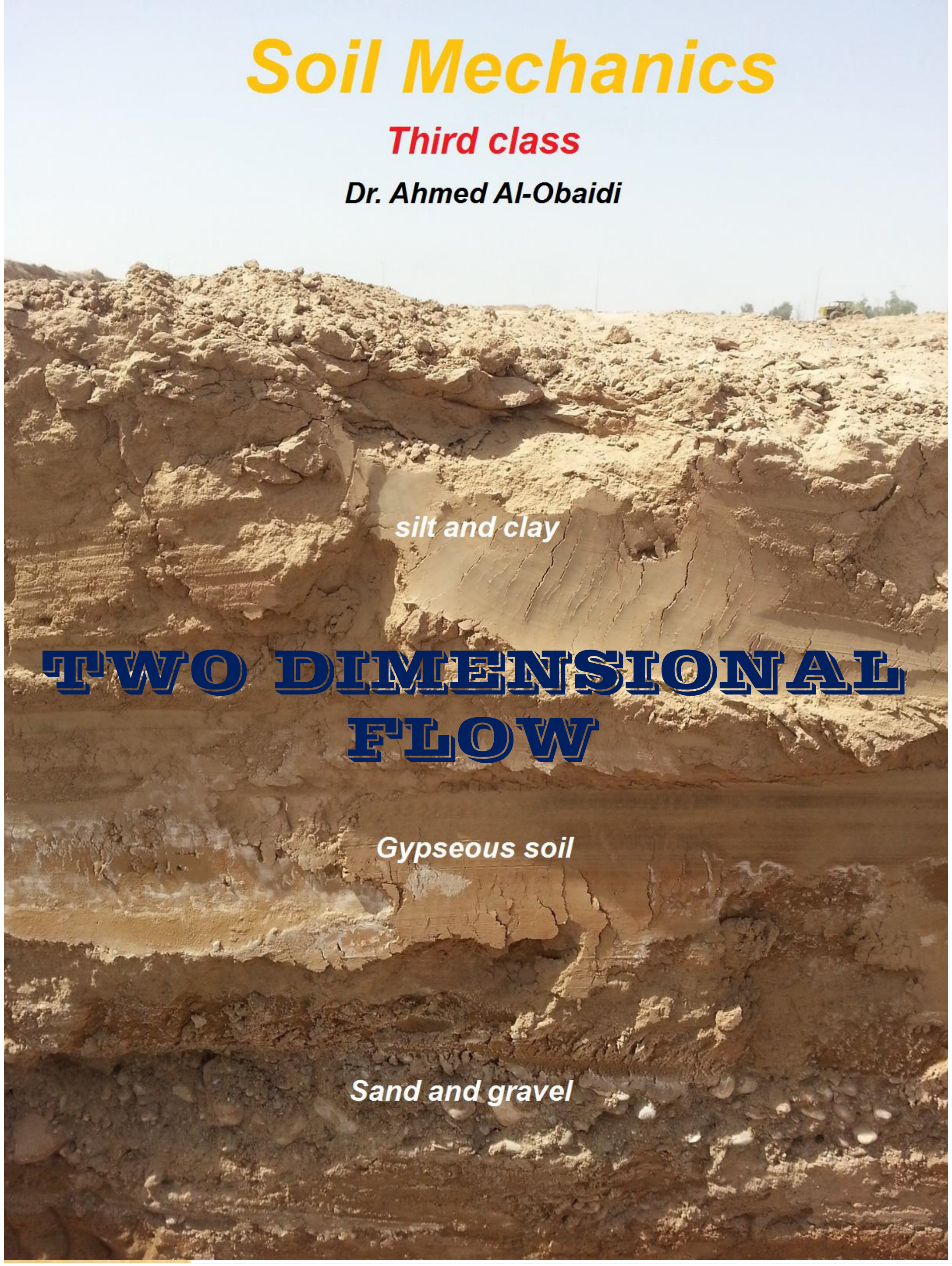


Tikrit University

College of Engineering

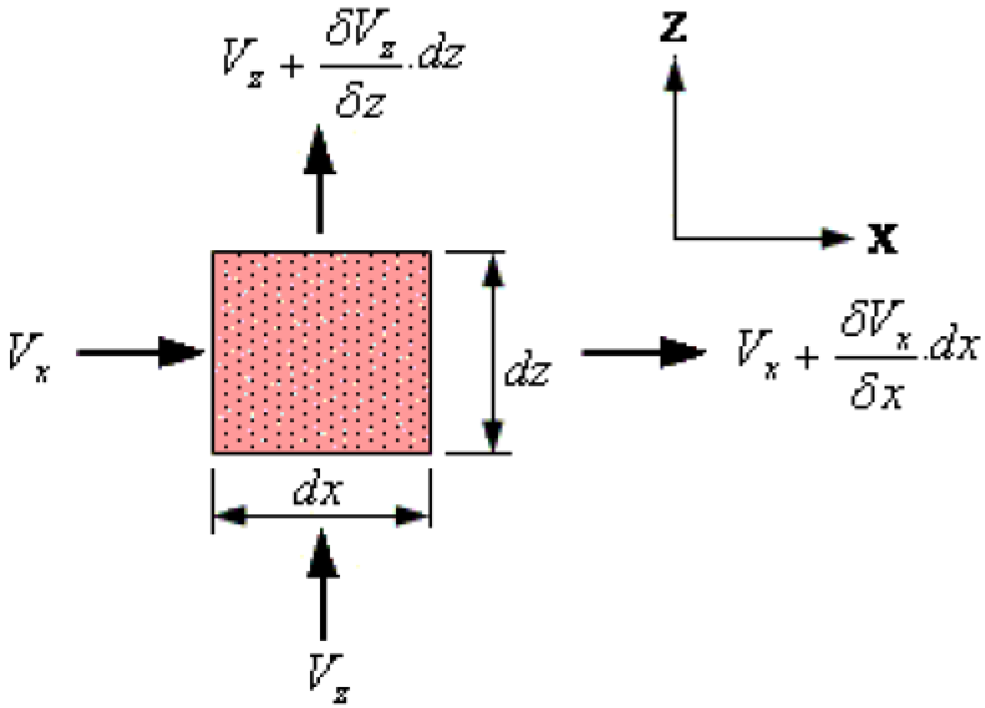
Civil engineering Department

Soil Mechanics

3rd Class

Lecture notes

Two Dimensional Flow



A rectangular soil element is shown with dimensions ***dx and dz in the plane, and thickness dy perpendicuar to this plane. Consider planar flow into the rectangular soil element.***

Two Dimensional Flow

In the **x-direction, the net amount of the water entering and leaving the element is**

$$\frac{\delta V_x}{\delta x}.dx.dy.dz$$

Similarly in the **z-direction, the difference between the water inflow and outflow is**

$$\frac{\delta V_z}{\delta z}.dz.dx.dy$$

For a two-dimensional steady flow of pore water, any imbalance in flows into and out of an element in the z-direction must be compensated by a corresponding opposite imbalance in the x-direction. Combining the above, and dividing by ***dx.dy.dz , the continuity equation is expressed as***

Two Dimensional Flow

$$\frac{\delta V_x}{\delta x} + \frac{\delta V_z}{\delta z} = 0$$

From Darcy's law, $V_x = k_x \cdot \frac{\delta h}{\delta x}$, $V_z = k_z \cdot \frac{\delta h}{\delta z}$, where h is the head causing flow.

When the continuity equation is combined with Darcy's law, the equation for flow is expressed as:

$$k_x \cdot \frac{\delta^2 h}{\delta x^2} + k_z \cdot \frac{\delta^2 h}{\delta z^2} = 0$$

Return

Two Dimensional Flow

For an isotropic material in which the permeability is the same in all directions (i.e. k x= k z), the **flow equation is**

$$\frac{\delta^2 h}{\delta x^2} + \frac{\delta^2 h}{\delta z^2} = 0$$

This is the **Laplace equation governing two-dimensional steady state flow. It can be solved *graphically, analytically, numerically, or analogically.***

For the more general situation involving ***three-dimensional steady flow, Laplace equation***

becomes: $$\frac{\delta^2 h}{\delta x^2} + \frac{\delta^2 h}{\delta y^2} + \frac{\delta^2 h}{\delta z^2} = 0$$

Two Dimensional Flow

One-dimensional Flow

For this, the **Laplace Equation** is $\frac{\delta^2 h}{\delta x^2} = 0$

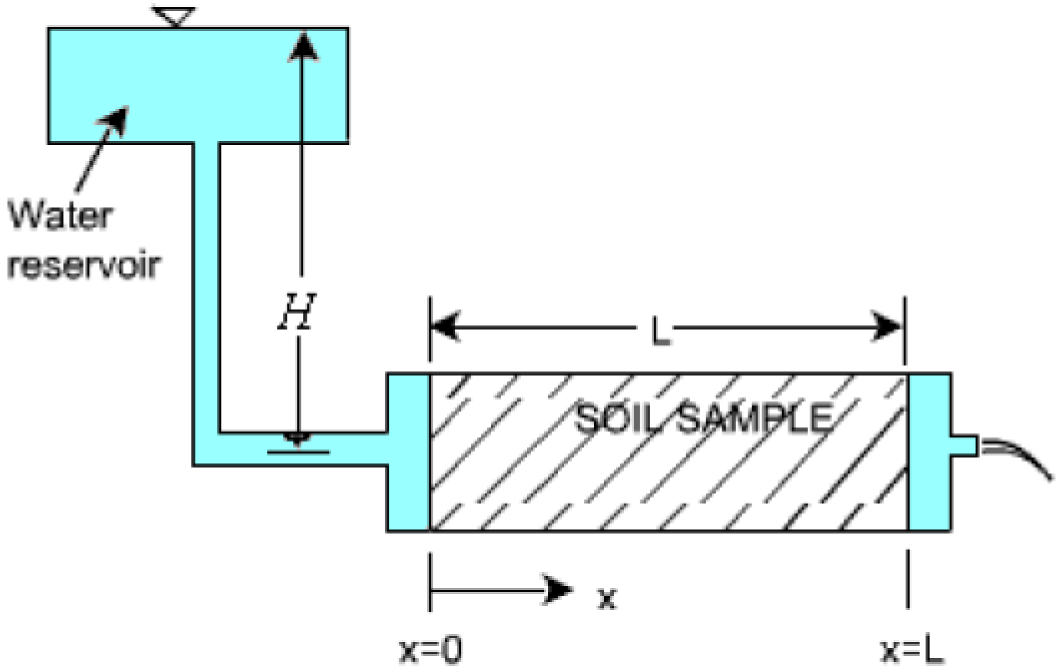
Integrating twice, a general solution is obtained.

$$\frac{\delta h}{\delta x} = c_1$$

$$h = c_2 + c_1 x$$

The values of constants can be determined from the specific boundary conditions.

Two Dimensional Flow



As shown, at x = 0, h = H , and at ***x* = *L*, *h* = *0***

Two Dimensional Flow

Substituting and solving,

$$c_2 = H \, , \quad c_1 = -\frac{H}{L}$$

The specific solution for flow in the above permeameter is

$$h = H - \frac{H}{L}x$$

which states that head is dissipated in a linearly uniform manner over the entire length of the permeameter.

Two Dimensional Flow

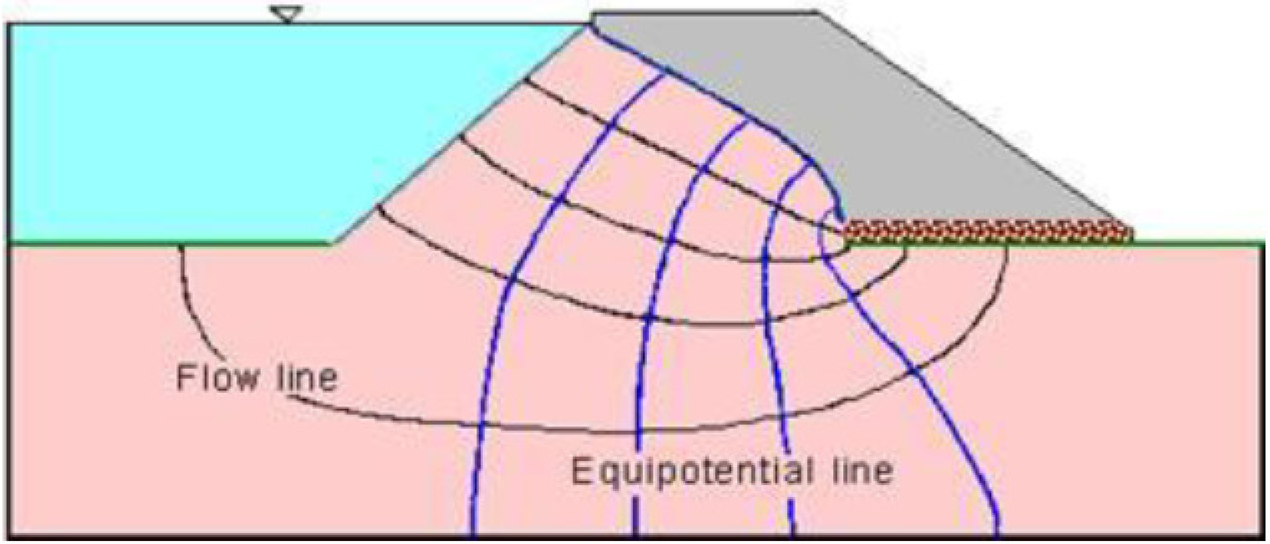
Two-dimensional Flow

Flow Nets Graphical form of solutions to Laplace equation for two-dimensional seepage can be presented as flow nets. Two orthogonal sets of curves form a flow net:

- ☐ **Equipotential lines connecting points of equal total head h**
- ☐ **Flow lines indicating the direction of seepage down a hydraulic gradient**

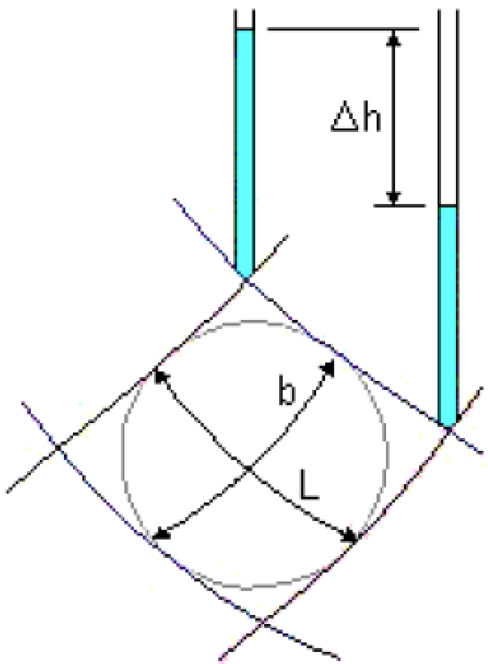
Two flow lines can never meet and similarly, two equipotential lines can never meet. The space between two adjacent flow lines is known as a **flow channel, and the figure formed on the flownet between any two adjacent flow lines and two adjacent equipotential lines is referred to as a field. Seepage through an embankment dam is shown.**

Two Dimensional Flow



Two Dimensional Flow

Calculation of flow in a channel If standpipe piezometers were inserted into the ground with their tips on a single equipotential line, then the water would rise to the same level in each standpipe. The pore pressures would be different because of their different elevations. There can be no flow along an equipotential line as there is no hydraulic gradient.



Two Dimensional Flow

Consider a field of length **L within a flow channel. There is a fall of total head Dh. The average hydraulic gradient is**

$$i = \frac{\Delta h}{L}$$

As the flow lines are **b apart and considering unit length perpendicuar to field, the flow rate is**

$$\Delta q = kb\frac{\Delta h}{L}$$

There is an advantage in sketching flow nets in the form of **curvilinear 'squares' so that a circle can be insrcibed within each four-sided figure bounded by two equipotential lines and two flow lines.**

Two Dimensional Flow

In such a square, b = L , and the flow rate is obtained as Dq = k.Dh Thus the flow rate through such a flow channel is the permeability **k multiplied by the uniform interval Dh between adjacent equipotential lines.**

Calculation of total flow For a complete problem, the flow net can be drawn with the overall head drop h divided into Nd so that Dh = h / Nd. If Nf is the no. of flow channels, then the total flow rate is

$$q = \Delta q . N_f = kh . \frac{N_f}{N_d}$$

Two Dimensional Flow

Procedure for Drawing Flow Nets At every point (x,z) where there is flow, there will be a value of head h(x,z). In order to represent these values, contours of equal head are drawn.

A flow net is to be drawn by trial and error. For a given set of boundary conditions, the flow net will remain the same even if the direction of flow is reversed. Flow nets are constructed

such that the head lost between successive **equipotential lines is the same, say Dh. It is useful in visualising the flow in a soil to plot the flow lines, as these are lines that are tangential to the flow at any given point. The steps of construction are:**

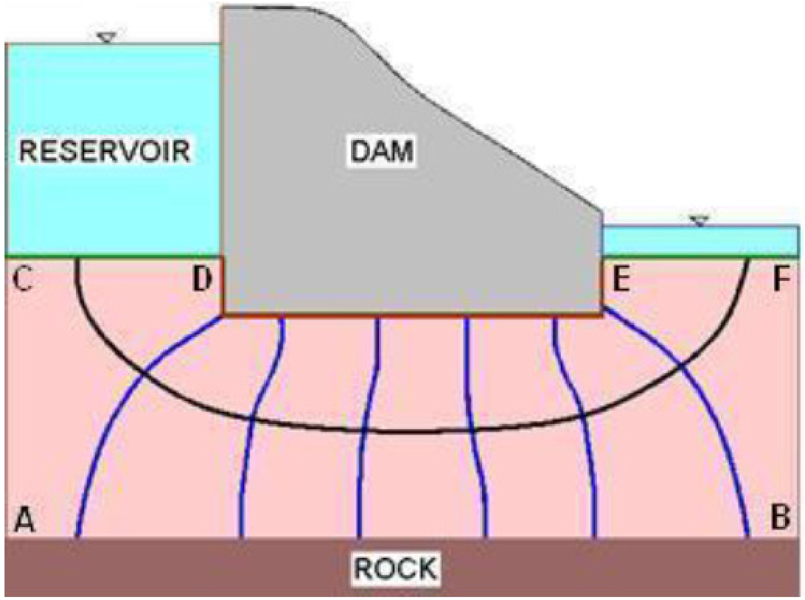
Two Dimensional Flow

1. Mark all boundary conditions, and draw the flow cross section to some convenient scale. 2. Draw a coarse net which is consistent with the boundary conditions and which has orthogonal equipotential and flow lines. As it is usually easier to visualise the pattern of flow, start by drawing the flow lines first. 3. Modify the mesh such that it meets the conditions outlined above and the fields between adjacent flow lines and equipotential lines are 'square'. 4. Refine the flow net by repeating step 3. The most common boundary conditions are:

Two Dimensional Flow

(a) A submerged permeable soil boundary is an equipotential line. This could have been determined by considering imaginary standpipes placed at the soil boundary, as for every point the water level in the standpipe would be the same as the water level. (Such a boundary is marked as CD and EF in the following figure.) (b) The boundary between permeable and impermeable soil materials is a flow line (This is marked as AB in the same figure). (c) Equipotential lines intersecting a phreatic surface do so at equal vertical intervals.

Uses of Flow Nets



Two Dimensional Flow

The graphical properties of a flow net can be used in obtaining solutions for many seepage problems such as: **1.** ***Estimation of seepage losses from reservoirs: It is possible to use the flow net in the transformed space to calculate the flow underneath the dam.***

2. Determination of uplift pressures below dams: From the flow net, the pressure head at any point at the base of the dam can be determined. The uplift pressure distribution along the base can be drawn and then summed up. 3. Checking the possibility of piping beneath dams: At the toe of a dam when the upward exit hydraulic gradient approaches unity, boiling condition can occur leading to erosion in soil and consequent piping. Many dams on soil foundations have failed because of a sudden formation of a piped shaped discharge channel. As the stored water rushes out, the channel widens and catastrophic failure results. This is also often referred to as piping failure

Two Dimensional Flow

Examples