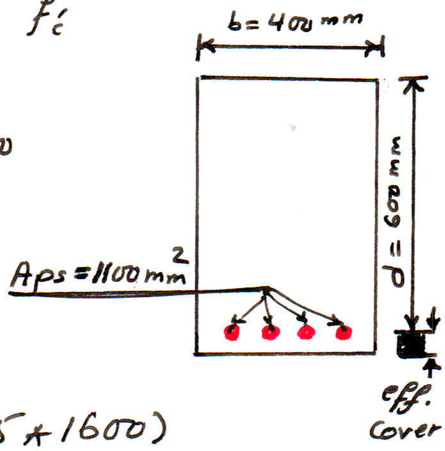
; steel $f_u = 1600 MPa$

$f_{ps} = 0.85 f_u$

Find M_u ?

solution :

$$* \quad M_u = 0.90 \rho f_{ps} \left(1 - 0.59 \rho \frac{f_{ps}}{f'_c}\right) . bd^2$$



Here $\rho = \frac{A_{ps}}{bd} = \frac{1100}{400 \times 600}$

$= 0.004583$

substitute :

$$M_u = 0.90 * 0.004583 * (0.85 * 1600) * \left(1 - 0.59 * 0.004583 * \frac{0.85 * 1600}{36}\right) * 400 * 600^2$$

$= 725.266 * 10^6$ N.mm

$= 725.266$ KN.m

$*$ Note : using $R \rightarrow \omega$; $\omega = \rho \frac{f_{ps}}{f'_c}$

$= 0.004583 * \frac{0.85 * 1600}{36}$

$= 0.1731$

This gives $R = 0.1554$

Then $M_u = 0.1554 * 0.9 * 36 * 400 * 600^2$

$= 725.034 * 10^6$ N.mm

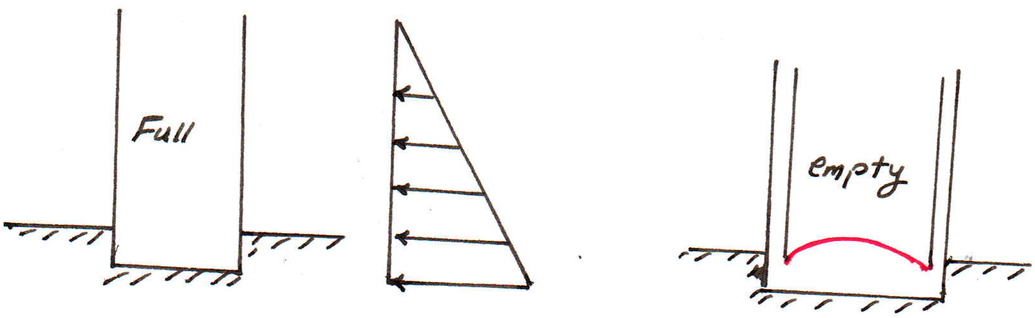
$= 725.034$ KN.m

Reinforced Concrete Tanks

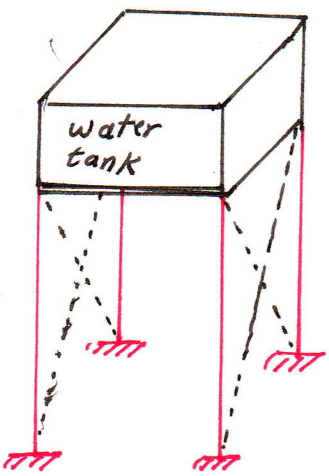
Types of Tanks

1. circular tanks.
2. rectangular tanks.
3. square tanks.
4. spherical tanks.
5. polygonal tanks.

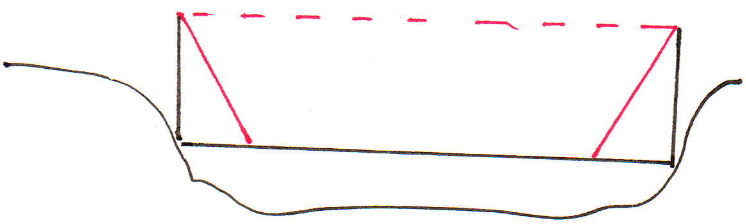
Tanks may be open or covered ontop. They may resting on ground, underground or may be elevated ones. Various types of water tanks, with different joint conditions,



(Tanks on ground)
(Fluid tank, Purification or settling)



(elevated tank)
- Tower -



(swimming tank)

General Design Requirements :

1. Permissible stresses in concrete
(For resistance of cracking) - due to bending & shear

2. Permissible stresses in steel

i. For resistance to cracking. when steel and concrete are assumed to act together, for checking the tensile stress in concrete for avoidance of cracking, the tensile stresses in steel will be limited by the requirement that the permissible tensile stress in the concrete is not exceeded, so that tensile stress in steel shall be equal to the product of modular ratio of steel and concrete, and the corresponding allowable tensile stress in concrete.

ii. For strength calculations. In strength calculations the permissible stresses in steel reinforcement shall be as :

a. Tensile stress in members in direct tension = 1000 kg/cm^2

b. Tensile stress in members in bending
- on liquid retaining face of members - 1000 kg/cm^2
- on face away from liquid for members less than 225mm thick - 1000 kg/cm^2
- on face away from liquid for members 225mm or more in thickness - 1250 kg/cm^2

c. Tensile stress in shear reinforcement
For members less than 225mm in thickness - 1000 kg/cm^2
For member 225mm or more in thickness - 1250 kg/cm^2

d. Compressive stress in columns subjected to directed load - 1250 kg/cm^2

Joints in water Tanks:

Joints to be provided in water tanks may be of following three types:

1. Movement joints.
2. Construction joints and.
3. Temporary joints.

1. **Movement joints**. In this joint relative movements between the sides of the joint are made possible. Special materials are used to maintain water-tightness. All the movement joints come under the category of flexible joints.

Movement joints may be further classified into the three types:

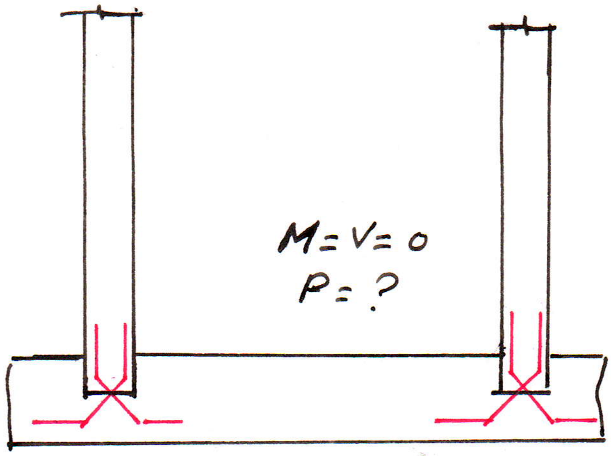
i. Contraction joint.
ii. Expansion joint and.
iii. sliding joint.

Design of Circular Tanks:

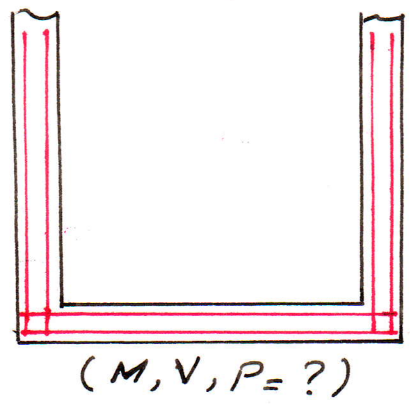
For design purpose, Circular tank may be divided into the following two categories:

1. Circular tank with flexible water-tight sliding joint between floor and vertical wall. Because of sliding flexible joint, tank wall is free to expand or contract independently of the floor slab.

2. Circular tank with rigid joint between floor and wall. In this case wall and floor are cast monolithically. In this case walls are not free to expand or contract.



Flexible sliding Joint (hinged)

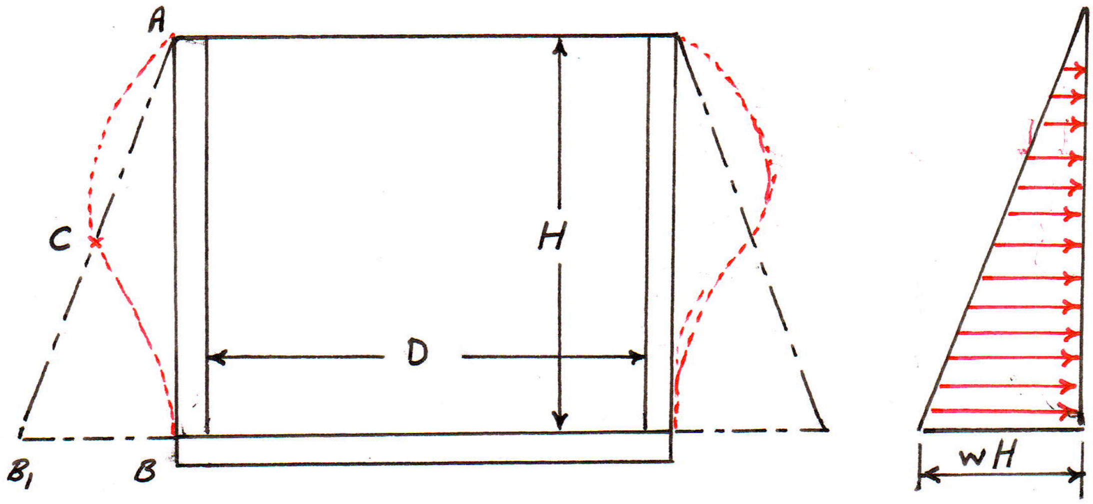


Rigid Joint (fixed)

Circular tank with flexible joint between floor slab and the wall :-

In this case, cylindrical wall of the tank is subjected to pure tension only and there will be no bending stresses. The wall is designed to take up tensile stresses developed in the wall due to hydrostatic pressure of water, filled in the tank. since hydrostatic pressure varies from zero at the top of the wall to maximum at the junction point with the floor slab, the amount of reinforcement will also vary from zero at the top of the wall to maximum at the bottom.

However minimum amount of steel is also provided at the top of the wall. Main reinforcement is in form of circular rings, generally provided at the middle thickness of the vertical wall. Distribution steel is provided in form of vertical bars touching the main reinforcement rings. A sliding flexible joint having effective water-tightness is provided at the junction point of the wall & the base.



Design Steps :

1. Let D be the diameter of the tank and p is the horizontal hydrostatic pressure due to water filled in the tank.

2. Calculate p at the bottom of the wall as follows.
$p = wH$, where H is the depth of water filled in the tank.

3. Calculate the maximum hoop tension (T) per meter height of the wall as follows:

$$T = \frac{wHD}{2}$$

4. Neglecting tension resistance of the concrete and whole of the tension being assumed taken by hoop steel only, calculate the area of hoop steel as follows. Permissible tensile stress in steel for water retaining structures is taken as 1000 Kg/cm^2.

$$A_t = \frac{T}{1000}$$

This area of steel may be provided at the centre of the wall if thickness of the wall is small. In case wall thickness is more than 22.5 cm, steel may be provided on each face keeping a minimum cover of 25 mm. The C/c spacing of the steel hoops should not be more than 3 times the thickness of the wall.

5. Though the reinforcement has been provided to take the entire hoop tension, but this does not mean that concrete does not take any tension. In fact tensile stresses are first set up in the concrete and then, because of perfect bond between steel & concrete, are transferred to the steel rings. Hence thickness of the wall should be so adopted that the tensile stress developend in the equivalent area of concrete does not exceed the ultimate tensile stress of the concrete.

then, $(100 * t + (m-1) A_t c_t = T)$

where t = thickness of wall in cm.

c_t = ultimate tensile stress in concrete which is normally taken $12 Kg/cm^2$ for M_{200} concrete.

A_t = area of steel per meter height of the wall in form of rings.

Thickness of the cylinderical wall can be fixed by following empirical formula also;

$$t = 3H + 5$$

where H is the depth of the liquid in the tank in meters & t the thickness of wall in centimeters.

$A_s = A_t = \frac{T}{f_s}$ A_t or A_s : Area of steel resist hoop tension per meter length of the wall.

f_s : Allowable tensile stress in steel.

sufficent Thickness of concrete is provided to resist this tension against cracking & steel provided for more strength.

Ex. Design a circular water tank for a storage capacity of 500000 liters. The joint between side walls & the base slab is flexible sliding type.

Solution: Volume of tank $= \frac{500000}{1000} = 500 m^3$

Let depth of water in the tank be 3.5m

Floor area of the tank $= \frac{500}{3.5} = 142.857 \, m^2 \approx 143 \, m^2$

Let D be the diameter of the tank

$$\frac{\pi D^2}{4} = 143$$

$$D = \sqrt{\frac{143 * 4}{\pi}} = 13.5 m$$

Hence adopt 13.5 m as the internal diameter of the tank and water filled to a depth of 3.5 m.

Total hoop tension per meter hight of the tank at the bottom

$$T = \frac{1}{2} w H D$$

$$= \frac{1}{2} * 1000 * 3.5 * 13.5$$

$$= 23625 \text{ kg}$$

where;

w: weight of cubic unit of water (1000 Kg/m^3) or (9.81kN/m^3).

T: hoop tension

Area of steel required for 1m height

$$A_t = \frac{23625}{1000} = 23.625 \text{ cm}^2$$

provide 18mm ϕ bars in form of hoops at c/c spacing of,

$$\frac{2.54 * 100}{23.625} = 10.75 \text{ cm.}$$

, No. of bars/m (hieght) $= \frac{23.625}{2.54} = 9.3$

These hoops will be provided at the middle thickness of the wall.

Let M_{200} concrete has been used in wall & let (t) can be the thickness of the wall & 13 as the value of modular ratio m.

Equivalent area of Concrete = $100t + (13-1)A_t$

$$\therefore \quad 100t + (13-1)\,23.625 = \frac{23625}{12}$$

$(100t + (m-1)A_t)C_t = T_{max}$

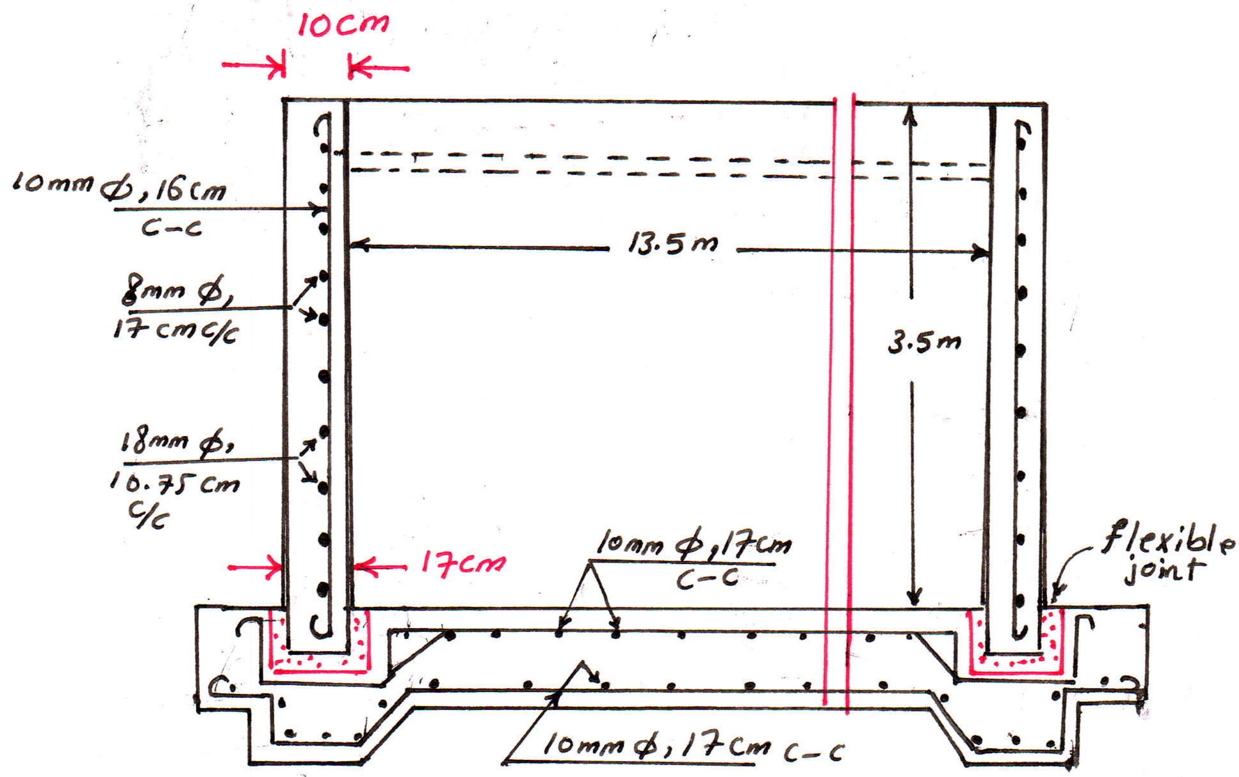
b = 100 cm ,1000mm

$t = 16.86$ cm

= say 17cm

Thickness of wall based on empirical formula

$$t = 3H + 5$$
$$= 3 * 3.5 + 5$$
$$= 15.5 \text{ cm.}$$



(circular water tank)

Hence adopt thickness of tank wall , 17 cm.

Thickness may be reduced towards the top and wall may tapering according to the depth of water. Since the pressure on the wall varies with the depth of water, the area of steel can be suitably reduced at lesser depths by increasing the c/c spacing of 18 mm ϕ bar hoops.

Let thickness of the vertical wall be reduced to 10 cm at the top , from 17 cm at the bottom. Theoretically no hoop steel is required at the top of the vertical wall, since there is no horizontal pressure. However minimum steel of 0.3% has to be provided for thickness of walls up ~~to~~ 10 cm & 0.2% for thickness of walls of 45 cm and above. Between thickness of walls of 10 cm & 45 cm, percentage of steel can be interpolated linearly.

Area of steel at top of the wall @ 0.3%

$$= \frac{0.3}{100} * 10 * 100 = 3 cm^2$$

provide 8mm ϕ bar hoops at c/c spacing

of $$\frac{0.5 * 100}{3} = 17 \text{ cm}$$

Hence area of hoop steel at bottom is 18mm ϕ bars at 10.75 cm c/c & at top of the wall it is 8 mm ϕ 17cm c/c. Area of steel can be suitably varied between these two limits.

Vertical reinforcement in the wall:

thick. bott. Thick. Top

Percentage of distribution steel $= 0.3 - 0.1\left(\frac{17-10}{35}\right)$

$= 0.28\%$

height of the tank 350 cm or $\frac{1}{10} * \frac{1}{35}$

Area of vertical steel / m length

; $A_s = \rho b d$;

$$= \frac{0.28}{100} * 17 * 100$$

$$= 4.76\ cm^2$$

provide 10 mm ϕ bars at c/c spacing of

$$= \frac{0.79 * 100}{4.76} = 16.6\ cm$$

say 16 cm.

Design of floor slab:

Adopt minimum thickness of the floor slab as 15cm & provide minimum steel of 0.3%,

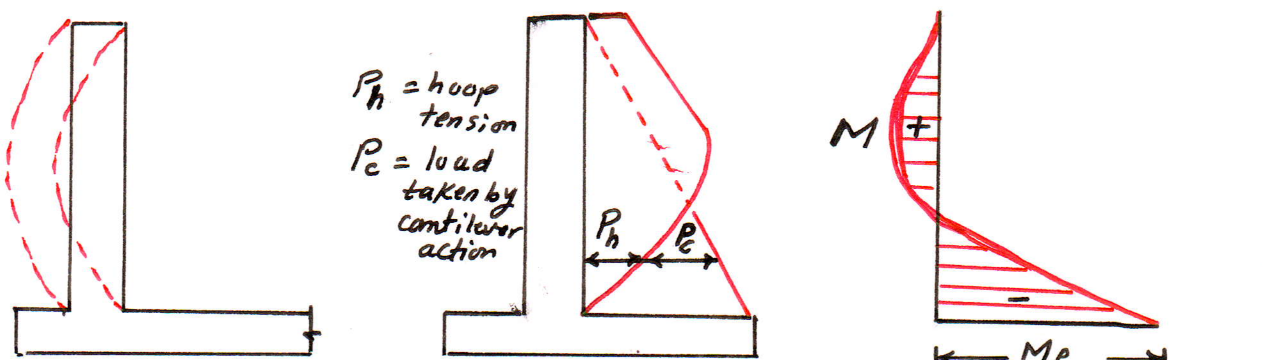
$$\frac{0.3 * 15 * 100}{100} = 4.5\ cm^2 \qquad ; (A_s = \rho b d)$$

provide 10mm ϕ bars 17 cm c/c both ways, both at top as well as at bottom of the slab.

Circular tank with rigid joint between floor & wall :-

If base slab of the tank and side walls are constructed monolith-ically, there is no possibility of horizontal displacement of the wall at the junction. In this case, because of monolithic character, the fixed bottom induces moments in the tank wall. The tank wall acts as a cantilever, fixed at the bottom for some height, and there is no hoop stress at bottom of the tank.

At certain height above the base slab, cantilever effect is reduced to zero & all the hydrostatic pressure induced by water is resisted by hoop stresses only. The cantilever action is predominant at & near the base but hoop action is predominant above the point, where cantilever action توقف ceases to occur. Hence in the analysis of such tanks, position of maximum hoop action & magnitudes of maximum hoop tension & B.M. at the base, due to cantilever action have to be determined. B.M. and hoop stress diagram for the vertical wall shown below.



(a) Deformed position of vertical wall

(b) Horizontal pressure distribution

(c) Bending moments at various positions

Analysis of such tanks can be done by, any design method below;

1. „Reissner's Method".
2. „Carpenter's Method".

1. Reissner's method. Reissner evolved (اُنشا اُوکړی) a term K by following relation:

$$\boxed{K = \frac{12H^4}{\left(\frac{D}{2}\right)^2 T^2} = \frac{48H^4}{D^2 T^2}}$$

He gave the following tables from which fixed moment & maximum hoop tension & its location from the base slab can be found out.

In this method D & H are known from the geometry of the tank because for known capacity, depth of filling & diameter of tank have been determined first. After this, thickness of the wall of tank is assumed by emperical rule $T = (3H+5)$ cm. Now putting the values of D, H & assumed value of T in the formula, value of K can be determined. The interpolation for K is done logarithmically. Deformation of wall, load distribution & B.M. along the height shown in figures above. P_h denotes hoop tension & P_c load taken by cantilever action. Maximum +B.M. may be taken between $\frac{M_f}{3}$ to $\frac{M_f}{4}$.

2. Carpenter's method. H. Carpenter gave Coefficients F & K corresponding to different ratios of $\frac{H}{T}$ & $\frac{H}{D}$. Using these Coefficients, values of maximum B.M. & position & magnitude of maximum hoop tension can be determinde as follows:

i. Position of maximum Hoop tension above base $= KH$

ii. Maximum Hoop tension $= w(H - KH)\frac{D}{2}$

iii. Maximum Cantilever B.M. at base $= FwH^3$

values of K & F can be read from Table (3).

Example: Design a circular tank of capacity 150,000 litres. The joint between vertical wall & the base slab is monolithic. A good soil of sufficient bearing capacity is available at 0.5m below the G.L.

solution: Capacity of the tank = 150,000 litres.

$$\text{Volume of the tank} = \frac{150,000}{1000}$$

$$= 150 \ m^3$$

Let water filling in the tank be 3m

$$\text{Area of the tank} = \frac{150}{3} = 50 m^2$$

$$\text{Tank diameter} = \sqrt{\frac{50 \times 4}{\pi}}$$

$$= 7.978 m \quad \text{say } 8m$$

Let t be the thickness of the tank

$$t = 3H + 5$$
$$= 3 \times 3 + 5$$
$$= 14 \ cm$$
$$= 0.14 m$$

(T : thickness or use t)

$$\therefore \frac{H}{D} = \frac{3}{8} = 0.375$$

$$\& \quad \frac{H}{t} = \frac{3}{0.14} = 21.43$$

Design has been done by H. Carpenters co-efficients

From Table (3) $\frac{H}{D} = 0.375$, $\frac{H}{t} = 21.43$

$$\Rightarrow F = 0.014642, \ K = 0.39392$$

i) Maximum hoop tension $= w(H-KH)\frac{D}{2}$

$$= \frac{wHD}{2}(1-K)$$

$$= \frac{1000*3*8}{2}(1-0.39392)$$

$$= 7273 \text{ kg}$$

ii) Maximum B.M. at junction of vertical wall & base slab

$$= FwH^3$$

$$= 0.014642 * 1000 * (3)^3$$

$$= 395 \text{ Kg.m}$$

$$= 39500 \text{ Kg.cm}$$

iii) Height of the point from base slab where maximum hoop tension occurs

$$= KH$$

$$= 0.39392 * 3$$

$$= 1.18 m.$$

Let M_{200} concrete be used for which

$c = 70 \text{ Kg/cm}^2$

$t = 1000 \text{ Kg/cm}^2$

$m = 13$

For these values of c, t & m

$$N = \frac{mc}{t+mc}$$

$$= \frac{13*70}{1000+13*70} = 0.476$$

$$J = 1 - \frac{N}{3}$$

$$= 1 - \frac{0.476}{3}$$

$$= 0.841$$

$$Q = \frac{1}{2} c N j$$

$$= 0.5 * 70 * 0.476 * 0.841$$

$$= 14.06 \ Kg/cm^2$$

wall thickness (effective),

$$d = \sqrt{\frac{Kg.cm}{\frac{Kg}{cm^2} \cdot cm}} \quad ; \qquad d = \sqrt{\frac{M}{Qb}}$$

$$= \sqrt{\frac{395 * 100}{14.06 * 100}}$$

$$= 5.3 \, cm$$

Hence adopted thickness of wall of 14 cm is adequate. If 4 cm effective concrete cover is left on the face of the wall coming in contact with water, the available effective thickness of wall

$$= 14 - 4 = 10 \ cm$$

Area of steel from B.M. point of view,

$$A_t = \frac{M}{tjd}$$

where;

$$\left\{ \begin{array}{l} j = 0.841 \\ t = 1000 \ Kg/cm^2 \\ d = 10 \ cm \end{array} \right\}$$

$$= \frac{395 * 1000}{0.841 * 10 * 1000}$$

$$= 4.7 \ cm^2.$$

provide 10 mm ϕ vertical bars at

$$\frac{0.79*100}{4.7} = 16.7 \text{ cm say } 16 \text{ cm c/c}$$

provide 10mm ϕ bars 16cm c/c vertically near the face coming in contact with water for a vertical height of 1.18m measured from base slab. Above 1.18m height curtail يقلص alternate bars, 10 mm ϕ bars at 32 cm c/c.

Design of wall for hoop tension:

Maximum hoop tension at 1.18 m from the base slab = 7273 kg

Area of steel, $A_t = \frac{7273}{1000} = 7.273 \text{ cm}^2$

provide 14 mm ϕ bars in form of hoops at c/c spacing of

$$\frac{1.54*100}{7.273} = 21 \text{ cm}$$

check for tension on composite section of wall:

Equivalent area of concrete $= 100*t + (m-1)A_t$

$= 100*14 + (13-1)*7.273$

$= 1487.276 \text{ cm}^2$

Tensile stress in concrete of composite section

$$= \frac{7273}{1487.276} = 4.89 \text{ Kg/cm}^2 < 12 \text{ Kg/cm}^2 \text{ (safe)}$$

c/c spacing of 14 mm ϕ bars (hoops) will be maintained constant up to 1.18 m from the base slab. Therefore the spacing may be increased depending upon the hoop tension.

Area of hoop steel at say 1.5 m below the top of the wall,

$$\text{Hoop tension} = \frac{whD}{2}$$

$$= \frac{1000 * 1.5 * 8}{2}$$

$$= 6000 \text{ kg}$$

$$\text{Area of steel} = \frac{6000}{1000} = 6 \text{ cm}^2$$

c/c spacing of 14 mm ϕ bars rings

$$= \frac{1.54 * 100}{6} = 25 \text{ cm c/c}$$

Similarly spacing may be varied as we go up towards the top of the wall. We have adopted a maximum spacing of the 30 cm at top of the wall.

Design of the base slab:

Since base slab is monolithic with the cylindrical wall, the B.M. transmitted to the base slab will be same as that at the bottom of the cylindrical wall, causing tension on the upper face of the base slab. But this B.M. is so small that it will require reinforcement even smaller than the minimum reinforcement to be provided @ of 0.3% of the section both ways.

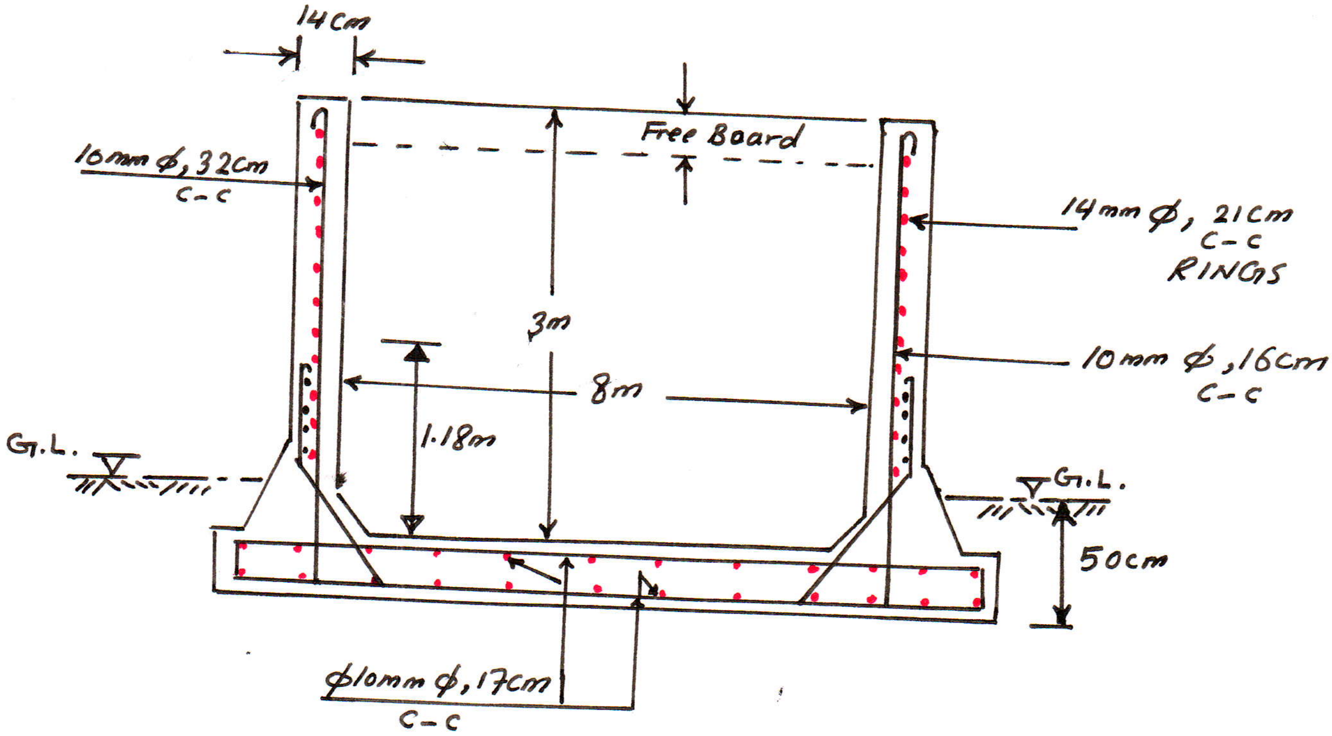
Adopt the 15 cm thick base slab, reinforced both at top & bottom both ways adopting area of steel 0.3% in each direction.

$$\text{Area of steel} = 0.3\%$$
$$= \frac{0.3}{100} * 15 * 100$$
$$= 4.5 \ cm^2$$

provide 10mm ϕ bars at 17 cm C/C with both ways, both near top & bottom surface of the base slab.

In addition to the above stated التسليح المبين أعلاه reinforcement provide 10mm ϕ bars at 17cm c/c as splay bars.

Arrangement of reinforcement as shown in fig. below.



(Details of reinforcement)

or;

$$A_s = \frac{M}{f_s J d} \quad , \quad d = \sqrt{\frac{M}{K b}} \quad , K = \frac{n}{n+r}$$

$$j = 1 - \frac{K}{3}$$

$$r = \frac{f_s}{f_c} \quad , n = \frac{E_s}{E_c}$$

$$f_c = 0.45 f'_c$$

Moments:

1. simply supported:-

$$M_R^+ = M_\theta^+ = \frac{3qR^2}{16} \quad \text{(center)}$$

$$M_R^- = 0 \ , \ M_\theta = \frac{qR^2}{16} \quad \text{(edge)}$$

2. Fixed edge (supported):-

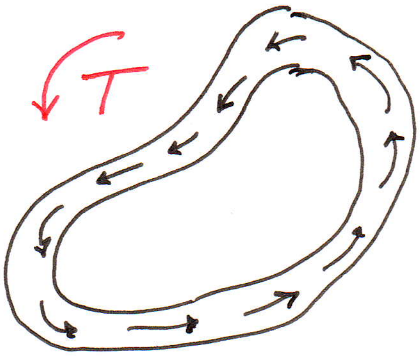
$$M_R^+ = M_\theta^+ = \frac{qR^2}{16} \quad \text{(center)}$$

$$M_\theta = 0 \quad , \ M_R = \frac{qR^2}{8} \quad \text{(edge)}$$

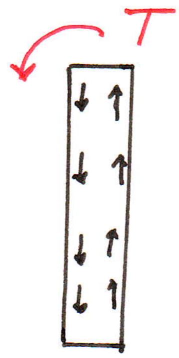
Ex: Design a cylindrical tank for $320 m^3$ water capacity, the wall is free at base $f'_c = 35$ MPa, $f_y = 414$ MPa, assume the bearing capacity $q = 100$ kN/m^2?

TORSION

Torsion of closed thin-walled sections (or cells) & torsion of thin rectangular sections are important in civil Engineering.



closed thin-walled section



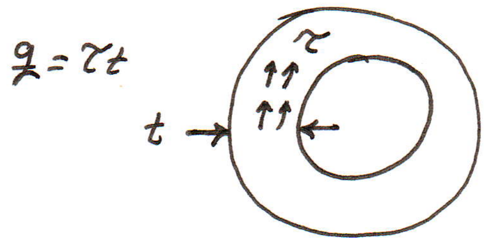
thin rectangular sections

closed thin-walled sections (or cells):

For this, a quantity (shear flow) is needed. shear flow in any position in the thin wall of a closed cell is defined as:-

$$\boxed{\text{shear flow} = \text{shear stress} * \text{Thickness of wall}}$$

$$q = \tau t \qquad \text{(force / unit length)}$$

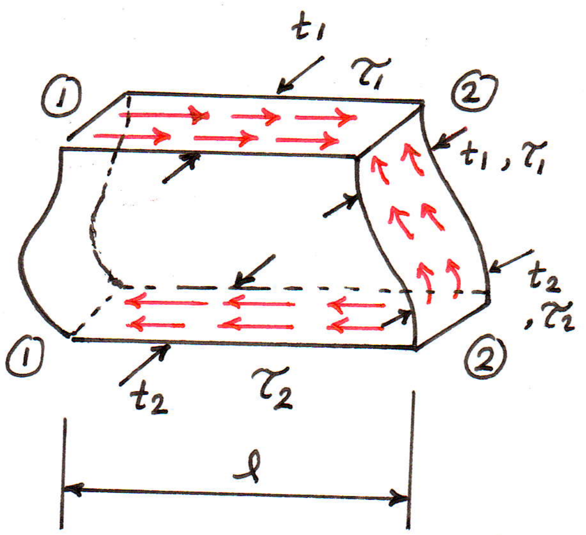


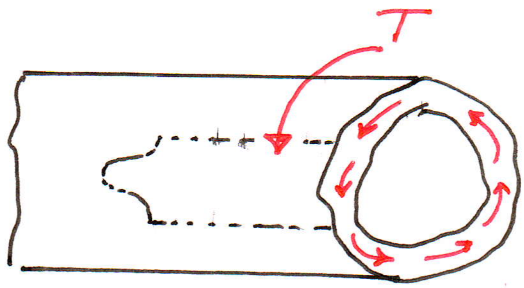
The following Theorems are needed:

1. The shear flow is constant in any closed thin-walled section. The shearing stress τ & the thickness t may vary in the wall, but $q = \tau t$ must remain constant. This is analogous to the flow of water in closed channel (flow = velocity * section area)

Proof:-

Take a portion cut out by longitudinal sections & cross sections in a tube:



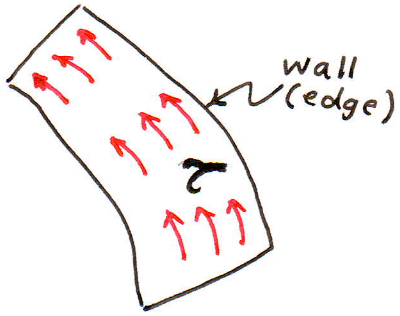


At edge 1, the shear flow is $q_1 = \tau_1 t_1$, At edge 2, the shear flow is $q_2 = \tau_2 t_2$. to prove $q_1 = q_2$, Consider the horizontal (or longitudinal) equilibrium. $\tau_1 * t_1 * \ell = \tau_2 * t_2 * \ell$, Thus The, $\tau_1 * t_1 = \tau_2 * t_2$, or $q_1 = q_2$.

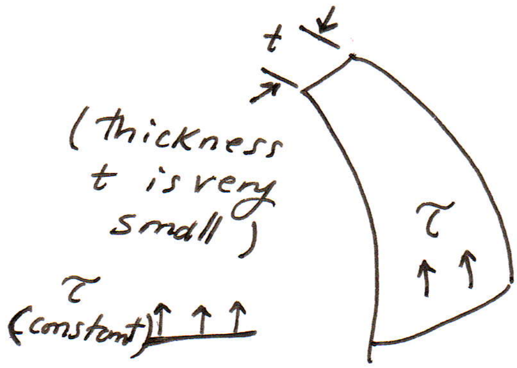
Important notes:

1. The shearing stress τ must be parallel to the edges of the wall.

   (The component normal to the edge does **not** exist).



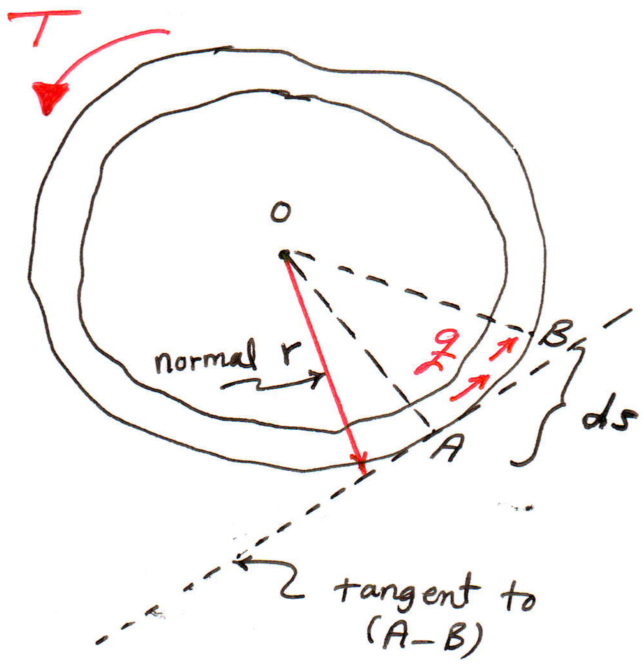
2. The shearing stress τ is constant a cross the small thickness (at any position). The thickness t is very small & τ can not vary in this thickness.



3. The torque T in the closed thin-walled section is;

$$T = 2qA$$

where A is the area enclosed by the wall of the section.



Take a small length (A-B) = ds in the wall. The shear flow $q = \tau t$ in (A-B) will produce torque;

moment arm force

$$dT = r \cdot \underbrace{\tau \cdot t}\, ds$$
$$= r \cdot q \cdot ds$$

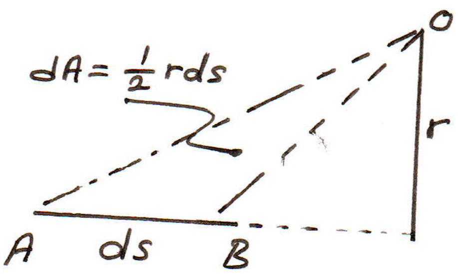
where r is the normal to (A – B) from origin O. The origin O can be any point inside (or outside) the section.

The area of the triangle OAB is $dA = \frac{1}{2} r\,ds$

Thus, $dT = 2q\,dA$

$$T = \int 2q\,dA$$
$$= 2q \int dA \quad \text{(as } q \text{ is constant)}$$

Thus $$\boxed{T = 2qA}$$



4. The torsion constant of a closed thin-walled section is:

$$J = \frac{4A^2}{\int_c \frac{ds}{t}} \quad (\text{length}^4)$$

where, $\int_c \frac{ds}{t}$ is the contour integral (taken round the section).

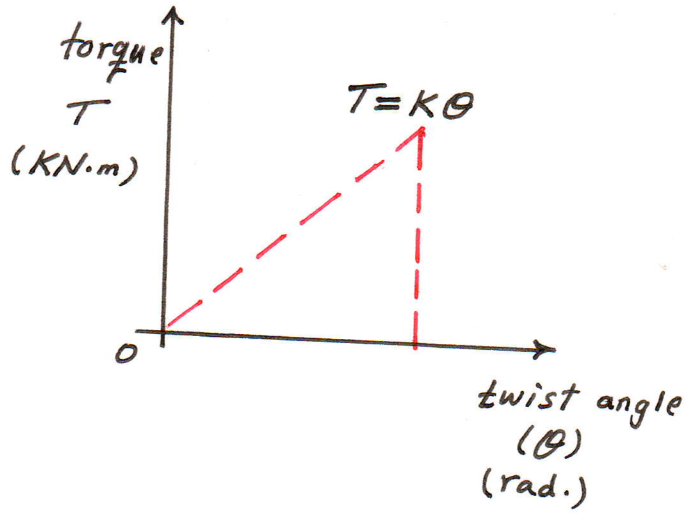
Definition: The torque – twist relation is linear (Hook's law).

The constant $K = \frac{T}{\theta}$ is torsional stiffness.

This is the torque to produce unit angle of twist usually,

$$T = \frac{GJ}{l} \cdot \theta \quad \text{or} \quad K = \frac{GJ}{l} \quad (\text{when } \theta = 1)$$

where l is the length of the tube & G the modulus of rigidity (or shear modulus of elasticity), & J is the torsion constant of the section. The product GJ is called torsional rigidity of the section. Notice that $GJ \rightarrow EI$ in the bending of beams. Here, prove that;



$$J = \frac{4A^2}{\oint_c \frac{ds}{t}}$$

proof: use the method of real work

External work = Internal strain energy

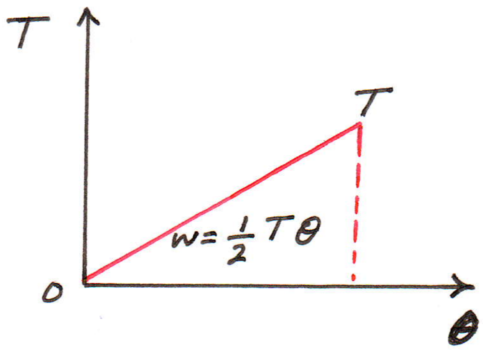
$(\tau \,\&\, t) \rightarrow (\text{varying})$

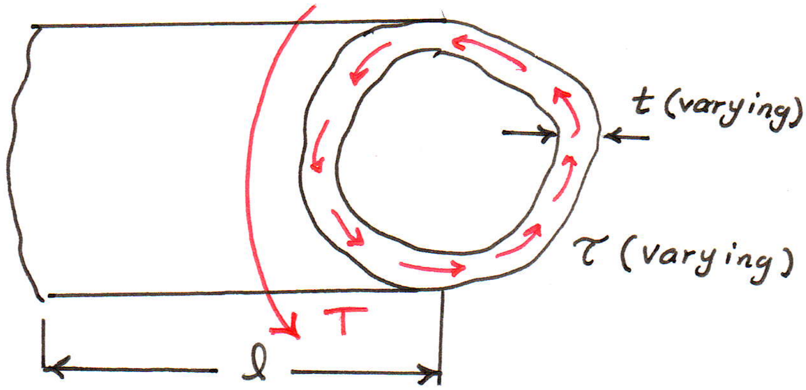
$(\tau * t) \rightarrow (\text{Constant})$

The external work is;

$$W = \frac{1}{2} T \theta$$

as $(T-\theta)$ relation is linear





The internal strain energy is;

$$U = \int_{vol} \frac{\tau^2}{2G} \, d(\text{vol.}) \qquad \text{(volume integral)}$$

as τ is the only stress in the tube.

Here;

$$d(\text{vol.}) = l.t.ds \quad \text{(volume of a strip)}$$

$$\tau = \frac{q}{t} = \frac{T/2A}{t}$$

substitute,

(countor integral)

$$U = \int_c \frac{\left(\frac{T}{2At}\right)^2}{2G} . l.t\, ds$$

$$= \int_c \frac{T^2 l}{8A^2 G t} ds = \frac{T^2 l}{8A^2 G} \int_c \frac{ds}{t}$$

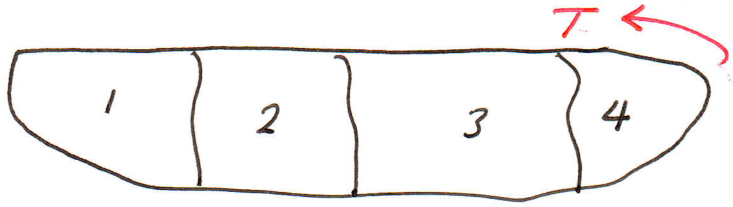
Use $W = U$, then

$$\frac{1}{2} T \theta = \frac{T^2 l}{8A^2 G} \int_c \frac{ds}{t}$$

or $$T = \frac{4A^2}{\int_c \frac{ds}{t}} \cdot \frac{G}{l} \cdot \theta$$

Compare to $T = \frac{GJ}{l} \cdot \theta$, then $J = \frac{4A^2}{\int_c \frac{ds}{t}}$

4. For multi-cell sections (statically indeterminate) the twist is the same for all cells;

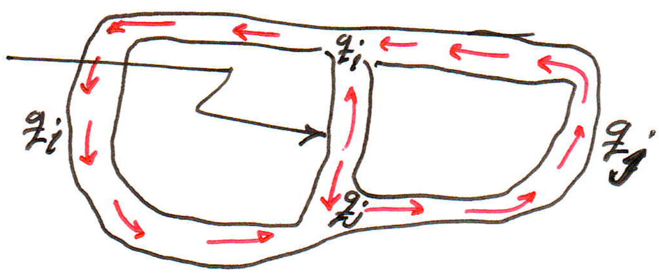


$$\theta_1 = \theta_2 = \theta_3 = \theta_4$$

The shear flow in an interior web is:

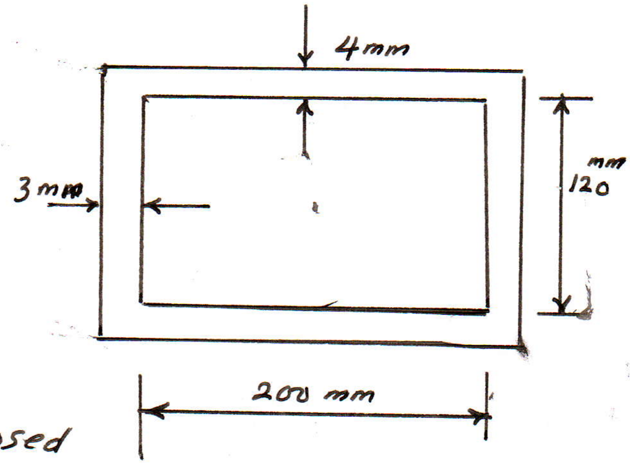
$$q_{i-j} = q_i - q_j$$

(in the web)
$q_{i-j} = q_i - q_j$
(shear flow)ال takes



Examples:- (closed thin-walled cells)

1. A tube of length 800 mm & rectangular cell (as shown) is under a torque 2 KN.m. Calculate the angle of twist & also the shearing stresses in the wall of the tube? G = 8000 N/mm^2?



Solution:- First find the enclosed area,

$$A = 200 * 120 = 24000 \text{ mm}^2$$

use $T = 2qA$

Then the constant shear flow is

$$q = \frac{T}{2A}$$

$$= \frac{2*10^6 \text{ (N.mm)}}{2*24000 \text{ (mm}^2)}$$

$$= \frac{1000}{24} \text{ N/mm}$$

$$= 41.667 \text{ N/mm}$$

The shearing stresses in the wall is;

$$\tau = \frac{q}{t}$$

In the wall of 4mm; $\tau = \dfrac{41.667}{4} = 10.417 \text{ N/mm}^2$

In the wall of 3mm; $\tau = \dfrac{41.667}{3} = 13.889 \text{ N/mm}^2$

To find the angle of twist, Calculate J?

Here $J = \dfrac{4A^2}{\int_c \frac{ds}{t}}$

$$= \frac{4 * (24000)^2}{2\left(\frac{200}{4} + \frac{120}{3}\right)} = 12.8 * 10^6 \ mm^4$$

use $\theta = \frac{Tl}{GJ} = \frac{2*10^6 * 800}{80*10^3 * 12.8 * 10^6} = 1.563 * 10^{-3}$ radians

always (T) represents force in laws?

$$= 1.563 * 10^{-2} * \frac{180}{\pi}$$

$$= 0.89553^\circ$$

2. A tube of length 1.8 m & cross-section as shown. The tube is under a torque 1.8 KN.m & axial tensile force 20 KN. Check the failure by Rankine, tresca & Von Mises theories. $G = 80 * 10^3 \ N/mm^2$ (Not needed), $f_y = 410$ MPa (yield)?

Hinte: shear flow → large stress

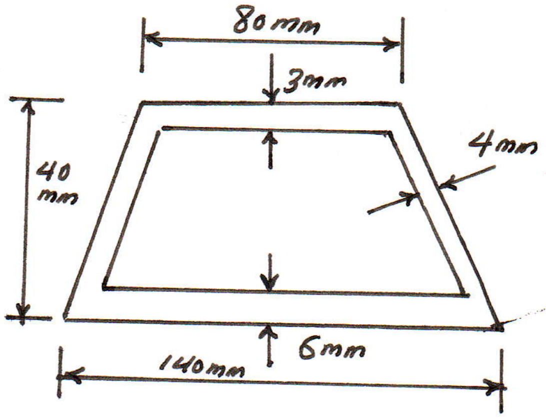
Here $\sigma_x = \frac{P}{\text{area of wall}}$

, $\sigma_y = \sigma_z = 0$

& $\tau_{xy} = \frac{q}{\text{Smallest thickness}}$

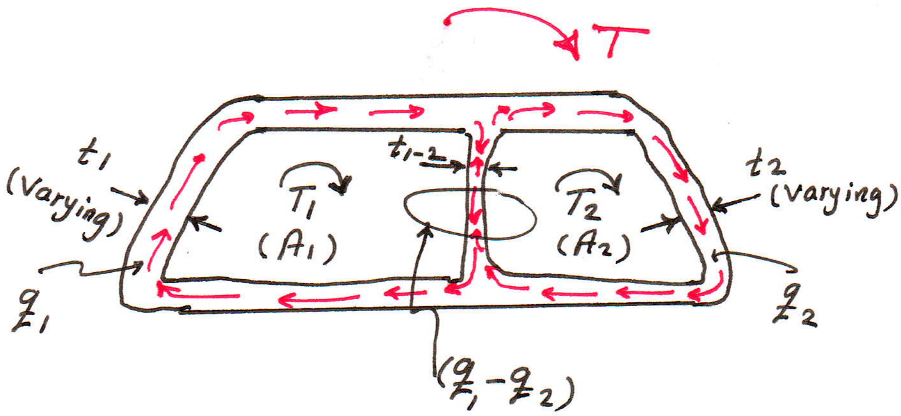
$\tau_{yz} = \tau_{zx} = 0$

Find the principal stresses σ_1, σ_2 & $\sigma_3 = 0$ (as 2-D stresses).



Torsion of thin-walled multi-cell sections (statically indeterminate)

Consider a tube of two cells under a torque T.



Let q_1 be the shear flow of cell 1 (with enclosing area A_1 & thickness t_1) & let q_2 be the shear flow in cell 2 (with thickness t_2 & enclosing area A_2).

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The diaphragms has a shear flow $(q_1 - q_2)$ & thickness t_{1-2}.

The torque T_1 in cell 1 by the shear flow q_1 is;

$$T_1 = 2A_1 q_1$$

The torque T_2 in cell 2 by the shear flow q_2 is;

$$T_2 = 2A_2 q_2$$

① Equilibrium gives; $T = T_1 + T_2$

or $$T = 2A_1 q_1 + 2A_2 q_2 \quad \text{-----} (1)$$