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CHAPTER 3

BEARING CAPACITY OF SHALLOW FOUNDATIONS

3.1 MODES OF FAILURE

Failure is defined as mobilizing the full value of soil shear strength accompanied with excessive settlements. For shallow foundations it depends on soil type, particularly its compressibility, and type of loading. Modes of failure in soil at ultimate load are of three types; these are (see **Fig. 1.5***):*

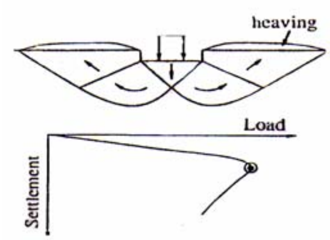
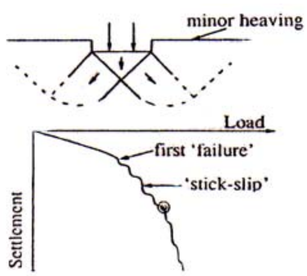
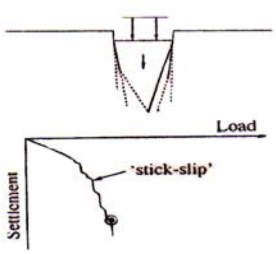
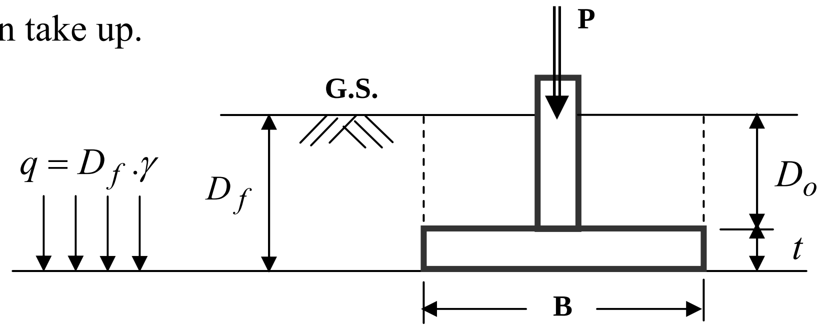
Mode of Failure	Characteristics	Typical Soils
1. General Shear failure  	• Well defined continuous slip surface up to ground level, • Heaving occurs on both sides with final collapse and tilting on one side, • Failure is sudden and catastrophic, • Ultimate value is peak value.	• Low compressibility soils • Very dense sands, • Saturated clays (NC and OC), • Undrained shear (fast loading).
2. local Shear failure (Transition)  	• Well defined slip surfaces only below the foundation, discontinuous either side, • Large vertical displacements required before slip surfaces appear at ground level, • Some heaving occurs on both sides with no tilting and no catastrophic failure, • No peak value, ultimate value not defined.	• Moderate compressibility soils • Medium dense sands,
3. Punching Shear failure  	• Well defined slip surfaces only below the foundation, non either side, • Large vertical displacements produced by soil compressibility, • No heaving, no tilting or catastrophic failure, no ultimate value.	• High compressibility soils • Very loose sands, • Partially saturated clays, • NC clay in drained shear (very slow loading), • Peats.

Fig. (3.1): Modes of failure.

3.2 BEARING CAPACITY CLASSIFICATION ***(According to column loads)***

- **Gross Bearing Capacity (q_{gross}):** It is the total unit pressure at the base of footing which the soil can take up.



q_{gross} = total pressure at the base of footing = $\sum P_{footing} / area.of.footing$.

where $\sum P_{footing} = p.(column.load)$ *+ own wt. of footing + own wt. of earth fill over the footing.* $q_{gross} = (P + \gamma_s.D_o.B.L + \gamma_c.t.B.L)/B.L$

$$q_{gross} = \frac{P}{B.L} + \gamma_s.D_o + \gamma_c.t \quad \textbf{(3.1)}$$

- **Ultimate Bearing Capacity ($q_{ult.}$):** It is the maximum unit pressure or the maximum gross pressure that a soil can stand without shear failure.

- **Allowable Bearing Capacity ($q_{all.}$):** It is the ultimate bearing capacity divided by a reasonable factor of safety.

$$q_{all.} = \frac{q_{ult.}}{F.S} \quad \textbf{(3.2)}$$

- **Net Ultimate Bearing Capacity:** It is the ultimate bearing capacity minus the vertical pressure that is produced on horizontal plain at level of the base of the foundation by an adjacent surcharge.

$$q_{ult.-net} = q_{ult.} - D_f.\gamma \quad \textbf{(3.3)}$$

- **Net Allowable Bearing Capacity ($q_{all.-net}$):** It is the **net safe bearing capacity** or the ultimate bearing capacity divided by a reasonable factor of safety.

Approximate: $$q_{all.-net} = \frac{q_{ult.-net}}{F.S} = \frac{q_{ult.} - D_f \cdot \gamma}{F.S} \quad \textbf{......(3.4)}$$

Exact: $$q_{all.-net} = \frac{q_{ult.}}{F.S} - D_f \cdot \gamma \quad \textbf{......(3.5)}$$

3.3 FACTOR OF SAFETY IN DESIGN OF FOUNDATION

The general values of safety factor used in design of footings are 2.5 to 3.0, however, the choice of factor of safety (F.S.) depends on many factors such as:

1. the variation of shear strength of soil,

2. magnitude of damages,

3. reliability of soil data such as uncertainties in predicting the $q_{ult.}$,

4. changes in soil properties due to construction operations,

5. relative cost of increasing or decreasing F.S., and

6. the importance of the structure, differential settlements and soil strata underneath the structure.

3.4 BEARING CAPACITY REQUIREMENTS

Three requirements must be satisfied in determining bearing capacity of soil. These are:

(1) ***Adequate depth;*** the foundation must be deep enough with respect to environmental effects; such as: frost penetration, seasonal volume changes in the soil, to exclude the possibility of erosion and undermining of the supporting soil by water and wind currents, and to minimize the possibility of damage by construction operations,

(2) ***<u>Tolerable settlements,</u>*** the bearing capacity must be low enough to ensure that both total and differential settlements of all foundations under the planned structure are within the allowable values,

(3) ***<u>Safety against failure,</u>*** this failure is of two kinds:

- the structural failure of the foundation; which may be occur if the foundation itself is not properly designed to sustain the imposed stresses, and
- the bearing capacity failure of the supporting soils.

3.5 FACTORS AFFECTING BEARING CAPACITY

- type of soil (cohesive or cohesionless).
- physical features of the foundation; such as size, depth, shape, type, and rigidity.
- amount of total and differential settlement that the structure can stand.
- physical properties of soil; such as density and shear strength parameters.
- water table condition.
- original stresses.

3.6 METHODS OF DETERMINING BEARING CAPACITY

(a) Bearing Capacity Tables

The bearing capacity values can be found from certain tables presented in building codes, soil mechanics and foundation books; such as that shown in **Table (3.1)**. They are based on experience and can be only used *<u>for preliminary design of light and small buildings as a helpful indication</u>*; *<u>however, they should be followed by the essential laboratory and field soil tests.</u>*

Table (3.1) neglects the effect of: **(i)** underlying strata, **(ii)** size, shape and depth of footings, **(iii)** type of the structures supported by the footings, **(iv)** there is no specification of the physical properties of the soil in question, and **(v)** assumes that the ground water table level is at foundation level or with depth less than width of footing. Therefore, if water table rises above the foundation level, the hydrostatic water pressure force which affects the base of foundation should be taken into consideration.

Table (3.1): Bearing capacity values according to building codes.

Soil type	Description	Bearing pressure (kg/cm^2)		Notes
Rocks	1. bed rocks. 2. sedimentary layer rock (hard shale, sand stone, siltstone). 3. shest or erdwas. 4. soft rocks.	70 30 20 13		Unless they are affected by water.
		Dry	submerged	
Cohesionless soil	1. well compacted sand or sand mixed with gravel. 2. sand, loose and well graded or loose mixed sand and gravel. 3. compacted sand, well graded. 4. well graded loose sand.	3.5-5.0 1.5-3.0 1.5-2.0 0.5-1.5	1.75-2.5 0.5-1.5 0.5-1.5 0.25-0.5	Footing width 1.0 ms.
Cohesive soil	1. very stiff clay 2. stiff clay 3. medium-stiff clay 4. low stiff clay 5. soft clay 6. very soft clay 7. silt soil	2-4 1-2 0.5-1 0.25-0.5 up to 0.2 0.1-0.2 1.0-1.5		It is subjected to settlement due to consolidation

(b) Field Load Test

This test is fully explained in **(chapter 2)**.

(c) Bearing Capacity Equations

Several bearing capacity equations were developed *for the case of **general shear failure** by many researchers as presented in Table (3.2); see **Tables** (3.3, 3.4 and 3.5) for related factors.*

Table (3.2): Bearing capacity equations by the several authors indicated.

- Terzaghi (see **Table 3.3** for typical values for $K_{P\gamma}$ values)

$$q_{ult.} = cN_c.S_c + \bar{q}N_q + 0.5.B.\gamma.N\gamma.S_\gamma$$

$$N_q = \frac{e^{2[0.75\pi.-\frac{\phi}{2}(\frac{\pi}{180})].\tan\phi}}{2\cos^2(45+\phi/2)}; \qquad N_c = (N_q - 1).\cot\phi; \qquad N_\gamma = \frac{\tan\phi}{2}\left(\frac{k_{P\gamma}}{\cos^2\phi} - 1\right)$$

where a close approximation of $k_{P\gamma} \approx 3.\tan^2\left(45 + \frac{(\phi+33)}{2}\right)$.

		Strip	circular	square	rectangular
S_c	=	1.0	1.3	1.3	(1+ 0.3 B / L)
S_γ	=	1.0	0.6	0.8	(1- 0.2 B / L)

- Meyerhof (see **Table 3.4** for shape, depth, and inclination factors)

Vertical load: $q_{ult.} = c.N_c.S_c.d_c + q.N_q.S_q.d_q + 0.5.B.\gamma.N_\gamma.S_\gamma.d_\gamma$

Inclined load: $q_{ult.} = c.N_c.d_c.i_c + q.N_q.d_q.i_q + 0.5.B.\gamma.N_\gamma.d_\gamma.i_\gamma$

$$N_q = e^{\pi.\tan\phi}\tan^2(45+\phi/2); \qquad N_c = (N_q - 1).\cot\phi; \qquad N_\gamma = (N_q - 1).\tan(1.4\phi)$$

- Hansen (see **Table 3.5** for shape, depth, and inclination factors)

For..$\phi > 0$: $q_{ult.} = cN_cS_cd_ci_cg_cb_c + qN_qS_qd_qi_qg_qb_q + 0.5.B.\gamma.N_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$

For..$\phi = 0$: $q_{ult.} = 5.14S_u(1 + S'_c + d'_c - i'_c - b'_c - g'_c) + \bar{q}$

$$N_q = e^{\pi.\tan\phi}\tan^2(45+\phi/2); \qquad N_c = (N_q - 1).\cot\phi; \qquad N_\gamma = 1.5(N_q - 1).\tan\phi$$

- Vesic (see **Table 3.5** for shape, depth, and inclination factors)

Use Hansen's equations above

$$N_q = e^{\pi.\tan\phi}\tan^2(45+\phi/2); \qquad N_c = (N_q - 1).\cot\phi; \qquad N_\gamma = 2(N_q + 1).\tan\phi$$

- *All the bearing capacity equations above are based on general shear failure in soil.*

- **Note:** *Due to <u>scale effects</u>,* N_γ *and then the ultimate bearing capacity decreases with increase in size of foundation.* Therefore, Bowle's (1996) suggested that for (**B > 2m**), with any bearing capacity equation of **Table (3.2)**, the term $(0.5B\gamma.N_\gamma S_\gamma d_\gamma)$ must be multiplied by a reduction factor: $r_\gamma = 1 - 0.25\log\left(\frac{B}{2}\right)$; i.e., $0.5B\gamma.N_\gamma S_\gamma d_\gamma r_\gamma$

B (m)	2	2.5	3	3.5	4	5	10	20	100
r_γ	1	0.97	0.95	0.93	0.92	0.90	0.82	0.75	0.57

Table (3.3): Bearing capacity factors for Terzaghi's equation.

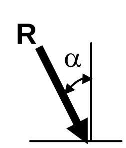
ϕ,..deg	N_c	N_q	N_γ	$K_{P\gamma}$
0	5.7$^+$	1.0	0.0	10.8
5	7.3	1.6	0.5	12.2
10	9.6	2.7	1.2	14.7
15	12.9	4.4	2.5	18.6
20	17.7	7.4	5.0	25.0
25	25.1	12.7	9.7	35.0
30	37.2	22.5	19.7	52.0
34	52.6	36.5	36.0	
35	57.8	41.4	42.4	82.0
40	95.7	81.3	100.4	141.0
45	172.3	173.3	297.5	298.0
48	258.3	287.9	780.1	
50	347.5	415.1	1153.2	800.0

$^+ = 1.5\pi + 1$

Table (3.4): Shape, depth and inclination factors for Meyerhof's equation.

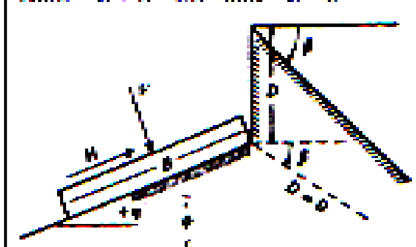
For	**Shape Factors**	**Depth Factors**	**Inclination Factors**
Any ϕ	$S_c = 1 + 0.2.K_P\frac{B}{L}$	$d_c = 1 + 0.2\sqrt{K_P}\frac{D_f}{B}$	$i_c = i_q = \left(1 - \frac{\alpha^\circ}{90^\circ}\right)^2$
$\phi \geq 10^\circ$	$S_q = S_\gamma = 1 + 0.1.K_P\frac{B}{L}$	$d_q = d_\gamma = 1 + 0.1\sqrt{K_P}\frac{D_f}{B}$	$i_\gamma = \left(1 - \frac{\alpha^\circ}{\phi^\circ}\right)^2$
$\phi = 0$	$S_q = S_\gamma = 1.0$	$d_q = d_\gamma = 1.0$	$i_\gamma = 0$

Where: $K_P = \tan^2(45 + \phi/2)$
$\alpha =$ angle of resultant measured from vertical without a sign.
B, L , D_f = width, length, and depth of footing.



Note:- When $\phi_{triaxial}$ is used for plan strain, adjust ϕ as: $\phi_{Ps} = (1.1 - 0.1\frac{B}{L})\phi_{triaxial}$

Table (3.5): Shape, depth, inclination, ground and base factors for use in Hansen or Vesic bearing capacity equations of Table (3.2). (1) Factors apply to either method unless subscripted with (H) or (V). **(2)** Use primed factors when $\phi = 0$.

Shape factors	Depth factors	Inclination factors	Ground Factors (Base on slope)
$S'_c = 0.2\dfrac{B}{L}$ $S_c = 1 + \dfrac{N_q}{N_c} \cdot \dfrac{B}{L}$ S_c = 1.0 for strip $S_q = 1 + \dfrac{B}{L}\tan\phi$ $S_\gamma = 1 - 0.4\dfrac{B}{L}$	$d'_c = 0.4k$ $d_c = 1 + 0.4k$ $d_q = 1 + 2\tan\phi(1-\sin\phi)^2 k$ d_γ = 1.0 for all ϕ $k = \dfrac{D_f}{B}$ for $\dfrac{D_f}{B} \le 1$ $k = \tan^{-1}\dfrac{D_f}{B}$ **(rad)** for $\dfrac{D_f}{B} > 1$	$i'_{c(H)} = 0.5 - 0.5\sqrt{1 - \dfrac{H}{A_f C_a}}$ $i'_{c(V)} = 1 - \dfrac{mH}{A_f C_a N_c}$ $i_c = i_q - \dfrac{1 - i_q}{N_q - 1}$ (Hansen and Vesic) $i_{q(H)} = \left[1 - \dfrac{0.5H}{V - A_f C_a \cot\phi}\right]^5$ $i_{q(V)} = \left[1 - \dfrac{H}{V - A_f C_a \cot\phi}\right]^m$ $i_{\gamma(H)} = \left[1 - \dfrac{0.7H}{V - A_f C_a \cot\phi}\right]^5$ for $(\eta = 0)$ $i_{\gamma(H)} = \left[1 - \dfrac{(0.7 - \eta^\circ/450)H}{V + A_f C_a \cot\phi}\right]^5$ for $(\eta > 0)$ $i_{\gamma(V)} = \left[1 - \dfrac{H}{V - A_f C_a \cot\phi}\right]^{m-1}$	$g'_c = \dfrac{\beta^\circ}{147^\circ}$ For Vesic use: $N_\gamma = -2\sin\beta$ for $\phi = 0$ $g_c = 1 - \dfrac{\beta^\circ}{147^\circ}$ $g_{q(H)} = g_{\gamma(H)} = (1 - 0.5\tan\beta)^5$ $g_{q(V)} = g_{\gamma(V)} = (1 - \tan\beta)^2$ **Base factors (Tilted base)** $b'_c = \dfrac{\eta^\circ}{147^\circ}$ $b_c = 1 - \dfrac{\eta^\circ}{147^\circ}$ $b_{q(H)} = \exp(-2\eta.\pi.\tan\phi/180)$ $b_{\gamma(H)} = \exp(-2.7\eta.\pi.\tan\phi/180)$ $b_{q(V)} = b_{\gamma(V)} = (1 - \eta.\pi.\tan\phi/180)^2$
Where $e_B \ldots e_L$ = Eccentricity of load from center of footing area A_f = Effective footing area $B' \times L'$ C_a = Adhesion to base = cohesion or a reduced value D_f = Depth of footing (used with B and not B') H = Horizontal component of load with $H \le C_a A_f + V\tan\delta$ V = Total vertical load on footing β = Slope of ground away from base with downward = (−) δ = Friction angle between base and soil: usually $\delta = \phi$ for concrete on soil η = Tilt angle of base from horizontal with (−) upward as usual case **GENERAL NOTES** 1. Do not use S_i in combination with i_i. 2. Can use S_i in combination with d_i, g_i, and b_i 3. For $L/B \le 2$ use ϕ_{tr} For $L/B > 2$ use $\phi_{PS} = 1.5\phi_{tr} - 17$ For $\phi \le 34^\circ$ $\phi_{PS} = \phi_{tr}$		$m = m_B = \dfrac{2 + B/L}{1 + B/L}$ for **H** parallel to **B** $m = m_L = \dfrac{2 + L/B}{1 + L/B}$ for **H** parallel to **L** **Note:** $i_q, i_\gamma > 0$	*Note:* $\beta + \eta < 90^\circ$ and $\beta < \phi$ 

3.7 WHICH EQUATIONS TO USE?

Of the bearing capacity equations previously discussed, the most widely used equations are Meyerhof's and Hansen's. While Vesic's equation has not been much used (but is the suggested method in the American Petroleum Institute, RP2A Manual, 1984).

Table (3.6) : Which equations to use.

Use	*Best for*
Terzaghi	• *Very cohesive soils where D/B ≤ 1 or for a quick estimate of $q_{ult.}$ to compare with other methods,* • *Somewhat simpler than Meyerhof's, Hansen's or Vesic's equations; which need to compute the shape, depth, inclination, base and ground factors,* • *Suitable for a concentrically loaded horizontal footing,* • *Not applicable for columns with moment or tilted forces,* • *More conservative than other methods.*
Meyerhof, Hansen, Vesic	• *Any situation which applies depending on user preference with a particular method.*
Hansen, Vesic	• *When base is tilted; when footing is on a slope or when D/B >1.*

3.8 EFFECT OF SOIL COMPRESSIBILITY <u>(local shear failure)</u>

1. For clays sheared in drained conditions, Terzaghi (1943) suggested that the shear strength parameters c and ϕ should be reduced as:

$$c^* = 0.67c' \quad \text{and} \quad \phi^* = tan^{-1}(0.67\, tan\,\phi') \qquad \textbf{(3.6)}$$

2. For loose and medium dense sands (when $D_r \le 0.67$), Vesic (1975) proposed:

$$\phi^* = tan^{-1}(0.67 + D_r - 0.75D_r^2)\, tan\,\phi' \qquad \textbf{(3.7)}$$

where D_r is the relative density of the sand, recorded as a fraction.

Note: <u>For dense sands ($D_r > 0.67$) the strength parameters need not be reduced, since the general shear mode of failure is likely to apply.</u>

BEARING CAPACITY EXAMPLES (1)

Example (1): **Determine the allowable bearing capacity of a strip footing shown below using Terzaghi and Hansen Equations if c = 0, $\phi = 30°$, D_f = 1.0m , B = 1.0m , $\gamma_{soil} = 19$ kN/m^3, the water table is at ground surface, and SF=3.**

Solution:

(a) By Terzaghi's equation:

$$q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma$$

Shape factors: from table (3.2), for strip footing $S_c = S\gamma = 1.0$

Bearing capacity factors: from table (3.3), for $\phi = 30°$, $N_q = 22.5,..N_\gamma = 19.7$

$q_{ult.} = 0 + 1.0\ (19\text{-}9.81)22.5 + 0.5x1(19\text{-}9.81)19.7x1.0 = 297\ kN/m^2$

$q_{all.}$=297/3 = $\boxed{99\ kN/m^2}$

(b) By Hansen's equation:

$for..\phi > 0:$

$$q_{ult.} = cN_cS_cd_ci_cg_cb_c + qN_qS_qd_qi_qg_qb_q + 0.5\gamma.B.N_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$$

Since c = 0, any factors with subscript c do not need computing. Also, all $g_i..and..b_i$ *factors are 1.0; with these factors identified the Hansen's equation simplifies to:*

$q_{ult.} = \overline{q}N_qS_qd_q + 0.5\gamma'.B.N_\gamma S_\gamma d_\gamma$

From table (3.5): $\begin{cases} for...\phi \le 34°..use..\phi_{ps} = \phi_{tr} \\ for\ L/B > 2..use..\phi_{ps} = 1.5\phi_{tr} - 17 \end{cases}$, $\therefore .use.\phi_{ps} = 1.5\phi_{tr} - 17$

$$\therefore .use.\phi_{ps} = 1.5\phi_{tr} - 17,\quad 1.5\ x\ 30 - 17 = 28°,$$

Bearing capacity factors: from table (3.4), for $\phi = 28°$, $N_q = 14.7,..N_\gamma = 10.9$

Shape factors: from table (3.5), $S_\gamma = S_q = 1.0,$

Depth factors: from table (3.5),

$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2 \frac{D}{B},$$

$$d_q = 1 + 2.\tan 28(1-\sin 28)^2 \frac{1}{1} = 1.29, \quad and \quad d_\gamma = 1.0$$

$q_{ult.} = 1.0\ (19\text{-}9.81)14.7x1.29 + 0.5x1(19\text{-}9.81)10.9x1.0 = 224.355\ kN/m^2$

$q_{all.}$=224.355/3 = $\boxed{74.785\ kN/m^2}$

Example (2): A footing load test produced the following data:

D_f **= 0.5m, B = 0.5m, L = 2.0m,** $\gamma'_{soil} = 9.31$ **kN/m^3,** $\phi_{tr} = 42.5°$**, c = 0,**

$P_{ult.}(measured) = 1863.kN$**,** $q_{ult.}(measured) = 1863/0.5x2 = 1863$ **kN/m^2.**

Required: compute $q_{ult.}$ by Hansen's and Meyerhof's equations and compare computed with measured values.

Solution:

(a) By Hansen's equation:

Since c = 0, and all $g_i..and..b_i$ *factors are 1.0; the Hansen's equation simplifies to:*

$$q_{ult.} = \overline{q}N_q S_q d_q + 0.5\gamma'.B.N_\gamma S_\gamma d_\gamma$$

From table (3.5): L/B = 2/0.5 = 4 > 2 $\therefore ..use..\phi_{ps} = 1.5\phi_{tr} - 17,$

$1.5 \text{ x } 42.5 - 17 = 46.75° \longrightarrow take...\phi = 47°$

Bearing capacity factors: from table (3.2)

$$N_q = e^{\pi.\tan\phi}..\tan^2(45+\phi/2), \quad N_\gamma = 1.5(N_q - 1)\tan\phi$$

for $\phi = 47°$: $\quad N_q = 187.2, \quad N_\gamma = 299.5$

Shape factors: from table (3.5),

$$S_q = 1 + \frac{B}{L}\tan\phi = 1 + \frac{0.5}{2.0}\tan 47 = 1.27, \qquad S_\gamma = 1 - 0.4\frac{B}{L} = 1 - 0.4\frac{0.5}{2.0} = 0.9$$

Depth factors: from table (3.5),

$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2\frac{D}{B}, d_q = 1 + 2\tan 47(1-\sin 47)^2\frac{0.5}{0.5} = 1.155, d_\gamma = 1.0$$

$$q_{ult.} = 0.5\ (9.31)187.2x1.27x1.155 + 0.5x0.5(9.31)299.5x0.9x1.0 = \boxed{1905.6\ kN/m^2}$$

versus 1863 kN/m² measured.

(b) By Meyerhof's equation:

From table (3.2) for vertical load with c = 0:

$$q_{ult.} = \bar{q}N_q S_q d_q + 0.5\gamma'.B.N_\gamma S_\gamma d_\gamma$$

From table (3.4): $\phi_{ps} = (1.1 - 0.1\frac{B}{L})\phi_{tr}$, $(1.1 - 0.1\frac{0.5}{2.0})42.5 = 45.7$, take... $\phi = 46°$

Bearing capacity factors: from table (3.2)

$$N_q = e^{\pi.\tan\phi}..\tan^2(45+\phi/2), \quad N_\gamma = (N_q - 1)\tan(1.4\phi)$$

for $\phi = 46°$: $N_q = 158.5$, $N_\gamma = 328.7$

Shape factors: from table (3.4)

$$K_p = \tan^2(45+\phi/2) = 6.13, \qquad S_q = S_\gamma = 1 + 0.1.K_p\frac{B}{L} = 1 + 0.1(6.13)\frac{0.5}{2.0} = 1.15$$

Depth factors: from table (3.4)

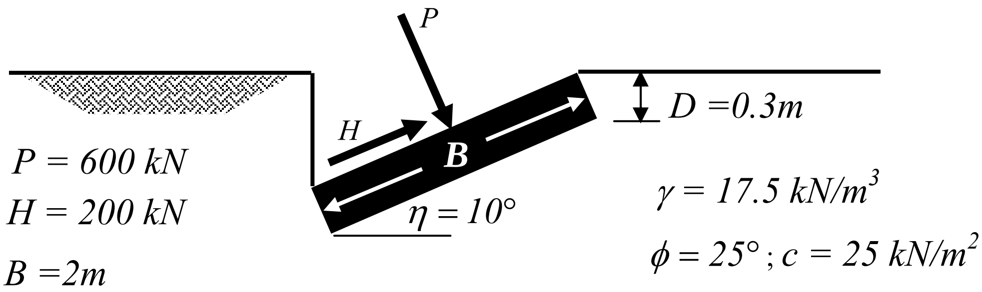
$$\sqrt{K_p} = 2.47, \qquad d_q = d_\gamma = 1 + 0.1.\sqrt{K_p}\frac{D}{B} = 1 + 0.1(2.47)\frac{0.5}{0.5} = 1.25$$

$$q_{ult.} = 0.5(9.31)158.5x1.15x1.25 + 0.5x0.5(9.31)328.7x1.15x1.25 = 2160.4\ kN/m^2$$

versus 1863 kN/m² measured

$\therefore$ **Both Hansen's and Meyerhof's eqs. give over-estimated** $q_{ult.}$ **compared with measured.**

Example (3): **A 2.0x2.0m footing has the geometry and load as shown below. Is the footing adequate with a SF=3.0?.**



Solution:

We can use either Hansen's, or Meyerhof's or Vesic's equations. An arbitrary choice is Hansen's method.

Check sliding stability:

use $\delta = \phi;\ C_a = c$ and $A_f = 2x2 = 4m^2$

$H_{max.} = A_f C_a + V \tan\delta = 4x25 + 600 \tan 25° = 280 > 200\ kN$ ***(O.K. for sliding)***

Bearing capacity By Hansen's equation:

$with..inclination..factors..all..S_i = 1.0$

$$q_{ult.} = cN_c.d_c.i_c.b_c + \bar{q}N_q.d_q.i_q.b_q + 0.5\gamma.B.N_\gamma.d_\gamma.i_\gamma.b_\gamma$$

Bearing capacity factors from table (3.2):

$N_c = (N_q - 1).\cot\phi$, $\quad N_q = e^{\pi.\tan\phi}..\tan^2(45 + \phi/2)$, $\quad N_\gamma = 1.5(N_q - 1)\tan\phi$

for $\phi = 25°$: $\quad N_c = 20.7$, $\quad N_q = 10.7$, $\quad N_\gamma = 6.8$

Depth factors from table (3.5):

for D =0.3m, and B = 2m, D/B = 0.3/2=0.15 < 1.0 (shallow footing)

$d_c = 1 + 0.4\dfrac{D}{B} = 1 + 0.4(0.15) = 1.06$, $d_q = 1 + 2\tan\phi(1 - \sin\phi)^2 \dfrac{D}{B} = 1 + 0.311(0.15) = 1.05$,

$d_\gamma = 1.0$

Inclination factors from table (3.5):

$$i_q = (1 - \frac{0.5H}{V + A_f.c.\cot\phi})^5 = (1 - \frac{0.5x200}{600 + 4x25x\cot 25})^5 = 0.52,$$

$$i_c = i_q - \frac{(1 - i_q)}{(N_q - 1)} = 0.52 - \frac{1 - 0.52}{10.7 - 1} = 0.47,$$

$$for..\eta > 0 : i_\gamma = (1 - \frac{(0.7 - \eta^\circ / 450)H}{V + A_f.c.\cot\phi})^5 = (1 - \frac{(0.7 - 10/450)200}{600 + 4x25x\cot 25})^5 = 0.40$$

The base factors $for..\eta = 10^\circ (0.175..radians)$ *from table (3.5):*

$$b_c = 1 - \frac{\eta^\circ}{147^\circ} = 1 - \frac{10}{147} = 0.93,$$

$$b_q = e^{(-2\eta\tan\phi)} = e^{(-2(0.175)\tan 25)} = 0.85, \quad b_\gamma = e^{(-2.7\eta\tan\phi)} = e^{(-2.7(0.175)\tan 25)} = 0.80$$

$$q_{ult.} = 25(20.7)(1.06)(0.47)(0.93) + 0.3(17.5)(10.7)(1.05)(0.52)(0.85)$$
$$+ 0.5(17.5)(2.0)(6.8)(1)(0.40)(0.80) = 304\ kN/m^2$$

$$q_{all.} = 304/3 = 101.3\ kN/m^2$$

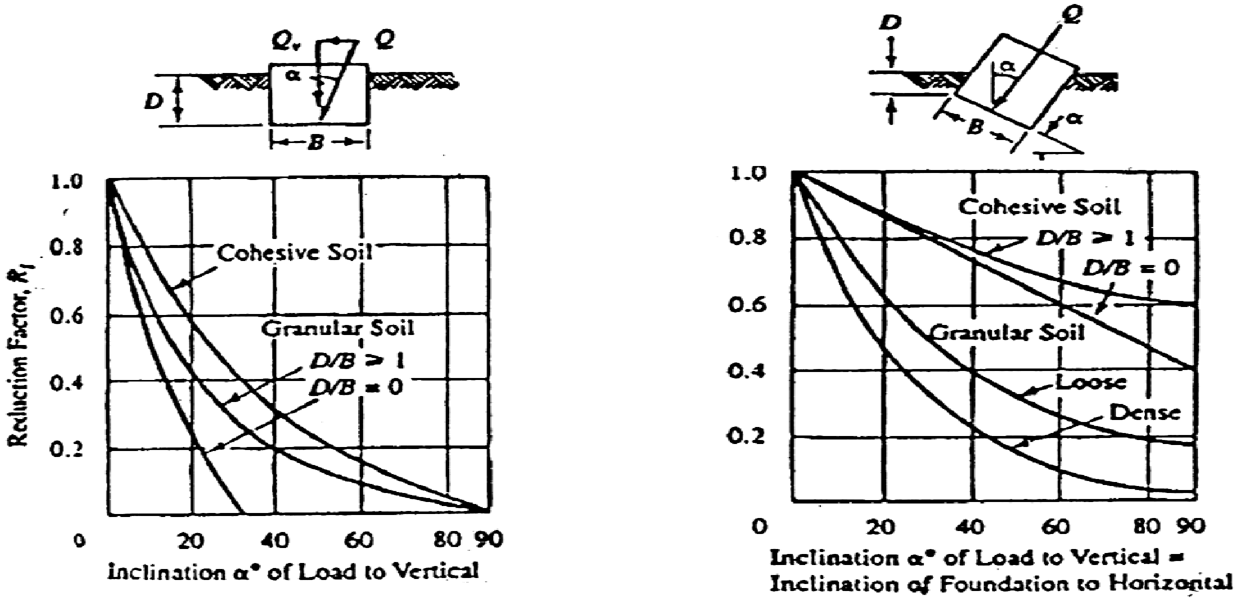
$P_{all.} = q_{all.}.A_f = 101.3(4) = 405.2\ kN < 600\ kN$ *(the given load),* $\therefore$ *B=2m is not adequate and, therefore it must be increased and* $P_{all.}$ *recomputed and checked.*

3.9 FOOTINGS WITH INCLINED OR ECCENTRIC LOADS

- **INCLINED LOAD:**

If a footing is subjected to an inclined load (see Fig.3.7), the inclined load Q can be resolved into vertical and horizontal components. The vertical component Q_v can then be used for bearing capacity analysis in the same manner as described previously (Table 3.2). After the bearing capacity has been computed by the normal procedure, it must be corrected by an R_i factor using Fig.(3.7) as:

$$\therefore \quad \boxed{q_{ult.(inclined..load)} = q_{ult.(vertical..load)}.x.R_i} \quad \text{..}(3.8)$$



(a) horizontal foundation ***(b) Inclined foundation***

Figure (3.7): Inclined load reduction factors.

Important Notes:

- *Remember that in this case, Meyerhof's bearing capacity equation for inclined load (from Table 3.2) can be used directly:*

$$\boxed{q_{ult.(inclined..load)} = cN_c d_c i_c + \overline{q} N_q d_q i_q + 0.5\gamma'.B.N_\gamma d_\gamma i_\gamma} \text{ ....(3.9)}$$

- The footings stability with regard to the inclined load's horizontal component also must be checked by calculating ***the factor of safety against sliding*** as follows:

$$Fs_{(slididing)} = \frac{H_{max.}}{H} \text{ ....(3.10)}$$

where:

H = *the inclined load's horizontal component,*

$\boxed{H_{max.} = the.\max imum.resisting.force = A'_f.C_a + \sigma' \tan\delta \text{ .... } for\ (c-\phi)\ soils;}$ *or*

$H_{max.} = A'_f.C_a$ .... *for the undrained case in clay* ($\phi_u = 0$); *or*

$H_{max.} = \sigma' \tan\delta$ .... *for a sand and the drained case in clay* $(c' = 0)$.

$A'_f = effective..area = B'.L'$

$C_a = adhesion = \alpha.C_u$

$where...\alpha = 1.0....for.soft.to.medium.clays.; and$

$.\alpha = 0.5....for.stiff.clays.$

σ' = the net vertical effective load = $Q_v - D_f.\gamma$; *or*

$\sigma' = (Q_v - D_f.\gamma) - u.A'_f$ *(if the water table lies above foundation level)*

δ = *the skin friction angle, which can be taken as equal to* (ϕ')*,and*

u = the pore water pressure at foundation level.

- **ECCENTRIC LOAD:**

Eccentric load result from loads applied somewhere other than the footing's centroid or from applied moments, such as those resulting at the base of a tall column from wind loads or earthquakes on the structure.

To provide adequate $SF_{(against.lifting)}$ *of the footing edge, it is recommended that the eccentricity* $(e \le B/6)$. Footings with eccentric loads may be analyzed for bearing capacity by two methods: (1) the concept of useful width and (2) application of reduction factors.

(1) Concept of Useful Width:

In this method, only that part of the footing that is symmetrical with regard to the load is used to determine bearing capacity by the usual method, with the remainder of the footing being ignored.

- *First, computes eccentricity and adjusted dimensions:*

$$e_x = \frac{M_y}{V}; \quad L' = L - 2e_x; \quad e_y = \frac{M_x}{V}; \quad B' = B - 2e_y; \quad A'_f = A' = B'.L'$$

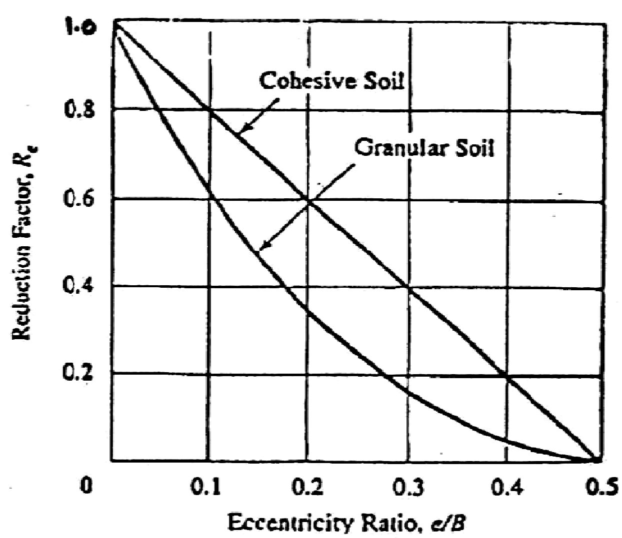
- *Second, calculates $q_{ult.}$ from Meyerhof's, or Hansen's, or Vesic's equations (Table 3.2) using B' in the $(\frac{1}{2}B.\gamma.N_\gamma)$ term and B' or/and L' in computing the shape factors and not in computing depth factors.*

(2) Application of Reduction Factors:

First, computes bearing capacity by the normal procedure (using equations of Table 3.2), assuming that the load is applied at the centroid of the footing. Then, the computed value is corrected for eccentricity by a reduction factor (R_e) obtained from Figure (3.8) or from Meyerhof's reduction equations as:

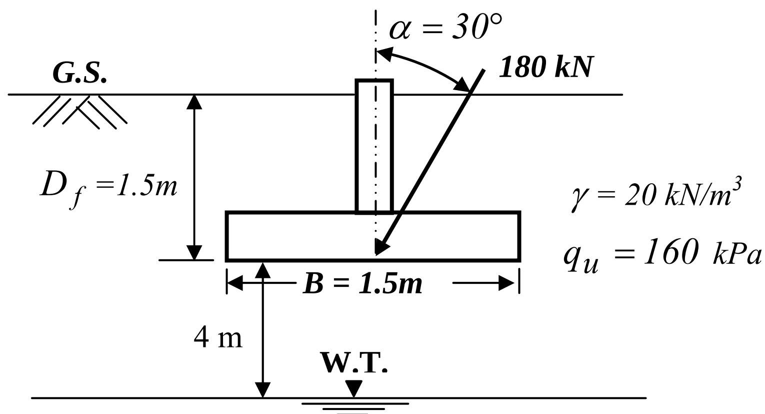
$$\left.\begin{aligned} R_e &= 1 - 2(e/B) \text{.........for..cohesive..soil} \\ R_e &= 1 - (e/B)^{1/2} \text{........for.cohesionless.soil} \end{aligned}\right\} \text{......................}\textbf{(3.11)}$$

$$\therefore \quad \boxed{q_{ult.(eccentric)} = q_{ult.(concentric)}.x.R_e} \text{..................................}\textbf{(3.12)}$$



Figure(3.8): Eccentric load reduction factors.

<u>Example (4):</u> A square footing of 1.5x1.5m is subjected to an inclined load as shown in figure below. What is the factor of safety against bearing capacity (use Terzaghi's equation).



Solution:

By Terzaghi's equation: $q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma$

Shape factors: from table (3.2) for square footing $S_c = 1.3; S\gamma = 0.8$, $c = q_u / 2 = 80$ *kPa*

Bearing capacity factors: from table (3.3) for $\phi_u = 0$: $N_c = 5.7, N_q = 1.0, N_\gamma = 0$

$q_{ult.(vertical.load)} = 80(5.7)(1.3) + 20(1.5)(1.0) + 0.5(1.5)(20)(0)(0.8) = 622.8\ kN/m^2$

From Fig.(3.7) with $\alpha = 30°$ *and cohesive soil, the reduction factor for inclined load is 0.42.*

$q_{ult.(inclined.load)} = 622.8(0.42) = 261.576\ kN/m^2$

$Q_v = Q.\cos 30 = 180\ (0.866) = 155.88\ kN$

Factor of safety (against bearing capacity failure) $= \frac{Q_{ult.}}{Q_v} = \frac{261.576(1.5)(1.5)}{155.88} = 3.77$

Check for sliding:

$Q_h = Q.\sin 30 = 180\ (0.5) = 90\ kN$

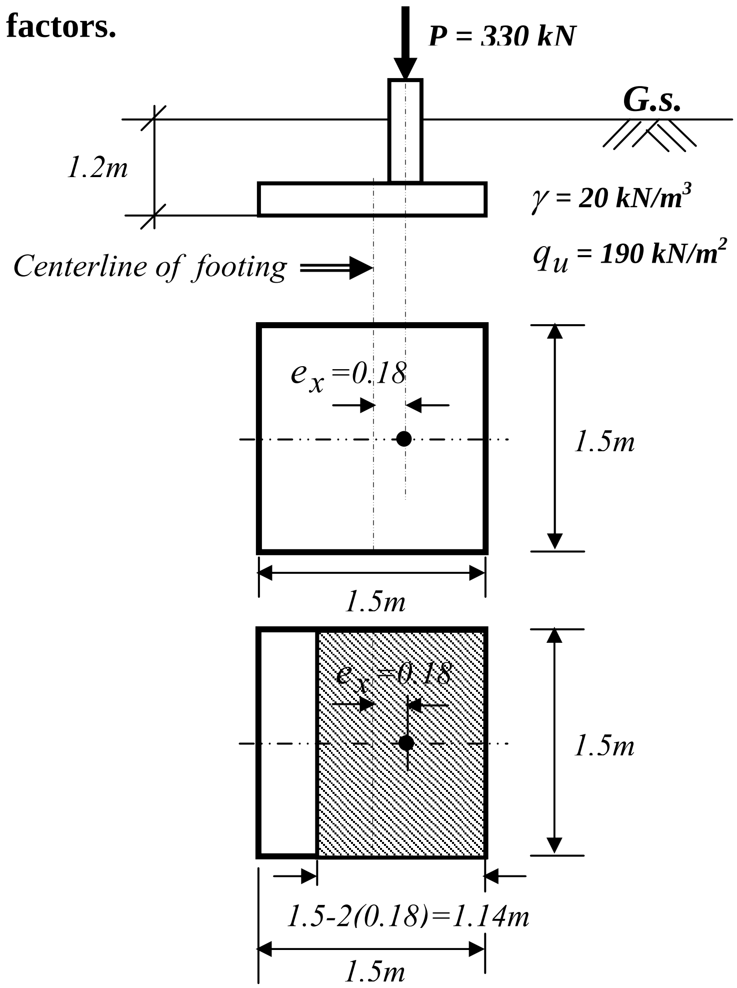
$H_{max.} = A'_f.C_a + \sigma' \tan\delta = (1.5)(1.5)(80) + (180)(\cos 30)(\tan 0) = 180\ kN$

Factor of safety (against sliding) $= \frac{H_{max.}}{Q_h} = \frac{180}{90} = 2.0$ **(O.K.)**

Example (5): **A 1.5x1.5m square footing is subjected to eccentric load as shown below. What is the safety factor against bearing capacity failure (use Terzaghi's equation):**

(a) By the concept of useful width, and

(b) Using Meyerhof's reduction factors.



Solution:

(1) Using concept of useful width:

from Terzaghi's equation:

$$q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B'.\gamma.N\gamma.S_\gamma$$

Shape factors: from table (3.2) for square footing $S_c = 1.3; S\gamma = 0.8,\ c = q_u / 2 = 95\ kPa$

Bearing capacity factors: from table (3.3) for $\phi_u = 0:\ N_c = 5.7, N_q = 1.0, N_\gamma = 0$

The useful width is: $B' = B - 2e_x = 1.5 - 2(0.18) = 1.14m$

$q_{ult.} = 95(5.7)(1.3) + 20(1.2)(1.0) + 0.5(1.14)(20)(0)(0.8) = 727.95\ kN/m^2$

Factor of safety (against bearing capacity failure) $= \frac{Q_{ult.}}{Q_v} = \frac{727.95(1.14)(1.5)}{330} = 3.77$

(2) *Using Meyerhof's reduction factors:*

In this case, $q_{ult.}$ *is computed based on the actual width: B = 1.5m*

from Terzaghi's equation:

$$q_{ult.} = 1.3cN_c + qN_q + 0.4B.\gamma.N\gamma$$

$$q_{ult.(concentric.load)} = 1.3(95)(5.7) + 20(1.2)(1.0) + 0.4(1.5)(20)(0) = 727.95\ kN/m^2$$

For eccentric load from figure (3.8):

$$\text{with Eccentricity ratio} = \frac{e_x}{B} = \frac{0.18}{1.5} = 0.12\ ;\ \text{and cohesive soil } R_e = 0.76$$

$$\therefore q_{ult.(eccentric.load)} = 727.95\ (0.76) = 553.242\ kN/m^2$$

$$\text{Factor of safety (against bearing capacity failure)} = \frac{Q_{ult.}}{Q_v} = \frac{553.242(1.5)(1.5)}{330} = 3.77$$

Example (6): **A square footing of 1.8x1.8m is loaded with axial load of 1780 kN and subjected to M_x = 267 kN-m and M_y = 160.2 kN-m moments. Undrained triaxial tests of unsaturated soil samples give $\phi = 36°$ and $c = 9.4$ kN/m^2. If D_f = 1.8m, the water table is at 6m below the G.S. and $\gamma = 18.1$ kN/m^3, what is the allowable soil pressure if SF=3.0 using (a) Hansen bearing capacity and (b) Meyerhof's reduction factors.**

Solution:

$$e_y = \frac{267}{1780} = 0.15m\ ; \qquad e_x = \frac{160.2}{1780} = 0.09m$$

$$B' = B - 2e_y = 1.8 - 2(0.15) = 1.5m\ ; \quad L' = L - 2e_x = 1.8 - 2(0.09) = 1.62m$$

(a) Using Hansen's equation:

$$(with...all...i_i, g_i..and...b_i..factors...are...1.0)$$

$$q_{ult.} = cN_c.S_c.d_c + \overline{q}N_q.S_q.d_q + 0.5\gamma.B'.N_\gamma.S_\gamma.d_\gamma$$

Bearing capacity factors from table (3.2):

$$N_c = (N_q - 1).\cot\phi, \qquad N_q = e^{\pi.\tan\phi}..\tan^2(45 + \phi/2), \qquad N_\gamma = 1.5(N_q - 1)\tan\phi$$

$for\ \phi = 36°:\quad N_c = 50.6,\qquad N_q = 37.8,\qquad N_\gamma = 40$

Shape factors from table (3.5):

$$S_c = 1 + \frac{N_q}{N_c}\frac{B'}{L'} = 1 + \frac{37.8}{50.6}\frac{1.5}{1.62} = 1.692,\qquad S_q = 1 + \frac{B'}{L'}\tan\phi = 1 + \frac{1.5}{1.62}\tan 36 = 1.673$$

$$S_\gamma = 1 - 0.4\frac{B'}{L'} = 1 - 0.4\frac{1.5}{1.62} = 0.629$$

Depth factors from table (3.5):

for D =1.8m, and B = 1.8m, D/B = 1.0 (shallow footing)

$$d_c = 1 + 0.4\frac{D}{B} = 1 + 0.4(1.0) = 1.4,$$

$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2\frac{D}{B} = 1 + 2\tan 36(1-\sin 36)^2(1.0) = 1.246,\quad d_\gamma = 1.0$$

$q_{ult.}$ = 9.4(50.6)(1.692)(1.4) + 1.8(18.1)(37.7)(1.673)(1.246)

+ 0.5(18.1)(1.5)(40)(0.629)(1)= 4028.635 kN/m^2

$q_{all.} = 4028.635 / 3 = 1342.878$ kN/m^2

Actual soil pressure ($q_{act.}$) = 1780/(1.5)(1.62)= 732.510 < 1342.878 **(O.K.)**

(b) Using Meyerhof's reduction:

$$R_{ex} = 1 - (\frac{e_x}{L})^{1/2} = 1 - (\frac{0.09}{1.8})^{0.5} = 0.78;\qquad R_{ey} = 1 - (\frac{e_y}{B})^{1/2} = 1 - (\frac{0.15}{1.8})^{0.5} = 0.72$$

Recompute $q_{ult.}$ as for a centrally loaded footing, since the depth factors are unchanged.

The revised Shape factors from table (3.5) are:

$$S_c = 1 + \frac{N_q}{N_c}\frac{B}{L} = 1 + \frac{37.8}{50.6}\frac{1.8}{1.8} = 1.75;\qquad S_q = 1 + \frac{B}{L}\tan\phi = 1 + \frac{1.8}{1.8}\tan 36 = 1.73$$

$$S_\gamma = 1 - 0.4\frac{B}{L} = 1 - 0.4\frac{1.8}{1.8} = 0.60$$

$$q_{ult.} = cN_c.S_c.d_c + \overline{q}N_q.S_q.d_q + 0.5\gamma.B.N_\gamma.S_\gamma.d_\gamma$$

$q_{ult.} = 9.4(50.6)(1.75)(1.4) + 1.8(18.1)(37.7)(1.73)(1.246)$

$+ 0.5(18.1)(1.8)(40)(0.60)(1) = 4212.403\ kN/m^2$

$q_{all.centrally.loaded.footing} = 4212.403 / 3 = 1404.134\ kN/m^2$

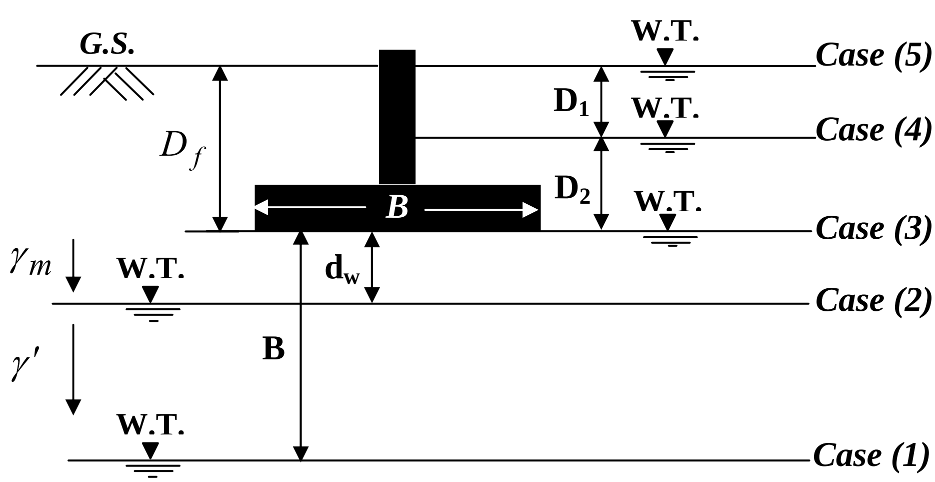
$q_{all.eccentric.loaded..footing} = q_{all.centrally.loaded.footing}(R_{ex})(R_{ey})$

$= 1404.134(0.78)(0.72) = 788.35\ kN/m^2$ *(very high)*

Actual soil pressure ($q_{act.}$) $= 1780/(1.8)(1.8) = 549.383 < 788.35$ **(O.K.)**

3.10 EFFECT OF WATER TABLE ON BEARING CAPACITY

Generally the submergence of soils will cause loss of all apparent cohesion, coming from capillary stresses or from weak cementation bonds. At the same time, the effective unit weight of submerged soils will be reduced to about one-half the weight of the same soils above the water table. Thus, through submergence, all the three terms of the bearing capacity (B.C.) equations may be considerably reduced. Therefore, it is essential that the B.C. analysis be made assuming the highest possible groundwater level at the particular location for the expected life time of the structure.



Case (1):

If the water table (W.T.) lies at B or more below the foundation base; no W.T. effect.

Case (2):

- *<u>(from Ref.;Foundation Engg. Hanbook):</u> if the water table (W.T.) lies within the depth (d_w<B) ; (i.e., between the base and the depth B), use* $\gamma_{av.}$ *in the term* $\frac{1}{2}\gamma.B.N_\gamma$ *as:*

$$\gamma_{av.} = \gamma' + (d_w / B)(\gamma_m - \gamma') \text{……..………..……….(from Meyerhof)}$$

- *<u>(from Ref.;Foundation Analysis and Design):</u> if the water table (W.T.) lies within the wedge zone* $\{H = 0.5B.tan(45+\phi/2)\}$*; use* $\gamma_{av.}$ *in the term* $\frac{1}{2}\gamma.B.N_\gamma$ *as:*

$$\gamma_{av.} = (2H - d_w)\frac{d_w}{H^2}.\gamma_{wet} + \frac{\gamma'}{H^2}(H - d_w)^2 \text{……….(from ,Bowles)}$$

where:

$H = 0.5B.tan(45+\phi/2)$.
γ' = *submerged unit weight =*($\gamma_{sat.} - \gamma_w$),
d_w = *depth to W.T. below the base of footing,*
$\gamma_m = \gamma_{wet}$ = *moist or wet unit weight of soil in depth* (d_w) *, and*

- *<u>Snice in many cases of practical purposes, the term</u>* $\frac{1}{2}\gamma.B.N_\gamma$ *<u>can be ignored for conservative results, it is recommended for this case to use</u>* $\gamma = \gamma'$ *<u>in the term</u>* $\frac{1}{2}\gamma.B.N_\gamma$ *<u>instead of</u>* $\gamma_{av.}$

$$(\gamma' < \gamma_{av.}(from..Meyerhof) < \gamma_{av.}(from..Bowles))$$

Case (3): *if* $d_w = 0$ *; the water table (W.T.) lies at the base of the foundation; use* $\gamma = \gamma'$

Case (4): *if the water table (W.T.) lies above the base of the foundation; use:*

$$q = \gamma_t.D_{1(above..W.T.)} + \gamma'.D_{2(below..W.T.)} \text{ and } \gamma = \gamma' \text{ in } \frac{1}{2}\gamma.B.N_\gamma \text{ term.}$$

Case (5): *if the water table (W.T.) lies at ground surface (G.S.); use:* $q = \gamma'.D_f$ *and*

$\gamma = \gamma'$ *in* $\frac{1}{2}\gamma.B.N_\gamma$ *term.*

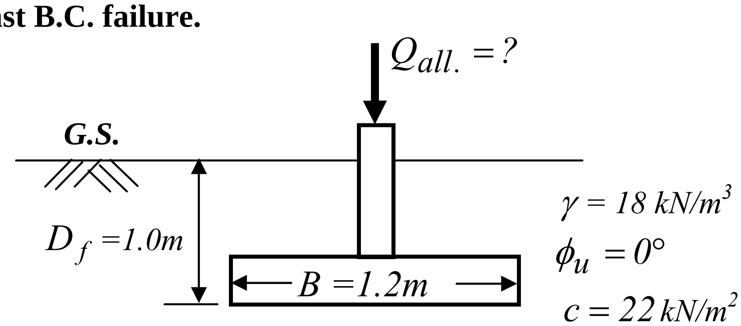
Note: *All the preceding considerations are based on the assumption that the seepage forces acting on soil skeleton are negligible. The seepage force adds a component to the body forces caused by gravity. This component acting in the direction of stream lines is equal to* $(i.\gamma_w)$*, where* i *is the hydraulic gradient causing seepage.*

Example (7): A (1.2x4.2)m rectangular footing is placed at a depth of (D_f =1m) below the G.S. in clay soil with $\phi_u = 0°$, $\gamma = 18$ kN/m^3, $C_u = 22$ kN/m^2. Find the allowable maximum load which can be applied under the following conditions:

(a) W.T. at base of footing with $\gamma_{sat} = 20$ kN/m^3,

(b) W.T. at 0.5m below the surface and $\gamma_{sat} = 20$ kN/m^3,

(c) If the applied load is 400kN and the W.T. at the surface what will be the factor of safety of the footing against B.C. failure.



Solution:

L/B = 4.2/1.2 = 3.5 < 5 ∴ rectangular footing,

D/B= 1/1.2 = 0.833 < 1.0 ∴shallow footing; therefore Terzaghi's equation is suitable.

By Terzaghi's equation: $q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma$

Shape factors: from table (3.2), for rectangular footing $S_c = (1 + 0.3\frac{B}{L})$; $S_\gamma = (1 - 0.2\frac{B}{L})$

Bearing capacity factors: from table (3.3), for $\phi = 0°$, $N_c = 5.7$, $N_q = 1.0$,..$N_\gamma = 0$

(a) for W.T. at base of footing:

$$q_{ult.} = (22)(5.7)\ (1+0.30\ \frac{1.2}{4.2}) + 1.0(18)(1)$$

$$+ 0.5(1.2)(20\text{-}10)(0)(1\text{-}0.20\frac{1.2}{4.2}) = 154.148\ kN/m^2$$

$q_{all.} = 154.148\ /3 = 51.388\ kN/m^2$

$Q_{all.} = 51.388(1.2x4.2) = \boxed{258.970\ kN}$

(b) for W.T. at 0.5m below the surface:

$$q = \gamma_t.D_{1(above..W.T.)} + \gamma'.D_{2(below..W.T.)}$$

$D_1 = 0.5$ and $D_2 = 0.5$; $q = 18(0.5) + (20 - 10)(0.5) = 14\ kN/m^2$

$$q_{ult.} = (22)(5.7)\ (1+0.30\ \frac{1.2}{4.2}) + 1.0(14)(1)$$

$$+ 0.5(1.2)(20\text{-}10)(0)(1\text{-}0.20\frac{1.2}{4.2}) = 150.148\ kN/m^2$$

$q_{all.} = 150.148\ /3 = 50.049\ kN/m^2$

$Q_{all.} = 50.049(1.2x4.2) = \boxed{252.249\ kN}$

(c) If the applied load is 400kN and the W.T. at the surface what will be the factor of safety of the footing against B.C. failure?.

$Q_{all.} = 400\ kN$; $q_{all.} = 400/(1.2(4.2) = 79.36\ kN/m^2$; $q = D_f.\gamma' = (1)(20\text{-}10) = 10\ kN/m^2$

$$q_{ult.} = (22)(5.7)\ (1+0.30\ \frac{1.2}{4.2}) + 10(1) + 0.5(1.2)(20\text{-}10)(0)(1\text{-}0.20\frac{1.2}{4.2}) = 146.14\ kN/m^2$$

$$SF = \frac{q_{ult.}}{q_{all.}} = \frac{146.14}{79.36} = \boxed{1.8}$$

3.11 Bearing Capacity For Footings On Layered Soils

Stratified soil deposits are of common occurrence. It was found that when a footing is placed on stratified soils and the thickness of the top stratum form the base of the footing (d_1 or H) is less than the depth of penetration [$H_{crit.} = 0.5B\tan(45+\phi/2)$]; in this case the rupture zone will extend into the lower layer (s) depending on their thickness and therefore require some modification of ultimate bearing capacity (qult.).

Several solutions have been proposed to estimate the bearing capacity of footings on layered soils, however, they are limited for the following three general cases:

Case (1): Footing on layered clays (all ϕ = 0):

(a) Top layer stronger than lower layer ($C_2/C_1 \leq 1$).

(b) Top layer weaker than lower layer ($C_2/C_1 > 1$).

For clays in undrained condition (ϕ_u= 0), the undrained shear strength (S_u or c_u) can be determined from unconfined compressive (q_u) tests. So that assuming a circular slip surface of the soil shear failure pattern, may give reasonably reliable results (***see figure (3.9)***).

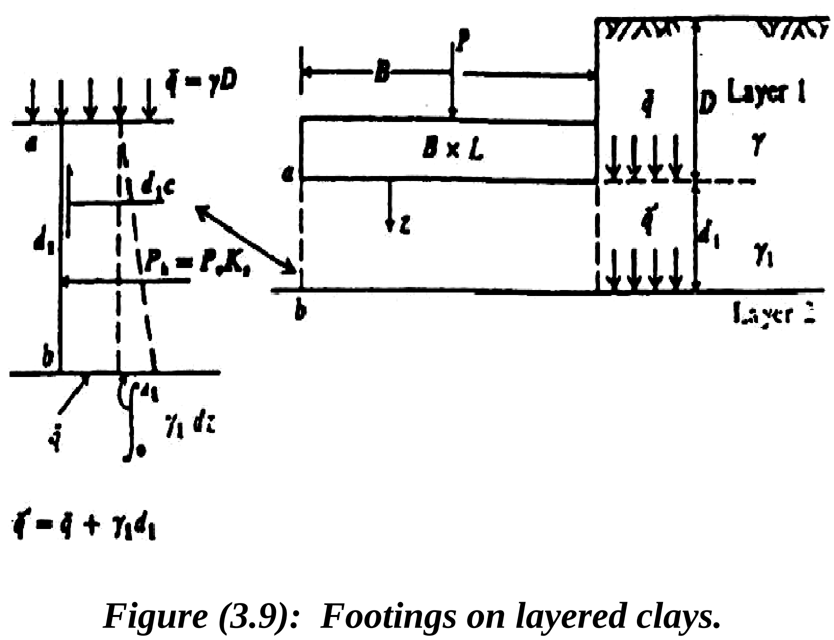


Figure (3.9): Footings on layered clays.

The first situation occurs when the footing is placed on a stiff clay or dense sand stratum followed by a relatively soft normally consolidated clay. The failure in this case is basically a punching failure. While, the second situation is often found when the footing is placed on a relatively thin layer of soft clay overlying stiff clay or rock. The failure in this case occurs, at least in part by lateral plastic flow ***(see Fig.(3.10))***.

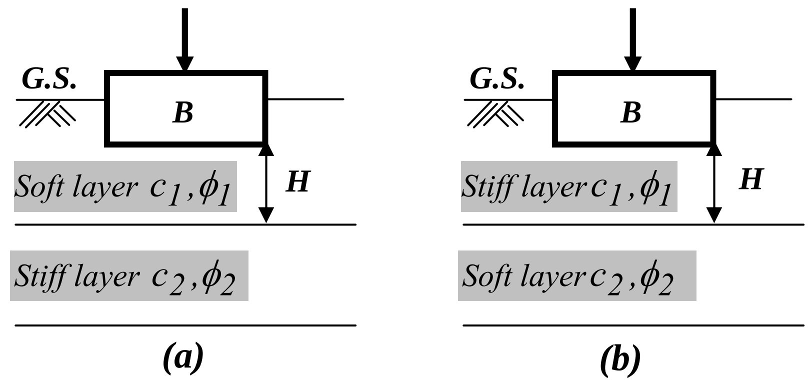


Figure (3.10): Typical two-layer soil profiles.

- **Hansen Equation** (Ref., Bowles's Book, 1996)

For both cases (a and b), the ultimate bearing capacity is calculated from Table (3.2) for (ϕ = 0) as:

$$q_{ult.} = S_u N_c (1 + S'_c + d'_c - i'_c - b'_c - g'_c) + q' \quad \textbf{(3.25)}$$

If the inclination, base and ground effects are neglected, then equation (3.25) will be:-

$$q_{ult.} = S_u N_c (1 + S'_c + d'_c) + q' \quad \textbf{(3.26)}$$

*where: S_u and N_c can be calculated by the following method (From **Bowles's** Book, 1996)**:***

In this method, S_u is calculated as an average value $C_{avg.}$ depending on the depth of penetration ($H_{crit.} = 0.5B\tan(45 + \phi/2)$, while $N_c = 5.14$. So that, equation (3.26) is written as:

$$\boxed{q_{ult.} = 5.14 C_{avg.} (1 + S'_c + d'_c) + q'} \quad \textbf{(3.26b)}$$

where: $S_u = C_{avg.} = \dfrac{C_1 H + C_2\ [Hcrit - H]}{Hcrit}$;

$$S'c = 0.2\frac{B}{L}; \quad d'c = 0.4\frac{Df}{B}\ for\ \frac{Df}{B} \le 1\ ; \quad and \quad d'_c = 0.4\tan^{-1}\frac{D}{B}\ for\ (D > B)$$

Case (2): Footing on layered $c-\phi$ soils as in Fig.(3.11):

(a) Top layer stronger than lower layer ($C_2/C_1 \le 1$).

(b) Top layer weaker than lower layer ($C_2/C_1 > 1$).

Figure (3.18) shows a foundation of any shape resting on an upper layer having strength parameters c_1, ϕ_1 and underlain by a lower layer with c_2, ϕ_2.

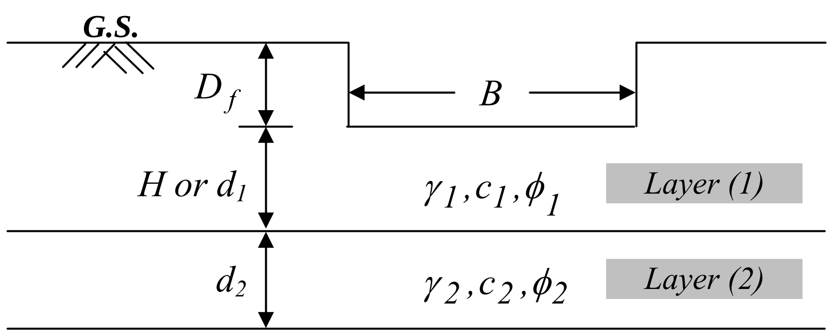


Figure (3.11): Footing on layered $c-\phi$ soils.

- **Hansen Equation** (Ref., Bowles's Book, 1996)

(1) *Compute* $H_{crit.} = 0.5B\tan(45 + \phi_1/2)$ *using* ϕ_1 *for the top layer.*

(2) *If* $H_{crit.} > H$ *compute the modified values of* c *and* ϕ *as:*

$$c^* = \frac{Hc_1 + (H_{crit.} - H)c_2}{H_{crit.}}; \qquad \phi^* = \frac{H\phi_1 + (H_{crit.} - H)\phi_2}{H_{crit.}}$$

Note: *A possible alternative for $c-\phi$ soils with a number of thin layers is to use average values of c and ϕ in bearing capacity equations of Table (3.2) as:*

$$c_{av.}=\frac{c_1H_1+c_2H_2+.....+c_nH_n}{\Sigma H_i};\qquad \phi_{av.}=tan^{-1}\frac{H_1\,tan\phi_1+H_2\,tan\phi_2+.....+H_n\,tan\phi_n}{\Sigma H_i}$$

(3) *Use Hansen's equation from Table (3.2) for* $q_{ult.}$ *with* c^* *and* ϕ^* *as:*

$$q_{ult.}=c^*N_cS_cd_ci_cg_cb_c+qN_qS_qd_qi_qg_qb_q+0.5\gamma BN_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma \quad \textbf{......(3.27)}$$

If the effects of inclination, ground and base factors are neglected, then equation (3.27) will takes the form:

$$\boxed{q_{ult.}=c^*N_cS_cd_c+qN_qS_qd_q+0.5\gamma BN_\gamma S_\gamma d_\gamma} \quad \textbf{.............................(3.28)}$$

where:

<u>*Bearing capacity factors*</u>*: from table (3.2)*

$$N_q=e^{\pi.tan\phi^*}tan^2(45+\phi^*/2),\; N_c=(N_q-1)cot\phi^*,\quad N_\gamma=1.5(N_q-1)tan\phi^*$$

<u>*Shape factors*</u> *from table (3.6):* $S_c=1+\frac{N_q}{N_c}\frac{B}{L},\quad S_q=1+\frac{B}{L}tan\phi^*,\quad S_\gamma=1-0.4\frac{B}{L}$

<u>*Depth factors*</u>*: from table (3.6)*

$$d_c=1+0.4k,\qquad d_q=1+2\,tan\,\phi^*(1-sin\,\phi^*)^2\,k,\qquad d_\gamma=1.0$$

where: $k=\frac{D}{B}$ *for* $\frac{Df}{B}\le 1$ *or* $k=tan^{-1}\frac{D}{B}(radian)$ *for* $\frac{Df}{B}>1$

<u>Case (3): Footing in layered sand and clay soils:</u>

(a) Sand overlying clay. *(b) Clay overlying sand.*

- <u>**Hansen Equation** (Ref., Bowles's Book, 1996)</u>

(1) *Compute* $H_{crit.}=0.5B\,tan(45+\phi_1/2)$ *using* ϕ_1 *for the top layer.*

(2) *If* $H_{crit.}>H$*, for both cases; sand overlying clay or clay overlying sand, estimate* $q_{ult.}$ *as follows:*

$$\boxed{q_{ult.}=q_b+\frac{p.Pv.K_s.tan\phi_1}{A_f}+\frac{p.d_1c_1}{A_f}\le q_t} \quad \textbf{.................(3.29)}$$

where: q_t, q_b *= ultimate bearing capacities of footing with respect to top and bottom soils ,*

for $\phi > 0$ ***(sand or clay)***

$$q_t = c_1 N_{c1} S_{c1} d_{c1} + \gamma_1 D_f N_{q1} S_{q1} d_{q1} + 0.5 B \gamma_1 N_{\gamma 1} S_{\gamma 1} d_{\gamma 1} \quad \textbf{......................(3.29a)}$$

$$q_b = c_2 N_{c2} S_{c2} d_{c2} + \gamma_1 (D_f + H) N_{q2} S_{q2} d_{q2} + 0.5 B \gamma_2 N_{\gamma 2} S_{\gamma 2} d_{\gamma 2} \quad \textbf{.........(3.29b)}$$

for $\phi_u = 0$ ***(clay in undrained condition)***

$$q_t = 5.14 S_u (1 + S'_c + d'_c) + \gamma_1 D_f \quad \textbf{...(3.29c)}$$

$$q_b = 5.14 S_u (1 + S'_c + d'_c) + \gamma_1 (D_f + H) \quad \textbf{......................................(3.29d)}$$

Hansen's bearing capacity factors from table (3.2) with $(\phi = \phi_i)$:

$$N_q = e^{\pi . \tan\phi} \tan^2(45 + \phi/2), \quad N_c = (N_q - 1)\cot\phi, \qquad N_\gamma = 1.5(N_q - 1)\tan\phi$$

Shape factors from table (3.5): $S_c = 1 + \dfrac{N_q}{N_c}\dfrac{B}{L}$, $S_q = 1 + \dfrac{B}{L}\tan\phi$, $S_\gamma = 1 - 0.4\dfrac{B}{L}$

Depth factors from table (3.5): $d_c = 1 + 0.4k$, $d_q = 1 + 2\tan\phi(1 - \sin\phi)^2 k$, $d_\gamma = 1.0$

where: $k = \dfrac{D}{B}$ for $\dfrac{Df}{B} \le 1$ or $k = \tan^{-1}\dfrac{D}{B}(radian)$ for $\dfrac{Df}{B} > 1$

p = total perimeter for punching = 2 (B+L) or $\pi.D$ *(diameter),*

P_v *= total vertical pressure from footing base to lower soil computed as:*

$$\int_0^{d_1} \gamma_1 h.dh + \bar{q} d_1 = \gamma_1 \frac{d_1^2}{2} + \gamma_1 D_f . d_1$$

K_s *= lateral earth pressure coefficient, which may range from* $\tan^2(45 \pm \phi/2)$ *or use* $K_o = 1 - \sin\phi$,

$\tan\phi$ *= coefficient of friction between* $P_v\, K_s$ *and perimeter shear zone wall,*

$p d_1 c_1$ *= cohesion on perimeter as a force,* A_f *= area of footing.*

(3) *Otherwise, if* $(H/B)_{crit.} \le (H/B)$*, then* $q_{ult.}$ *is estimated as the bearing capacity of the first soil layer whether it is sand or clay.*

BEARING CAPACITY EXAMPLES (3)

Footings on layered soils

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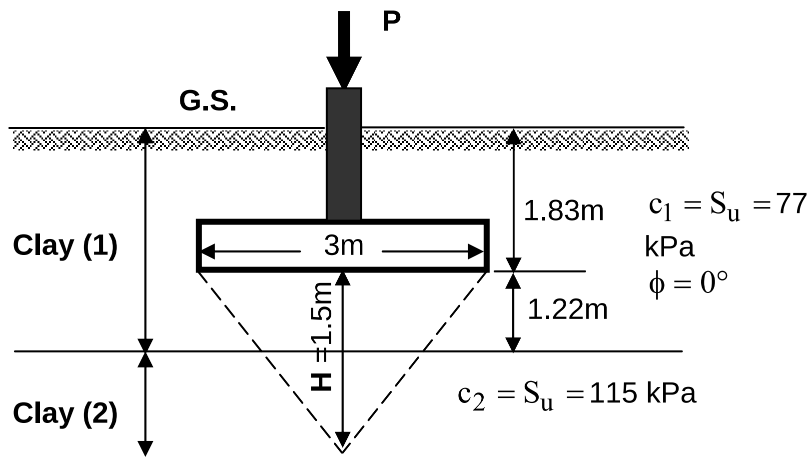
Example (8): *(footing on layered clay)*

A rectangular footing of 3.0x6.0m is to be placed on a two-layer clay deposit as shown in figure below. Estimate the ultimate bearing capacity.



Solution:

$$H_{crit.} = 0.5B\tan(45 + \phi/2) = 0.5(3)\tan 45 = 1.5m > 1.22m$$

$\therefore$ *the critical depth penetrated into the* $2^{nd.}$ *layer of soil.*

For case(1); clay on clay layers using Hansen's equation:

- ***F**rom Bowles's Book, 1996:*

$$q_{ult.} = 5.14.C_{avg.}(1 + S'_c + d'_c) + q'$$

where:

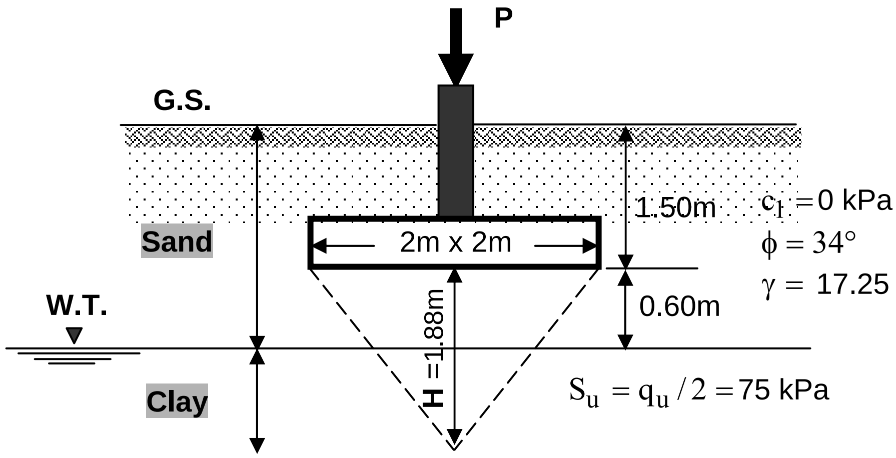
$$S_u = C_{avg.} = \frac{C_1 H + C_2\,[Hcrit - H]}{Hcrit} = \frac{77(1.22) + 115(1.5 - 1.22)}{1.5} = 84.093$$

$$S'_c = 0.2B/L = 0.2(3/6) = 0.1;\quad for\ Df/B \le 1:\quad d'_c = 0.4D/B = 0.4(1.83/3) = 0.24$$

$$\therefore \quad q_{ult.} = 5.14(84.093)(1+0.1+0.24) + 1.83(17.26) = \boxed{610.784\ kPa}$$

Example (9): *(footing on sand overlying clay)*

A 2.0x2.0m square footing is to be placed on sand overlying clay as shown in figure below. Estimate the allowable bearing capacity of soil?.



Solution:

$$H_{crit.} = 0.5B\tan(45+\phi_1/2) = 0.5(2)\tan(45+34/2) = 1.88\,m > 0.6m$$

$\therefore$ *the critical depth* $H_{crit.} > H$ *penetrated into the* $2^{nd.}$ *layer of soil.*

For case (3); sand overlying clay using Hansen's equation:

$$q_{ult.} = q_b + \frac{p.Pv.K_s.\tan\phi_1}{A_f} + \frac{p.d_1c_1}{A_f} \le q_t$$

where:

- ***for sand layer:***

$$q_t = \gamma_1 D_f N_{q1} S_{q1} d_{q1} + 0.5B\gamma_1 N_{\gamma 1} S_{\gamma 1} d_{\gamma 1}$$

Hansen's bearing capacity factors from Table (3.2) with $(\phi = 34°)$:

$$N_q = e^{\pi\tan 34}\tan^2(45+34/2) = 29.4\,, \qquad N_\gamma = 1.5(29.4-1)\tan 34 = 28.7$$

Shape factors from Table (3.5): $S_q = 1+\frac{B}{L}\tan\phi = 1.67\,,\quad S_\gamma = 1-0.4\frac{B}{L} = 0.6$

Depth factors from Table (3.5):

$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2 \frac{D_f}{B} = 1 + 2\tan 34(1-\sin 34)^2 \frac{1.5}{2} = 1.2,$$

$$d_\gamma = 1.0$$

$$\therefore \quad q_t = 17.25(1.5)(29.4)(1.67)(1.2) + 0.5(2)17.25)(28.7)(0.6)(1.0) = \boxed{1821.5\ kPa}$$

for clay layer:

$$q_b = 5.14 S_u (1 + S'_c + d'_c) + q'$$

$$S'_c = 0.2\frac{B}{L} = 0.2\frac{2}{2} = 0.2;$$

$$for\ \frac{Df}{B} > 1: \quad d'_c = 0.4\tan^{-1}\frac{D_f}{B} = 0.4\tan^{-1}\left(\frac{1.5+0.6}{2}\right) = 0.32\ ;$$

$$S_q = d_q = 1$$

$$\therefore \quad qb = 5.14(75)(1+0.2+0.32)+(1.5+0.6)(17.25) = \boxed{622\ kPa}$$

Now, obtain the punching contribution:

$$P_v = \int_0^{d_1} \gamma_1 h.dh + \overline{q} d_1 = \gamma_1 \frac{d_1^2}{2}\Bigg]_0^{0.6} + \gamma_1 D_f d_1 = 17.25\frac{0.6^2}{2} + 17.25(1.5)(0.6) = 18.6\ kN/m$$

$$K_o = 1 - \sin\phi = 1 - \sin 34 = 0.44,$$

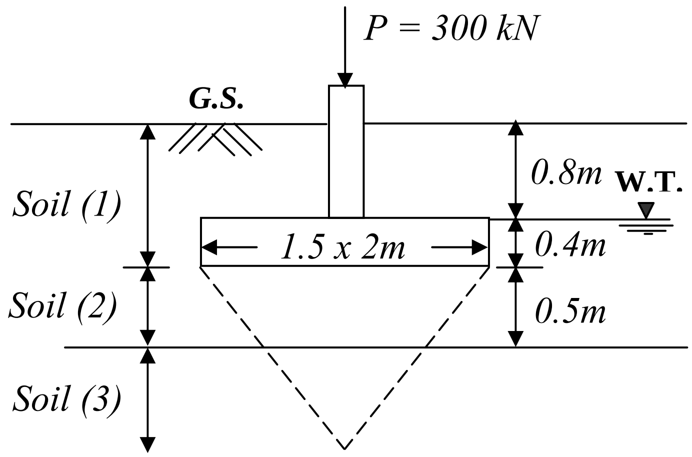
$$\therefore q_{ult.} = 622 + \frac{2(2+2)(18.6)(0.44)\tan 34}{2x2} + \frac{2(2+2)(0.6)(0)}{2x2} = 633\ kPa < q_{ult.} = 1821.5\ kPa$$

$$q_{all.} = 633/3 = \boxed{211\ kPa}$$

Example (10): *(footing on $c-\phi$ soils)*

Check the adequacy of the rectangular footing 1.5x2.0m shown in figure below against shear failure (use F.S.= 3.0), γ_w =10 kN/m^3 .

parameter	Soil (1)	Soil (2)	Soil (3)
Gs	2.70	2.65	2.75
e	0.8	0.9	0.85
c (kPa)	10	60	80
$\phi°$	35	0	0



Solution:

$$\gamma_{d1} = \frac{G_s \cdot \gamma_w}{1+e} = \frac{2.70(10)}{1+0.8} = 15\ kN/m^3$$

$$\gamma_{sat\,1} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.70+0.8)10}{1+0.8} = 19.4\ kN/m^3$$

$$\gamma_{d2} = \frac{G_s \cdot \gamma_w}{1+e} = \frac{2.65(10)}{1+0.9} = 18.7\ kN/m^3$$

$$\gamma_{sat2} = \frac{(2.75+0.85)10}{1+0.85} = 19.45\ kN/m^3$$

$$H_{crit.} = 0.5B\tan(45+\phi/2) = 0.5(1.5)\tan 45 = 0.75m > 0.50m$$

$\therefore$ *the critical depth penetrated into the soil layer (3).*

Since soils (2) and (3) are of clay layers, therefore; by using Hansen's equation:

- *From Bowles's Book, 1996:*

$$q_{ult.} = 5.14C_{avg.}(1+S'_c+d'_c)+q'$$

where:

$$C_{avg.} = \frac{C_1H + C_2\ [Hcrit - H]}{Hcrit} = \frac{60(0.5) + 80(0.75 - 0.50)}{0.75} = 66.67$$

$S'_c = 0.2B / L = 0.2(1.5 / 2) = 0.15$;

for $Df / B \leq 1\ \ d'_c = 0.4D / B = 0.4(1.2 / 1.5) = 0.32$

$\therefore\quad q_{ult.} = 5.14(66.67)(1+0.15+0.32)+0.8(15)+0.4(19.45-10) = 519.5\ kPa$

$$q_{all}(_{net}) = \frac{519.5}{3} - 15.78 = \boxed{157.4\ \ kPa}$$

$$q_{applied} = \frac{300}{1.5x2} = 100\ \ kPa < q_{all}(_{net}) = 157.4\ \ kPa \quad \therefore \textbf{(O.K.)}$$

<u>Check for squeezing:</u>

For no squeezing of soil beneath the footing: $(q_{ult.} > 4c_1 + \overline{q})$

$4c_1 + \overline{q} = 4(60) + 0.8(15) + 0.4(19.45-10) = 255.78\ kPa < 519.5\ kPa \ \therefore$ **(O.K.)**

3.12 Skempton's Bearing Capacity Equation

- **<u>Footings on Clay and Plastic Silts:</u>**

  From Terzaghi's equation, the ultimate bearing capacity is:

$$q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma \quad \textbf{.......................................(3.12)}$$

<u>For saturated clay and plastic silts:</u> ($\phi_u = 0$ and $N_c = 5.7, N_q = 1.0,.and.N_\gamma = 0$),

<u>For strip footing:</u> $S_c = S_\gamma = 1.0$

$$\boxed{q_{ult.} = cN_c + \overline{q}} \quad \textbf{...(3.30)}$$

$$q_{all.} = \frac{q_{ult.}}{3} \quad and \quad q_{all.(net)} = q_{all.} - \overline{q}$$

$$\therefore \qquad q_{all.(net)} = \frac{q_{ult.}}{3} - \overline{q} = \frac{cN_c + \overline{q}}{3} - \overline{q} = \frac{cN_c}{3} + (\frac{\overline{q}}{3} - \overline{q}) \quad \textbf{.......................(3.30a)}$$

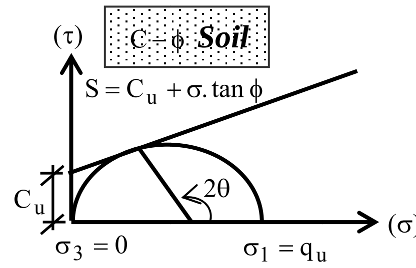
where: N_c = *bearing capacity factor obtained from* **figure (3.12)** *depending on shape of footing and* $\frac{D_f}{B} \cdot (\frac{\bar{q}}{3} - \bar{q})$ *is a small value can be neglected.*

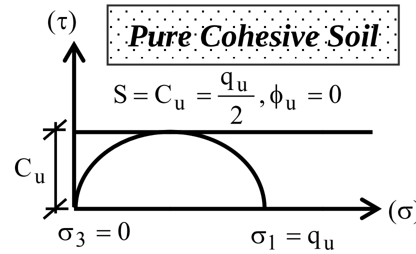
for $c - \phi$ **soil**: $\boxed{\sigma_1 = \sigma_3 \tan^2(45 + \phi/2) + 2c \tan(45 + \phi/2)}$

for UCT: $\sigma_1 = q_u$ *and* $\sigma_3 = 0$; *then* $\boxed{q_u = 2c \tan(45 + \phi/2)}$

or $\phi_u = 0$; $c = \frac{q_u}{2}$ *and equation (3.30a) will be:*

$$q_{all.(net)} = q_u \frac{N_c}{6} \quad \text{.......................................(3.30b)}$$





From **figure (3.12)** *for* $\frac{D_f}{B} = 0$: $N_c = 6.2$ *for square or circular footings; 5.14 for strip or continuous footings* *If* $N_c = 6.0$, *then:*

$$\boxed{q_{all.(net)} \approx q_u} \quad \text{...(3.31)}$$

See figure (3.13) *for net allowable soil pressure for footings on clay and plastic silt.*

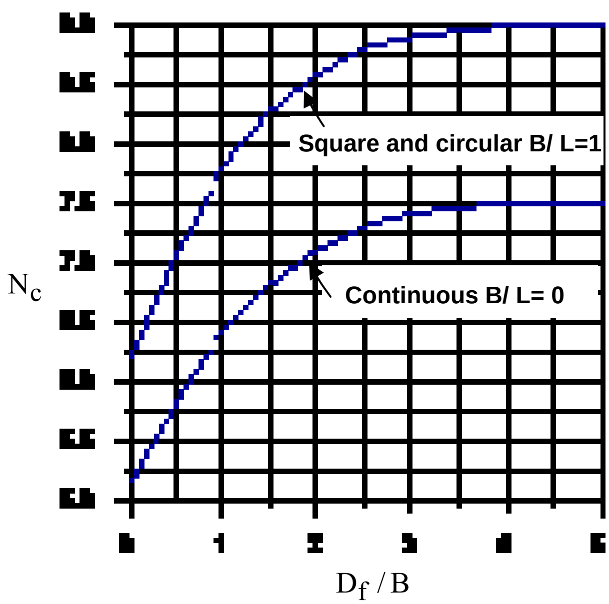


Figure (3.12): N_c **bearing capacity factor for Footings on clay under** $\phi = 0$ **conditions** (After Skempton, 1951).

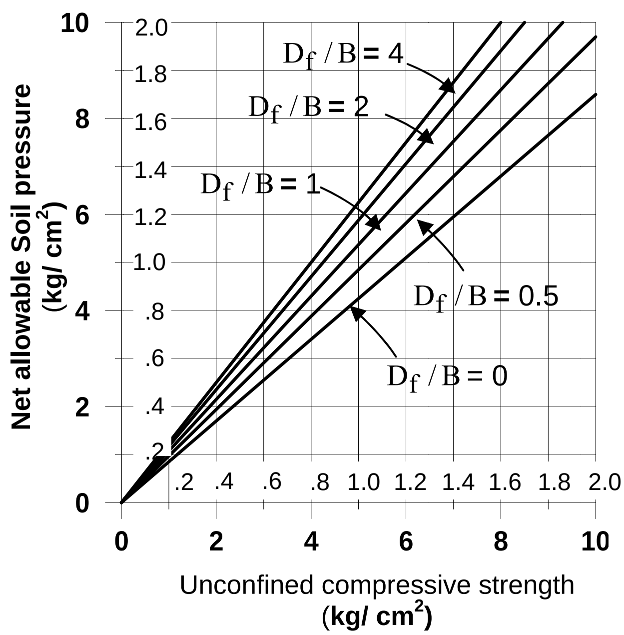
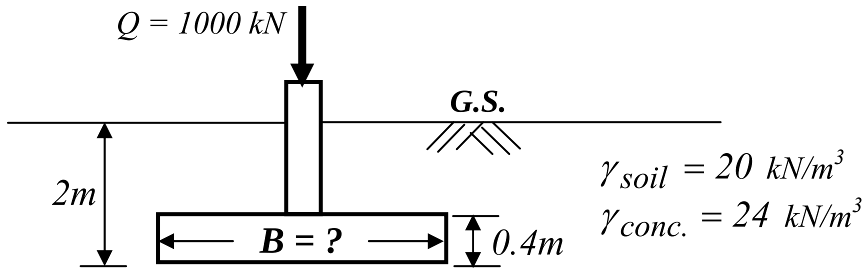


Figure (3.13): Net allowable soil pressure for footings on clay and plastic silt, determined for a factor of safety of 3 against bearing capacity failure (ϕ = 0 conditions). Chart values are for strip footings (B/L=0); and for other types of footings multiply values by (1+ 0.2B/L).

$$\boxed{N_{c(net)} = N_{c(strip)}(1 + 0.2\frac{B}{L}) \quad \text{or} \quad N_{c(net)} = N_{c(square)}(0.84 + 0.16\frac{B}{L})}$$

Example (11): *(footing on clay)*

Determine the size of the square footing shown in figure below. If q_u = 100 kPa and F.S.= 3.0?



Solution:

Assume B =3.5m, D/B *= 2/3.5 = 0.57 then from* **figure (3.12)**: $N_c = 7.3$

$q_{ult.} = cN_c + \overline{q}$ = 50(7.3) + 2(20) = 405 kPa

$$q_{all.(net)} = \frac{q_{ult.}}{3} - \overline{q} = \frac{405}{3} - 20(1.6) - 24(0.4) = 93.4 \ kPa$$

Area=1000/93.4 = 10.71 m²; for square footing: $B = \sqrt{10.71} = 3.27 < 3.5m$

$\therefore$ *take B =3.25m , and* D/B *= 2/3.25 = 0.61 then from figure (3.15):* $N_c = 7.5$

$q_{ult.} = cN_c + \overline{q}$ = 50(7.5) + 2(20) = 415 kPa

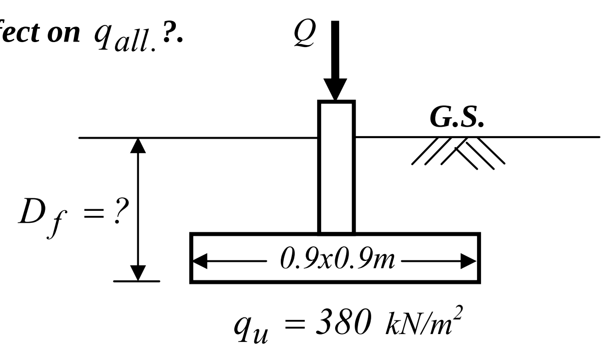
$$q_{all.(net)} = \frac{q_{ult.}}{3} - \overline{q} = \frac{415}{3} - 20(1.6) - 24(0.4) = 96.73 \ kPa$$

Area=1000/96.73 = 10.34 m²; $B = \sqrt{10.34} = 3.21 \approx 3.25m$ **(O.K.)**

$\therefore$ *use B x B = (3.25 x 3.25)m*

Example (12): *(footing on clay)*

For the square footing shown in figure below. If q_u = 380 kPa and F.S.= 3.0, determine $q_{all.}$ and $D_f(min.)$ which gives the maximum effect on $q_{all.}$?.



Solution:

From Skempton's equation:

For strip footing: $q_{all.(net)} = \frac{cN_c}{3}$

For square footing: $q_{all.(net)} = \frac{cN_c}{3} x1.2$

From Skempton's **figure (3.12)** *at* $D_f / B = 4$ *and* $B/L=1$ *(square footing):* $N_c = 9$

$$\therefore \; q_{all.(net)} = \frac{\frac{380}{2}(9)}{3} = \boxed{570 \; kPa} \quad and \quad D_f = 4(0.9) = \boxed{3.6m}$$

• Rafts on Clay:

$$If \;\; q_b = \frac{\Sigma Q}{A} = \frac{Total.load(D.L. + L.L.)}{area} > q_{all.}$$ *use pile or floating foundations.*

From Skempton's equation, the ultimate bearing capacity (for strip footing) is:

$$q_{ult.} = cN_c + \bar{q} \quad \textbf{..(3.30)}$$

$$q_{ult.(net)} = cN_c \, , \qquad q_{all.(net)} = \frac{cN_c}{F.S.} \quad or \quad F.S. = \frac{cN_c}{q_{all.(net)}}$$

Net soil pressure = $q_b - D_f.\gamma$

$$\therefore \quad \boxed{F.S. = \frac{cN_c}{q_b - Df.\gamma}} \quad \textbf{..(3.32)}$$

Notes:

(1) *If* $q_b = D_f.\gamma$ *(i.e.,* $F.S. = \infty$*) the raft is said to be fully compensated foundation (in this case, the weight of foundation (D.L.+ L.L.) = the weight of excavated soil).*

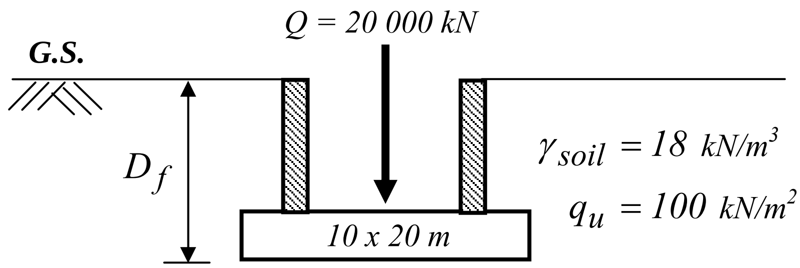
(2) *If* $q_b > D_f.\gamma$ *(i.e.,* $F.S. = certain.value$*) the raft is said to be partially compensated foundation such as the case of storage tanks.*

Example (13): *(raft on clay)*

Determine the F.S. for the raft shown in figure for the following depths: D_f ***=1m,2m, and 3m?.***

Solution:

$$F.S. = \frac{cN_c}{q_b - Df.\gamma}$$



- For $D_f = 1m$:

From figure (3.12) $D_f / B = 1/10 = 0.1$ *and* $B/L = 0$:

$$N_{c\,strip} = 5.4 \text{ and } \boxed{N_{c\,rectangular} = N_{c\,strip}(1+0.2B/L)} = 5.4\,(1+0.2\frac{10}{20}) = 5.94$$

$$\therefore\ F.S. = \frac{cN_c}{q_b - Df.\gamma} = \frac{(100/2)5.94}{\frac{20000}{10x20} - 1(18)} = \frac{50(5.94)}{100-18} = 3.62$$

- For $D_f = 2m$:

From figure (3.12) $D_f / B = 2/10 = 0.2$ *and* $B/L = 0$:

$$N_{c\,strip} = 5.5 \quad and \quad N_{c\,rectangular} = 5.5\,(1+0.2\frac{10}{20}) = 6.05$$

$$\therefore\ F.S. = \frac{cN_c}{q_b - Df.\gamma} = \frac{(100/2)6.05}{\frac{20000}{10x20} - 2(18)} = \frac{50(6.05)}{100-36} = 4.72$$

- For $D_f = 3m$:

From figure (3.12) $D_f / B = 3/10 = 0.3$ *and* $B/L = 0$:

$$N_{c\,strip} = 5.7 \quad and \quad N_{c\,rectangular} = 5.7\,(1+0.2\frac{10}{20}) = 6.27$$

$$\therefore\ F.S. = \frac{cN_c}{q_b - Df.\gamma} = \frac{(100/2)6.27}{\frac{20000}{10x20} - 3(18)} = \frac{50(6.27)}{100-54} = 6.81$$

3.13 Design Charts for Footings on Sand and Nonplastic Silt

From Terzaghi's equation, the ultimate bearing capacity is:

$$q_{ult.} = cN_c.S_c + \overline{q}N_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma \quad \textbf{………………………………………………(3.12)}$$

<u>For sand</u> $(c = 0)$ and <u>for strip footing</u> $(S_c = S_\gamma = 1.0)$, then, Eq.(3.12) will be:

$$q_{ult.} = \overline{q}N_q + \frac{1}{2}B.\gamma.N_\gamma \quad \textbf{…………………………………………………………(3.33)}$$

$$q_{ult.(net)} = \overline{q}N_q + \frac{1}{2}B.\gamma.N_\gamma - \overline{q}$$

$$q_{ult.(net)} = D_f.\gamma.N_q + \frac{1}{2}B.\gamma.N_\gamma - D_f.\gamma$$

$$q_{ult.(net)} = D_f.\gamma(N_q - 1) + \frac{1}{2}B.\gamma.N_\gamma = B\left[\frac{D_f.\gamma}{B}(N_q - 1) + \frac{1}{2}\gamma.N_\gamma\right]$$

$$\boxed{q_{all.(net)} = \frac{B}{F.S.}\left[\frac{D_f.\gamma}{B}(N_q - 1) + \frac{1}{2}\gamma.N_\gamma\right]} \quad \textbf{……………………………………(3.34)}$$

<u>Notes:</u>

*(1) the allowable bearing capacity shown by (**Eq.3.34**) is derived from the frictional resistance due to: (i) the weight of the sand below the footing level; and (ii) the weight of the surrounding surcharge or backfill.*

(2) the $q_{ult.}$ of a footing on sand depends on:

(a) width of the footing, B

(b) depth of the surcharge surrounding the footing, D_f

(c) angle of internal friction, ϕ

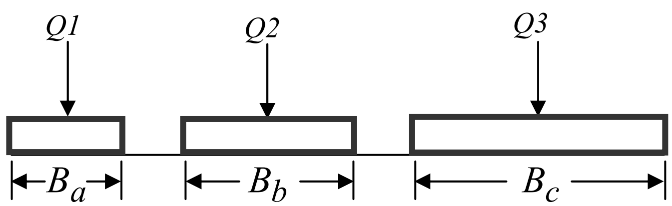
(d) relative density of the sand, D_r

(e) standard penetration resistance, N-value and

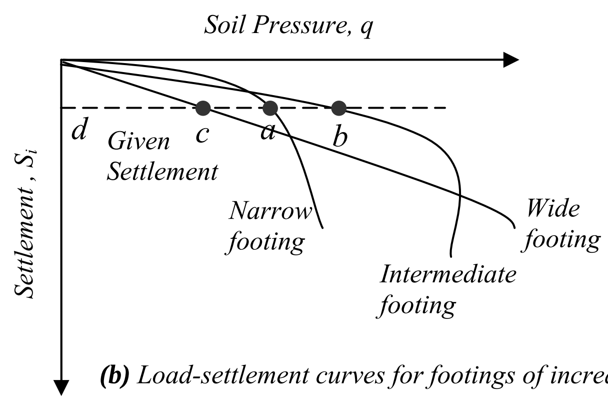
(f) water table position.

(3) *the wider the footing, the greater* $q_{ult.}$*/unit area. However, for a given settlement* S_i *such as (1 inch or 25mm), the soil pressure is greater for a footing of intermediate width* B_b *than for a large footing with a width* B_c *or for a narrow footing with width* B_a *(**see figure 3.14a**).*

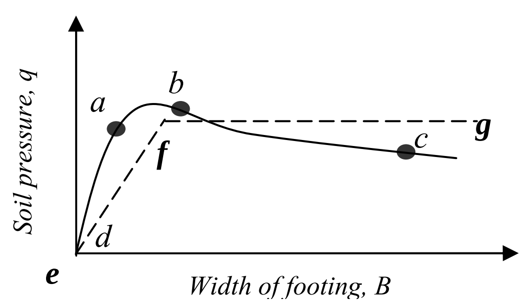
(4) *for* $\frac{D_f}{B}$ *= constant and a given settlement on sand, there is an actual relationship between* $q_{all.}$ *and B represented by (solid line)* ***(see figure 3.14b).*** *However, as basis for design a substitute relation (dashed lines) can be used as shown in* ***(figure 3.14c).*** *The error for footings of usual dimensions is less than ± 10%. The position of the broken line efg is differs for different sands.*



(a) *Footings of different widths.*



(b) *Load-settlement curves for footings of increasing widths.*



(c) *Variation of soil pressure with B for given settlement, Si.*

Figure (3.14): Footings on sand.

(5) <u>the design charts for proportioning shallow footings on sand and nonplastic silts are shown in</u> ***<u>Figures (3.15, 3.16 and 3.17).</u>***

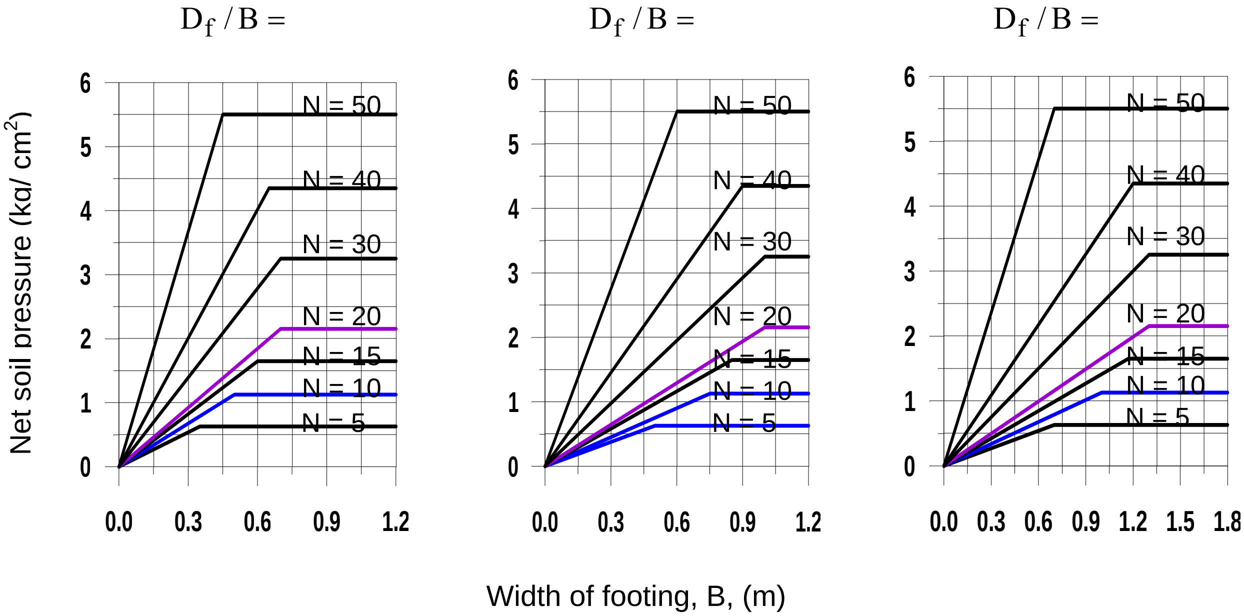


Fig.(3.15): Design charts for proportioning shallow footings on sand.

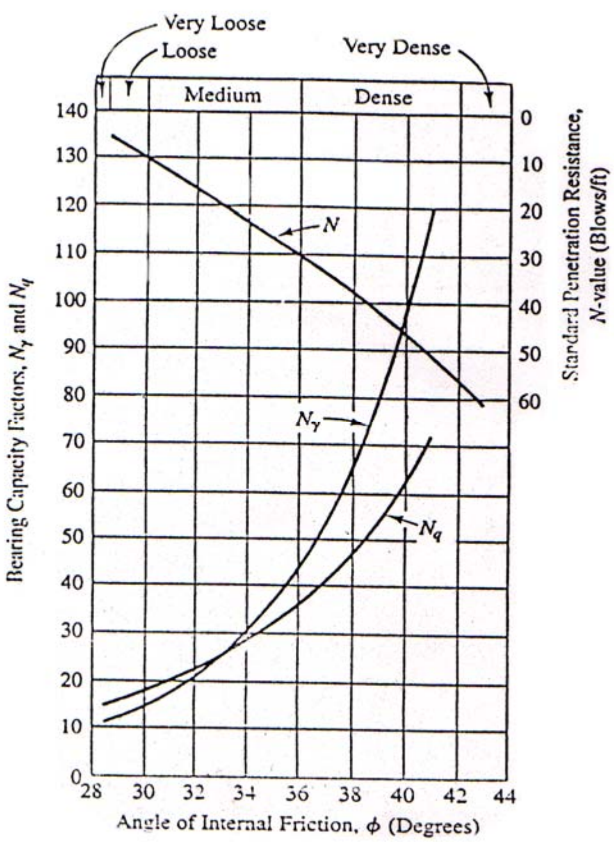


Fig.(3.16): Relationship between bearing capacity factors and ϕ***.***

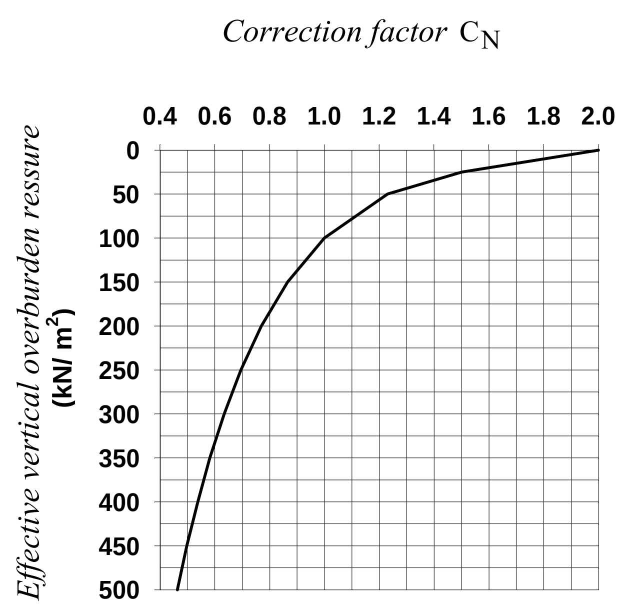


Fig.(3.17): Chart for correction of N-values in sand for overburden pressure.

Limitations of using charts (3.15, 3.16 and 3.17):

- *These charts are for strip footing, while for other types of footings multiply* $q_{all.}$ *by (1+ 0.2 B/L).*
- *The charts are derived for shallow footings* $(D_f / B \le 1)$; $\gamma = 100$ *Ib/ft³; settlement = 1.0 (inch); F.S. = 2.0; no water table (far below the footing); and corrected N-values.*
- *N-values must be corrected for:*

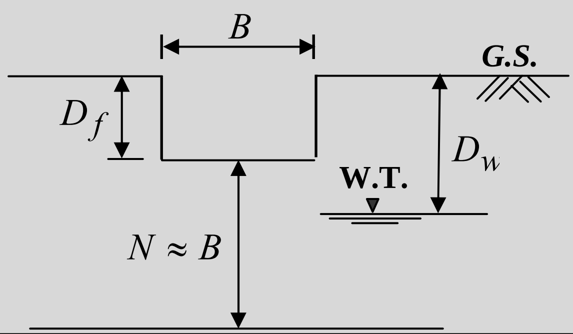
  (i) overburden pressure effect using figure (3.17) or the following formulas:

$$C_N = 0.77 \log \frac{20}{\overline{P}_o (Tsf)} \qquad or \qquad C_N = 0.77 \log \frac{2000}{\overline{P}_o (kPa)}$$

  If $\overline{p}_o < 0.25(Tsf)$ **or** $< 25(kPa)$**, (no need for overburden pressure correction).**

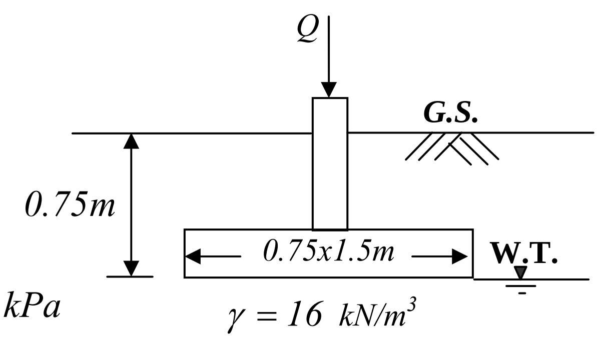
  (ii) and water table effect:

$$C_w = 0.5 + 0.5 \frac{D_w}{B + D_f}$$



Example (14): *(footing on sand)*

Determine the gross bearing capacity and the expected settlement of the rectangular footing shown in figure below. If $N_{avg.}$ ***(not corrected) =22 and the depth for correction = 6m?.***



Solution:

$P'_o = 0.75(16) + 5.25(16\text{-}9.81) = 44.5\ kPa > 25\ kPa$

$$C_N = 0.77 \log \frac{2000}{\overline{P}_o(kPa)} = 0.77 \log \frac{2000}{44.5} = 1.266$$

$$C_w = 0.5 + 0.5\frac{D_w}{B + D_f} = 0.5 + 0.5\frac{0.75}{0.75 + 0.75} = 0.75$$

$N_{corr.}$=22(1.266)(0.75)= 20.8 (use N = 20)

From **figure (3.15)** *for footings on sand: at* $D_f / B = 1$ *and B = 0.75m (2.5ft) and N 20* *<u>for strip footing:</u>* $q_{all.(net)} = 2.2(Tsf)x105.594 = 232.307\ kPa$

<u>for rectangular footing:</u> $q_{all.(net)}$ = 232.307 x (1+0.2B/L) = 255.538 kPa

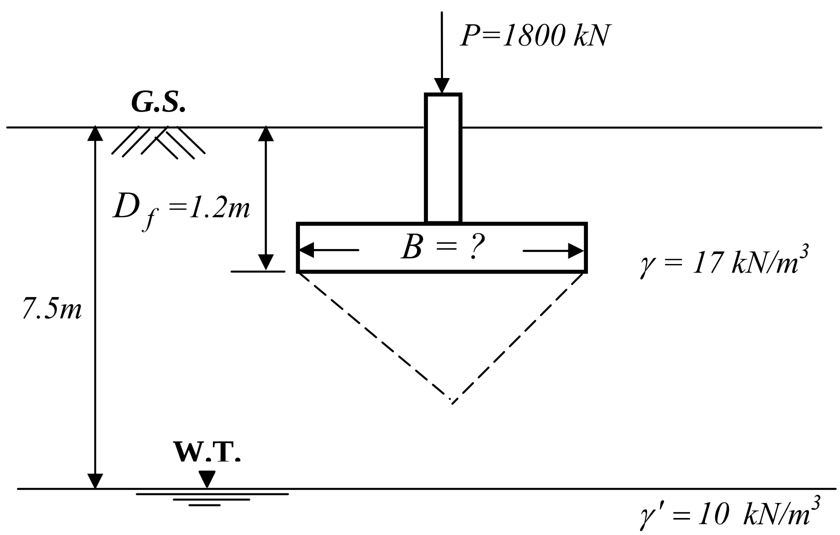
$q_{gross} = q_{all.(net)} + D_f.\gamma$ = 255.538 + 0.75(16) = $\boxed{267.538\ kPa}$

And the maximum settlement is not more than (1 inch or 25mm).

<u>Example (15):</u> *(bearing capacity from field tests)*

SPT results from a soil boring located adjacent to a planned foundation for a proposed warehouse are shown below. If spread footings for the project are to be found (1.2m) below surface grade, what foundation size should be provided to support (1800 kN) column load? Assume that 25mm settlement is tolerable, W.T. encountered at (7.5m).

SPT sample depth (m)	N_{field}
0.3	9
1.2	10
2.4	15
3.6	22
4.8	19
6	29
7.5	33
10	27



<u>Solution:</u>

Find σ'_o *at each depth and correct* N_{field} *values.*   *Assume B = 2.4 m*

At depth B below the base of footing (1.2+2.4) = 3.6m; $N'_{avg.} = (15+19+25)/3 = 20$

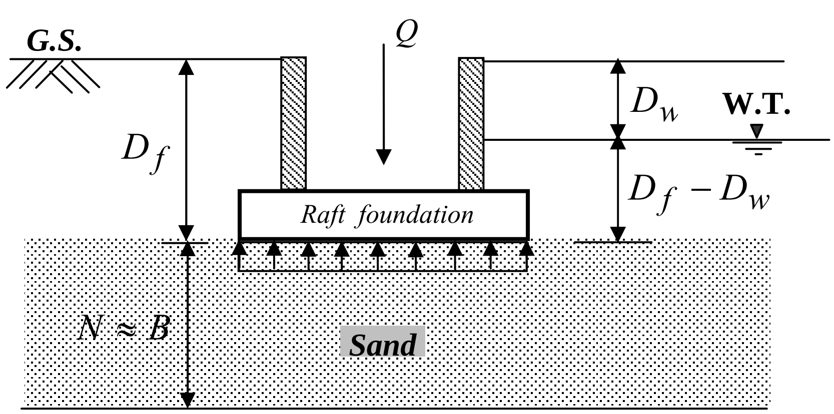
For $N'_{avg.} = 20$, *and* $D_f / B = 0.5$; $q_{all.}$ *=2.2 T/ft²* *= 232.31 kPa from Fig.***(3.15).**

SPT sample depth (m)	N_{field}	σ'_o (kN/m²)	σ'_o (T/ft²)	C_N **(Fig.3.17)**	$N' = C_N . N_{field}$
0.3	9				
1.2	10	20.4	0.21	1.55	15
2.4	15	40.8	0.43	1.28	19
3.6	22	61.2	0.64	1.15	25
4.8	19	81.6	0.85	1.05	20
6	29	102	1.07	0.95	27
7.5	33	127.5	1.33	0.90	30
10	27	152.5	1.59	0.85	23

Say B = 2.5 m, $q_{all.} = \dfrac{P}{B.x.L}$, $L = \dfrac{1800}{232.31X2.5} = 3.10m$, $\therefore$ *use* **(2.5 x 3.25)m** *footing.*

- ## Rafts on Sand:

For allowable settlement = 2 (inch) and differential settlement >3/4 (inch) provided that $D_f \geq (8ft).or.(2.4m)min.$ *the allowable net soil pressure is given by:*



$$\boxed{q_{all.(net)} = C_w \frac{S_{all.}(N)}{9}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \; for 5 \leq N \leq 50 \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{(3.35)}$$

If C_w*=1 and* $S_{all.} = 2''$; *then* $\boxed{q_{all.(net)}} = 1.0\dfrac{2.0(N)}{9} = \boxed{0.22N(Tsf) = 23.23N(kPa)}$

and $q_{gross} = q_{all.(net)} + D_f.\gamma = \dfrac{\Sigma Q}{Area}$

where: $D_f.\gamma = D_w\gamma + (D_f - D_w)(\gamma - \gamma_w) + (D_f - D_w)\gamma_w$

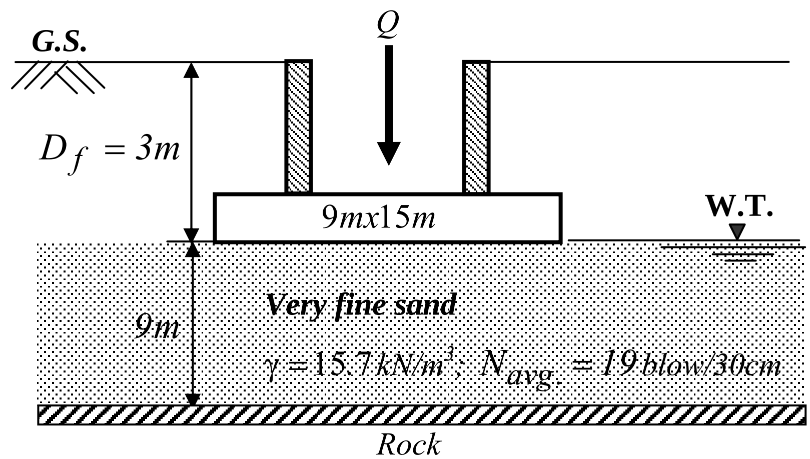
$$C_w = 0.5 + 0.5\frac{D_w}{B + D_f} = \text{(correction for water table)}$$

N = SPT number (corrected for both W.T. and overburden pressure).

Hint: *A raft-supported building with a basement extending below water table is acted on by hydroustatic uplift pressure or buoyancy equal to* $(D_f - D_w)\gamma_w$ *per unit area.*

Example (16): *(raft on sand)*

Determine the maximum soil pressure that should be allowed at the base of the raft shown in figure below If $N_{avg.}$(corrected) =19?.



Solution:

For raft on sand: $\boxed{q_{all.(net)} = 23.23N(kPa)} = 23.23(19) = 441.37$ kPa

Correction for water table: $C_w = 0.5 + 0.5\dfrac{D_w}{B + D_f} = 0.5 + 0.5\dfrac{3}{9+3} = 0.625$

$\therefore\ q_{all.(net)} = 441.37(0.625) = 275.856$ kPa

The surcharge = $D_f.\gamma = 3(15.7) = 47.1$ kPa

and $q_{gross} = q_{all.(net)} + D_f.\gamma = 275.856 + 47.1 = 323$ kPa

3.14 Bearing Capacity of Footings on Slopes

If footings are on slopes, their bearing capacities are less than if the footings were on level ground. In fact, bearing capacity of a footing is inversely proportional to ground slope.

- <u>**Meyerhof's Method:**</u>

In this method, the ultimate bearing capacity of footings on slopes is computed using the following equations:

$$(q_{ult.})_{continuous.footing.on.slope} = cN_{cq} + \frac{1}{2}\gamma.B.N_{\gamma q} \quad \textbf{......(3.36)}$$

$$(q_{ult.})_{c.or.s.footing.on.slope} = (q_{ult.})_{continuous.footing.on.slope}\left[\frac{(q_{ult.})_{c.or.s.footing.on.level.ground}}{(q_{ult.})_{continuous.footing.on.level.ground}}\right] \quad \textbf{......(3.37)}$$

where:

N_{cq} *and* $N_{\gamma q}$ *are bearing capacity factors for footings on or adjacent to a slope; <u>determined from **figure (3.18)**.</u>*

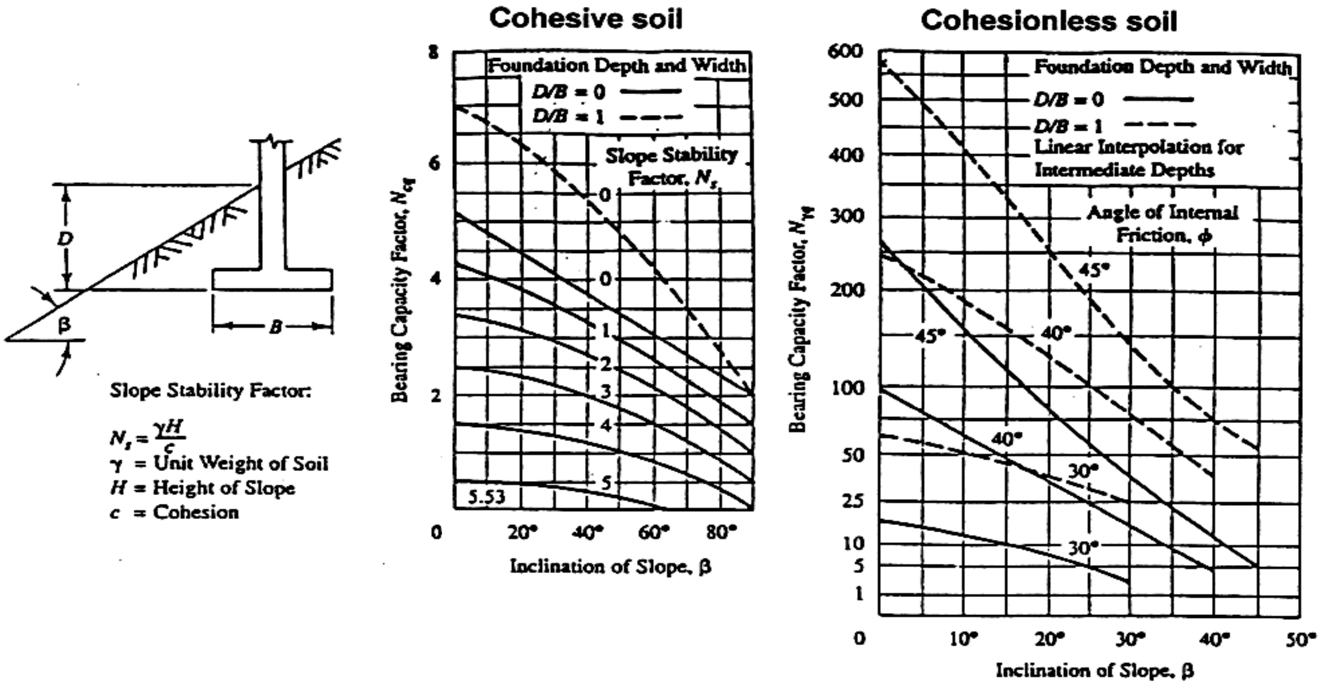
c or s footing denotes either circular or square footing, and

$(q_{ult.})$ *<u>of footing on level ground is calculated from Terzaghi's equation.</u>*

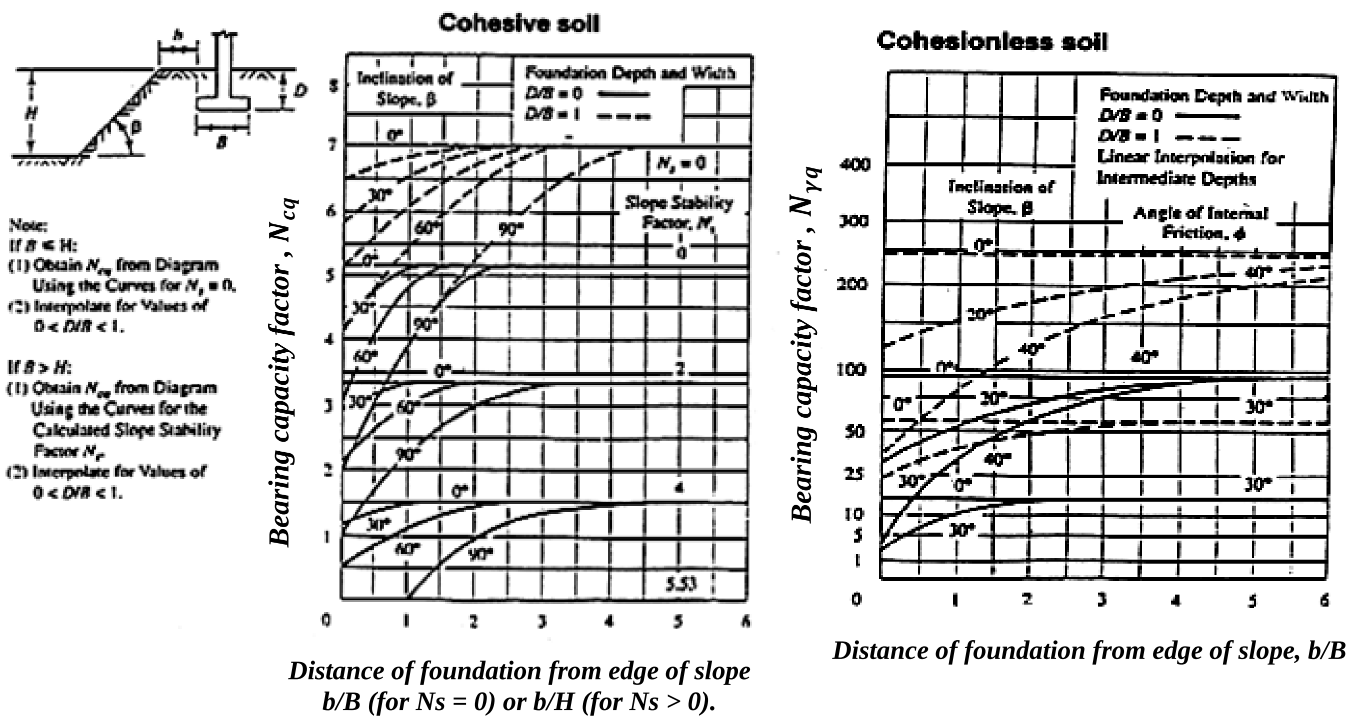
<u>***Notes:***</u>

(1) <u>A $\phi_{triaxial}$ should not be adjusted to</u> ϕ_{ps}, since the slope edge distorts the failure pattern such that plane-strain conditions may not develop except for large b/B ratios.

(2) For footings on or adjacent to a slope, <u>the overall slope stability</u> should be checked for the footing load using a slope-stability program or other methods *such as **method of slices by Bishop's***.



(a) on face of slope.



(b) on top of slope.

Figure (3.18): bearing capacity factors for continuous footing *(after Meyerhof).*

BEARING CAPACITY EXAMPLES (4)

Footings on slopes

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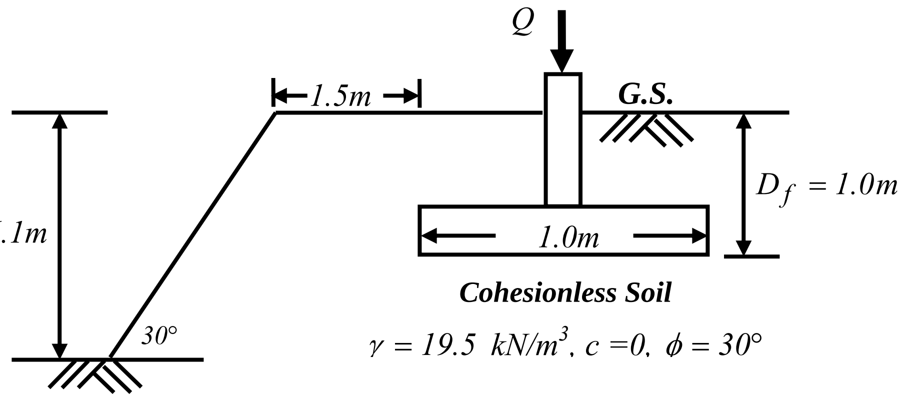
Tikrit University

<u>Example (17):</u> *(footing on top of a slope)*

A bearing wall for a building is to be located close to a slope as shown in figure. The ground water table is located at a great depth. Determine the allowable bearing capacity by Meyerhof's method using F.S. =3?.



<u>Solution:</u>

$$(q_{ult.})_{continuous.footing.on.slope} = cN_{cq} + \frac{1}{2}\gamma.B.N_{\gamma q} \quad \textbf{(3.36)}$$

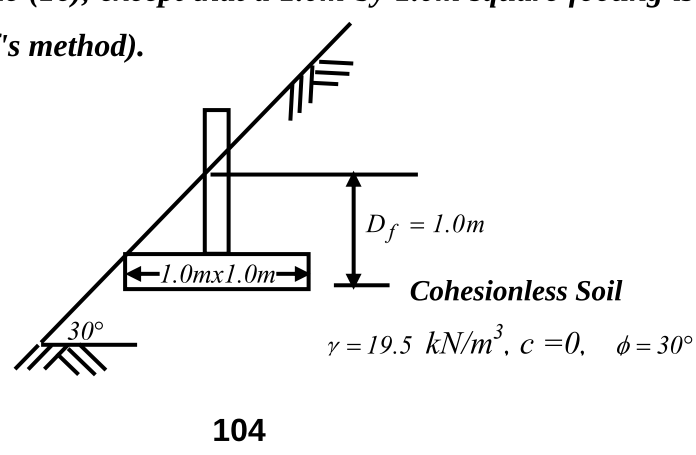
From figure (3.18-b): with $\phi = 30°$, $\beta = 30°$, $\frac{b}{B} = \frac{1.5}{1.0} = 1.5$, *and* $\frac{D_f}{B} = \frac{1.0}{1.0} = 1.0$ *(use the dashed line)* $\longrightarrow N_{\gamma q} = 40$

$$(q_{ult.})_{continuous.footing.on.slope} = (0)N_{cq} + \frac{1}{2}(19.5)(1.0)(40) = 390\ kN/m^2$$

$$q_{all.} = 390/3 = \boxed{130\ kN/m^2.}$$

<u>Example (18):</u> *(footing on face of a slope)*

Same conditions as example (16), except that a 1.0m-by 1.0m square footing is to be constructed on the slope (use Meyerhof's method).



Solution:

$$(q_{ult.})_{c.or.s.footing.on.slope} = (q_{ult.})_{continuous.footing.on.slope}\left[\frac{(q_{ult.})_{c.or.s.footing.on.level.ground}}{(q_{ult.})_{continuous.footing.on.level.ground}}\right] \quad \textbf{.....(3.37)}$$

$$(q_{ult.})_{continuous.footing.on.slope} = (0)N_{cq} + \frac{1}{2}(19.5)(1.0)(25) = \boxed{243.75 kN/m^2}$$

$(q_{ult.})$ *of square or strip footing on level ground is calculated from Terzaghi's equation:*

$$q_{ult.} = cN_c S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma$$

Bearing capacity factors from table (3.3): for $\phi = 30°$; $N_c = 37.2,..N_q = 22.5,..N_\gamma = 19.7$

Shape factors table (3.2): for square footing $S_c = 1.3, S\gamma = 0.8$; strip footing $S_c = S_\gamma = 1.0$

$$(q_{ult.})_{square.footing.on.level.ground} = 0 + 1.0\,(19.5)(22.5) + 0.5(1.0)(19.5)(19.7)(0.8) = 592.4 \ kN/m^2$$

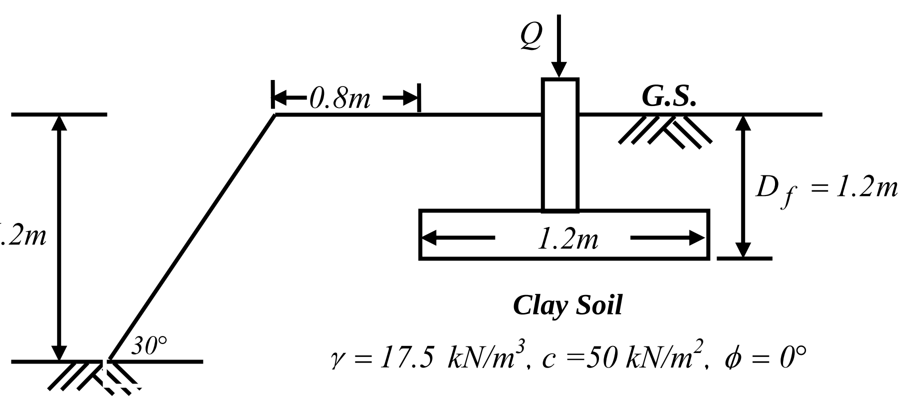
$$(q_{ult.})_{continuous.footing.on.level.ground} = 0 + 1.0\,(19.5)(22.5) + 0.5(1.0)(19.5)(19.7)(1.0) = 630.8 \ kN/m^2$$

$$\therefore \quad (q_{ult.})_{square.footing.on.slope} = 243.75\frac{592.4}{630.8} = 228.912 \ kN/m^2$$

and $(q_{all.})_{square.footing.on.slope} = \frac{228.912}{3} = \boxed{76 \ kN/m^2}$

Example (19): *(footing on top of a slope)*

A shallow continuous footing in clay is to be located close to a slope as shown in figure. The ground water table is located at a great depth. Determine the gross allowable bearing capacity using F.S. = 4



Solution:

Since B<H assume the stability number $N_s = 0$ *and for* **purely cohesive soil**, $\phi = 0$

$$\boxed{(q_{ult.})_{continuous.footing.on.slope} = cN_{cq}}$$

From figure (3.18-b) for cohesive soil: with $\phi = 30°$**,** $N_s = 0$**,** $\frac{b}{B} = \frac{0.8}{1.2} = 0.67$**,** *and*

$\frac{D_f}{B} = \frac{1.2}{1.2} = 1.0$ *(use the dashed line)* $\longrightarrow$ $N_{cq} = 6.3$

$(q_{ult.})_{continuous.footing.on.slope} = (50)(6.3) = 315\ kN/m^2$

$q_{all.} = 315/4 = \boxed{78.8\ kN/m^2.}$

3.15 Foundation on Rock

It is common to use the building code values for the allowable bearing capacity of rocks (***see Table 3.8***)**.** However, there are several significant parameters which should be taken into consideration together with the recommended code value; such as site geology, rock type and quality (as RQD).

Usually, the shear strength parameters c and ϕ of rocks are obtained from high Pressure Triaxial Tests. However, for most rocks $\phi = 45°$ except for limestone or shale $\phi = (38° - 45°)$ can be used. Similarly in most cases we could estimate $c = 5$ MPa with a conservative value.

Table (3.8): Allowable contact pressure $q_{all.}$ ***of jointed rock.***

RQD %	$q_{all.}$ **(T/ft²)**	$q_{all.}$ **(kN/m²)**	**Quality**
100	*300*	*31678*	*Excelent*
90	*200*	*21119*	*Very good*
75	*120*	*12671*	*Good*
50	*65*	*6864*	*Medium*
25	*30*	*3168*	*Poor*
0	*10*	*1056*	*Very poor*
1.0 (T/ft²) = 105.594 (kN/m²)			

Notes:

(1) *If* $q_{all.}(tabulated) > q_u(unconfined..compressive..strength)$ *of intact rock sample, then take* $q_{all.} = q_u$.

(2) *The settlement of the foundation should not exceed (0.5 inch) or (12.7mm) even for large loaded area.*

(3) *If the upper part of rock within a depth of about B/4 is of lower quality, then its RQD value should be used or that part of rock should be removed.*

Any of the bearing capacity equations from Table (3.2) with specified shape factors can be used to obtain $q_{ult.}$ of rocks, but with bearing capacity factors for sound rock proposed by *(Stagg and Zienkiewicz, 1968)* as:

$$N_c = 5\tan^4(45 + \phi/2), \qquad N_q = \tan^6(45 + \phi/2), \qquad N_\gamma = N_q + 1$$

Then, $q_{ult.}$ must be reduced on the basis of RQD as:

$$q'_{ult.} = q_{ult.}(RQD)^2$$

and

$$q_{all.} = \frac{q_{ult.}(RQD)^2}{F.S.}$$

where: *F.S.=safety factor dependent on RQD. It is common to use F.S. from (6-10) with the higher values for RQD less than about 0.75.*

- **Rock Quality Designation (RQD):**

It is an index used by engineers to measure the quality of a rock mass and computed from recovered core samples as:

$$RQD = \frac{\sum lengths..of..int\,act..pieces..of..core > 100mm}{length..of..core..advance}$$

Example (20): *(RQD)*

A core advance of 1500mm produced a sample length of 1310mm consisting of dust, gravel and intact pieces of rock. The sum of pieces 100mm or larger in length is 890mm.

Solution:

$$\text{The recovery ratio } (L_r) = \frac{1310}{1500} = 0.87\,;\ \text{ and } \ (RQD) = \frac{890}{1500} = 0.59$$

Example (21): *(foundation on rock)*

A pier with a base diameter of 0.9m drilled to a depth of 3m in a rock mass. If RQD = 0.5, $\phi = 45°$ ***and c = 3.5 MPa ,*** γ_{rock} ***= 25.14 kN/m^3, estimate*** $q_{all.}$ ***of the pier using Terzaghi's equation.***

Solution:

By Terzaghi's equation: $$q_{ult.} = cN_c.S_c + qN_q + \frac{1}{2}.B.\gamma.N\gamma.S_\gamma$$

Shape factors: from table (3.2) for circular footing: $S_c = 1.3\,;\quad S_\gamma = 0.6$

Bearing capacity factors: $N_c = 5\tan^4(45+\phi/2)\,,\quad N_q = \tan^6(45+\phi/2)\,,\quad N_\gamma = N_q + 1$

$$for\ \phi = 45°,\quad N_c = 170,\quad N_q = 198,\quad N_\gamma = 199$$

$$q_{ult.} = (3.5x10^3)(170)(1.3) + (3)(25.14)(198) + 0.5(25.14)(0.9)(199)(0.6) = 789.78\ \ MPa$$

and

$$q_{all.} = \frac{q_{ult.}(RQD)^2}{F.S.} = \frac{789.78(0.5)^2}{3.0} = 65.815..MPa$$