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CHAPTER 4

STRESSES IN SOIL MASS

4.1 DEFIFINTIONS

- **VERTTICAL STRESS**
  Occurs due to internal or external applied load such as, overburden pressure, weight of structure and earthquake loads.

- **HORIZONTAL STRESS**
  Occurs due to vertical stress or earth pressure, water pressure, wind loads or earthquake horizontal loads.

- **ISOBAR**
  It is a contour connecting all points below the ground surface of equal intensity of pressure.

- **PRESSURE BULB**
  The zone in a loaded soil mass bounded by an isobar of a given pressure intensity is called a pressure bulb for that intensity.

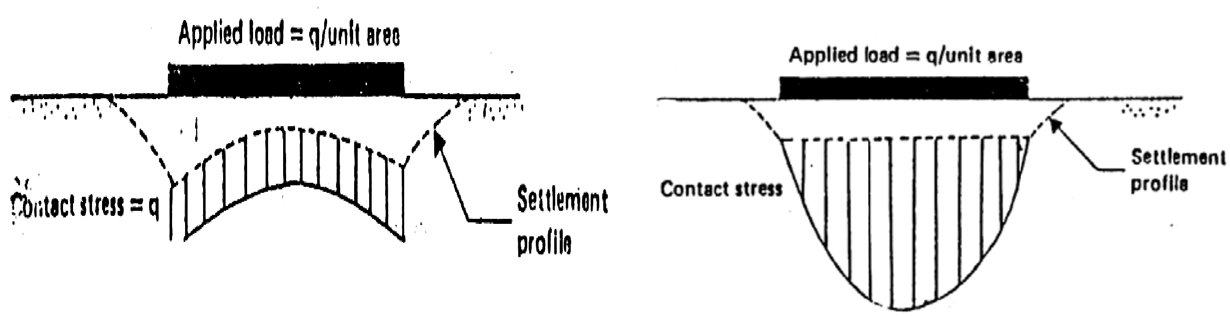
4.2 CONTACT PRESSURE

The analysis of Borowicka (1938) shows that the distribution of contact stress was dependent on a non-dimensional factor defined as:

$$k_r = \frac{1}{6}\left(\frac{1-\nu^2{}_s}{1-\nu^2{}_f}\right)\left(\frac{E_f}{E_s}\right)\left(\frac{T}{b}\right)^3 \quad \text{..}\mathbf{(4.1)}$$

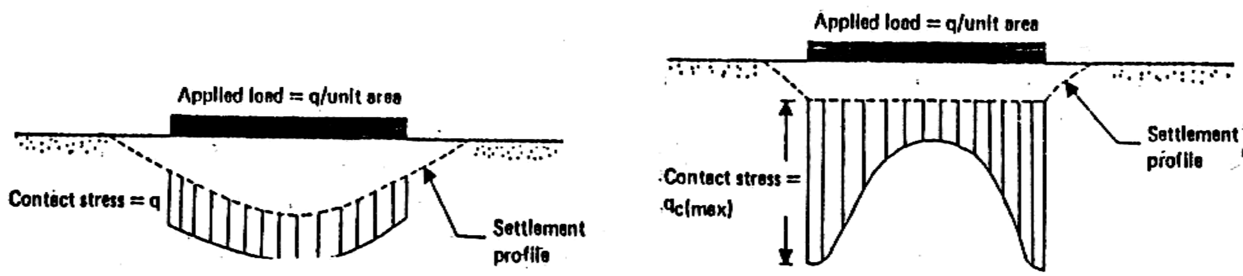
where: ν_s and ν_f = Poisson's ratio for soil and foundation materials, respectively,
E_s and E_f = Young's modulus for soil and foundation materials, respectively,
T = thickness of foundation,
B = half-width for strip footing; or radius for circular footing,
k_r = 0 indicates a perfectly flexible foundation; or
∞ means a perfectly rigid foundation.

The actual soil pressures distributions of rigid and flexible footings resting on sand and clay soils are shown in **Figures (4.1 and 4.2)**.



(a) Flexible **(b) Rigid**

Figure (4.1): Foundations on sand.



(a) Flexible **(b) Rigid**

Figure (4.2): Foundations on clay.

4.3 ASSUMPTIONS: The soil is assumed as:

(1) Semi-infinite in extent; x and y are infinite but the depth z has a limit value (Half-space),
(2) Isotropic; the soil has same properties in all directions,
(3) Homogeneous,
(4) Elastic and obeys Hook's law; the soil has linear relationship,
(5) Stresses at a point due to more than one surface load are obtained by superposition, and
(6) Negative values of loading can be used if the stresses due to excavations were required or the principle of superposition was used.

STRESS INCREASE DUE TO DIFFERENT LOADING

(1) POINT LOAD

- **BOUSSINESQ METHOD FOR HOMOGENEOUS SOIL:**

This method can be used for point loads acts directly at or outside the center. For a central load acting on the surface, its nature at depth z and radius r according to (simple radial stress distribution) is a cylinder in two-dimensional condition and a sphere in three-dimensional case.

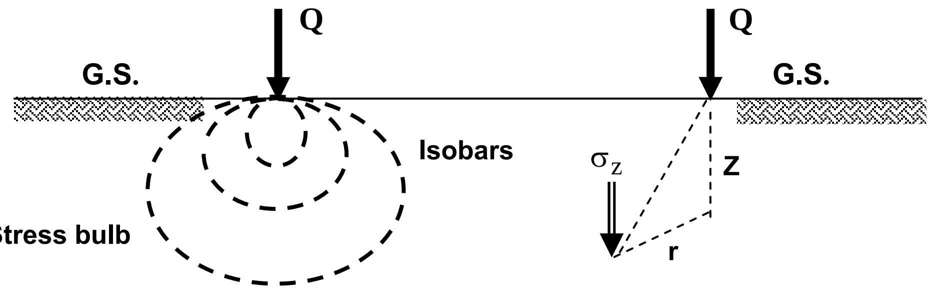


Figure (4.3): Vertical stress due to point load.

The vertical stress increase below or outside the point of load application is calculated as:

$$\sigma_z = \frac{Q}{.z^2} A_b \quad \text{...}\textbf{(4.2)}$$

where: $A_b = \dfrac{3/2\pi.}{\left[1+(\frac{r}{z})^2\right]^{5/2}}$

- **WESTERGAARD METHOD FOR STRATIFIED SOIL:**

$$\sigma_z = \frac{Q}{2\pi.z^2} \frac{\sqrt{(1-2\mu)/(2-2\mu)}}{\left\{\left[(1-2\mu)/(2-2\mu)\right]+(\frac{r}{z})^2\right\}^{3/2}}$$

$$\text{...............................}\textbf{(4.3a)}$$

where: $\mu = \text{Poisson's ratio.}$

when $\mu = 0$: $\sigma_z = \dfrac{Q}{.z^2} A_W \quad \text{...}\textbf{(4.3b)}$

where: $A_W = \dfrac{1/\pi.}{\left[1+2(\frac{r}{z})^2\right]^{3/2}}$; Values of A_W can be tabulated for different values of μ as:

r/z	A_W	
	$\mu = 0$	$\mu = 0.4$
0.0	0.3183	0.9549
0.2	0.2836	0.6916
0.8	0.0925	0.0897
1.0	0.0613	0.0516
2.0	0.0118	0.0076
3.0	0.0038	0.0023
4.0	0.0017	0.0010

Note that :
At (r/z ≈ 1.8) both Boussinessq and Westergaard methods give equal values of σ_z.

Example (4.1): A concentrated point load Q acts vertically at the ground surface. Determine the vertical stress σ_z for each of the following cases:

a. Along the depth for r = 2m, and
b. At depth z = 2m.

Solution: From Boussinesq's equation: $\sigma_z = \frac{Q}{.z^2} A_b$ where: $A_b = \frac{3/2\pi.}{\left[1+(\frac{r}{z})^2\right]^{5/2}}$

(a) For r = 2m, the values of σ_z at various arbitrarily selected depths are given in the following table and the distribution of σ_z with depth is shown in **Figure (4.4 a)**.

z (m)	r/z	A_b	z^2	Q/z^2	σ_z (Q/m^2)
0.0	∞	0	0	∞	Indeterminate
0.4	5.0	0.00014	0.16	6.250Q	0.0009Q
0.8	2.5	0.00337	0.64	1.563Q	0.0053Q
1.2	1.67	0.01712	1.44	0.694Q	0.0119Q
1.6	1.25	0.04543	2.56	0.391Q	0.0178Q
2.0	1.00	0.08440	4.00	0.250Q	0.0211Q
2.4	0.83	0.12775	5.76	0.174Q	0.0222Q
2.8	0.71	0.17035	7.84	0.128Q	0.217Q
3.6	0.56	0.24372	12.96	0.0772Q	0.0188Q
5.0	0.40	0.32946	25.00	0.0400Q	0.0132Q
10.0	0.20	0.43287	100.00	0.0100Q	0.0043Q

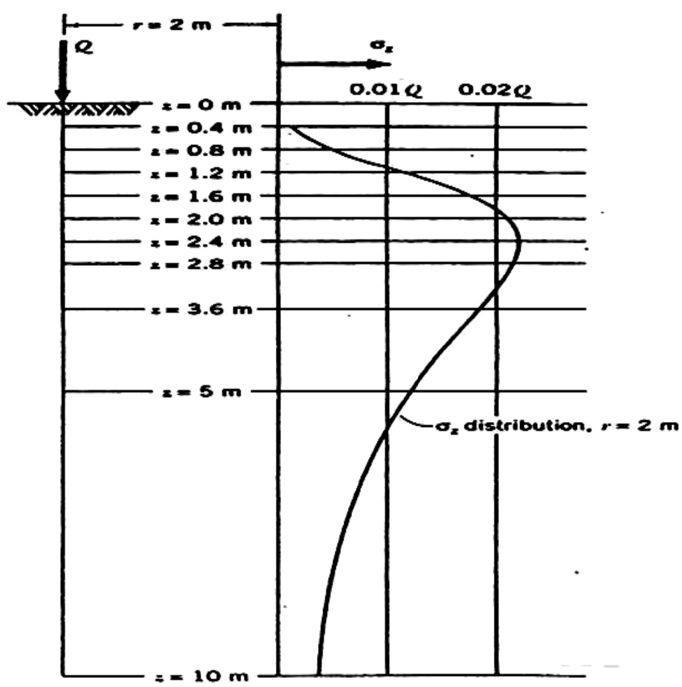


Figure (4.4 a): σ_z distribution with depth at a fixed radial distance from point of surface load.

(b) At depth z = 2m, the values of σ_z for various horizontal distances of r are given in the following table and the distribution of σ_z with r is shown in **Figure (4.4 b)**.

r (m)	r/z	A_b	z^2	Q/z^2	$\sigma_z \quad (Q/m^2)$
0.0	0	0.47746	4.0	0.25Q	0.1194Q
0.4	0.2	0.43287	4.0	0.25Q	0.1082Q
0.8	0.4	0.32946	4.0	0.25Q	0.0824Q
1.2	0.6	0.22136	4.0	0.25Q	0.0553Q
1.6	0.8	0.13862	4.0	0.25Q	0.0347Q
2.0	1.0	0.08440	4.0	0.25Q	0.0211Q
2.4	1.2	0.05134	4.0	0.25Q	0.0129Q
2.8	1.4	0.03168	4.0	0.25Q	0.0079Q
3.6	1.8	0.01290	4.0	0.25Q	0.0032Q
5.0	2.5	0.00337	4.0	0.25Q	0.0008Q
10.0	5.0	0.00014	4.0	0.25Q	0.0001Q

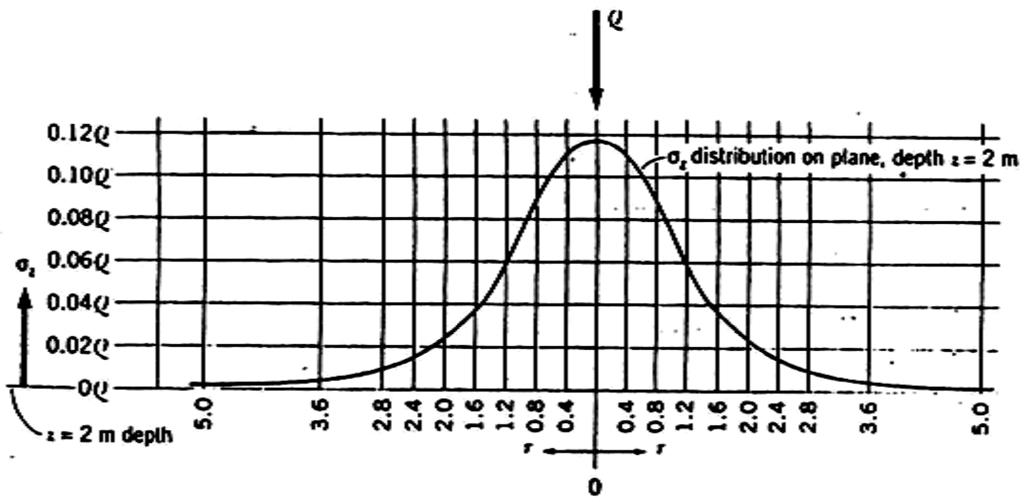


Figure (4.4 b): σ_z distribution with depth at a fixed radial distance from point of surface load.

Example (4.2): Q, is a concentrated point load acts vertically at the ground surface. Determine the vertical stress σ_z for various values of horizontal distances r and at z = 1, 2, 3, and 4m, then plot the σ_z distribution for all z depths.

Solution: From Boussinesq's equation: $\sigma_z = \frac{Q}{.z^2} A_b$ where: $A_b = \frac{3/2\pi.}{\left[1+(\frac{r}{z})^2\right]^{5/2}}$

σ_z for z = 1, 2, 3, and 4m depths is given in the following tables and their distributions with horizontal distances are shown in **Figure (4.5)**.

Z = 1 m					
r (m)	r/z	A_b	z^2	Q/z^2	σ_z (Q/m^2)
0.0	0	0.47746	1.0	Q	0.47746Q
0.4	0.4	0.32946	1.0	Q	0.32946Q
0.8	0.8	0.13862	1.0	Q	0.13862Q
1.2	1.2	0.05134	1.0	Q	0.05134Q
1.6	1.6	0.01997	1.0	Q	0.01997Q
2.0	2.0	0.00854	1.0	Q	0.00854Q
2.4	2.4	0.00402	1.0	Q	0.00402Q
2.8	2.8	0.00206	1.0	Q	0.00206Q
3.6	3.6	0.00066	1.0	Q	0.00066Q
5.0	5.0	0.00014	1.0	Q	0.00014Q

Z = 2 m					
r (m)	r/z	A_b	z^2	Q/z^2	σ_z (Q/m^2)
0.0	0	0.47746	4.0	0.25Q	0.1194Q
0.4	0.2	0.43287	4.0	0.25Q	0.1082Q
0.8	0.4	0.32946	4.0	0.25Q	0.0824Q
1.2	0.6	0.22136	4.0	0.25Q	0.0553Q
1.6	0.8	0.13862	4.0	0.25Q	0.0347Q
2.0	1.0	0.08440	4.0	0.25Q	0.0211Q
2.4	1.2	0.05134	4.0	0.25Q	0.0129Q
2.8	1.4	0.03168	4.0	0.25Q	0.0079Q
3.6	1.8	0.01290	4.0	0.25Q	0.0032Q
5.0	2.5	0.00337	4.0	0.25Q	0.0008Q

Z = 3 m					
r (m)	r/z	A_b	z^2	Q/z^2	σ_z (Q/m^2)
0.0	0	0.47746	9.0	Q/9	0.0531Q
0.4	0.1333	0.45630	9.0	Q/9	0.0507Q
0.8	0.2666	0.40200	9.0	Q/9	0.0447Q
1.2	0.4000	0.32950	9.0	Q/9	0.0366Q
1.6	0.5333	0.25555	9.0	Q/9	0.0284Q
2.0	0.6666	0.19060	9.0	Q/9	0.0212Q
2.4	0.8000	0.13862	9.0	Q/9	0.0154Q
2.8	0.9333	0.09983	9.0	Q/9	0.0111Q
3.6	1.2000	0.05134	9.0	Q/9	0.0057Q
5.0	3.3333	0.01710	9.0	Q/9	0.0019Q

Z = 4 m					
r (m)	r/z	A_b	z^2	Q/z^2	σ_z (Q/m^2)
0.0	0	0.47746	16	Q/16	0.02984Q
0.4	0.1	0.46573	16	Q/16	0.02911Q
0.8	0.2	0.43287	16	Q/16	0.02705Q
1.2	0.3	0.38492	16	Q/16	0.02406Q
1.6	0.4	0.32946	16	Q/16	0.02059Q
2.0	0.5	0.27332	16	Q/16	0.01708Q
2.4	0.6	0.22136	16	Q/16	0.01384Q
2.8	0.7	0.17619	16	Q/16	0.01101Q
3.6	0.9	0.10833	16	Q/16	0.00677Q
5.0	1.25	0.04543	16	Q/16	0.00284Q

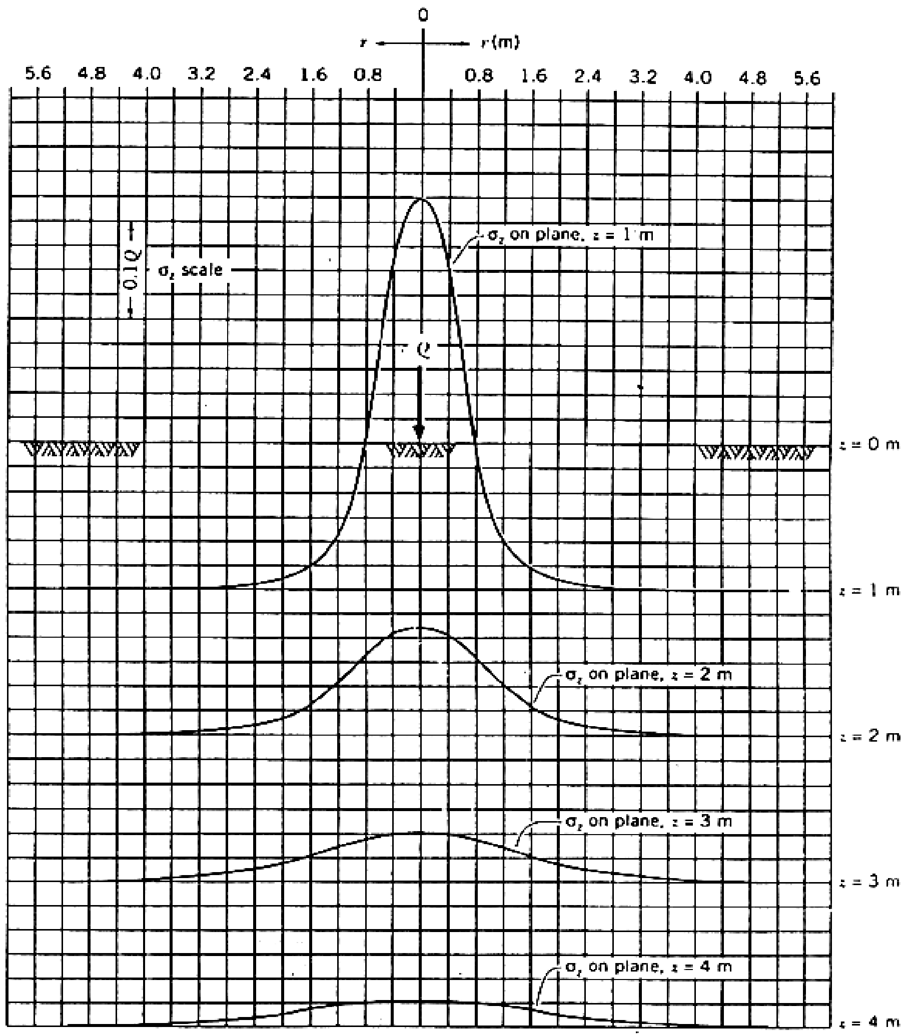
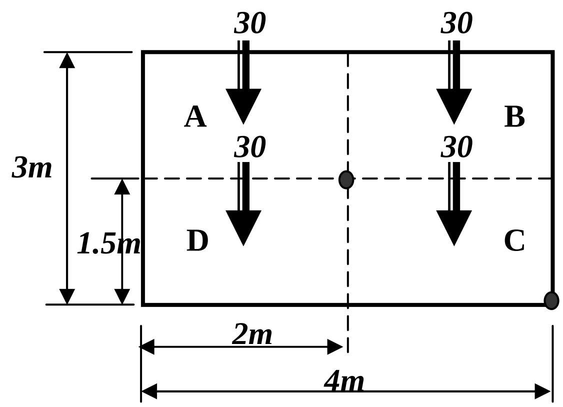


Figure (4.5): σ_z distribution with horizontal distance from point of surface load at several depths.

Example (4.3): An elastic soil medium of (4m x 3m) rectangular area is shown in figure. If the area is divided into 4 elementary areas of (2m x 1.5m) each that subjected at its surface a concentrated loads of (30 ton) at its centroid, use the Boussinesq's equation to find the vertical pressure at a depth of 6m below:

1. the center of the area,
2. one corner of the area.



Solution:

From Boussinesq's equation: $\sigma_z = \frac{Q}{.z^2} A_b$ where: $A_b = \frac{3/2\pi.}{\left[1+(r/z)^2\right]^{5/2}}$

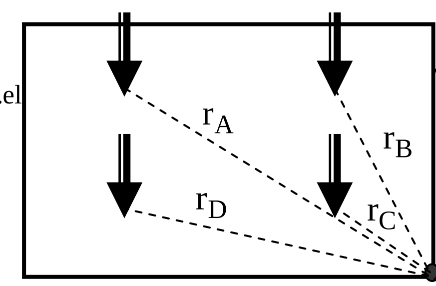
(a) At the center of the area:

r = 1.25m, r/z = 1.25/6 = 0.208

$A_b = 3/2\pi./.\left[1+(0.208)^2\right]^{5/2} = 0.4293,$ $\sigma_{z(One.el}$ 5

T/m^2

$\sigma_{z(Total)} = (4).0.35775 = 1.43\,T/m^2$



At one corner of the area:

Elementary area	r (m)	r/z	A_b	$\sigma_z = \frac{Q}{.z^2} A_b$
A	3.750	0.625	0.209	0.174
B	2.462	0.410	0.324	0.270
C	1.250	0.208	0.429	0.358
D	3.092	0.515	0.265	0.221
$\sum$				1.023 T/m2

(2) 2:1 APPROXIMATION METHOD *for depths < 2.5 (width of loaded area):*

Total load on the surface = q.B.L; and Area at depth z = (L + z) (B + z)

$$\sigma_z = \frac{q.B.L}{(L+z)(B+z)} \quad \text{......................(4.4)}$$

Type of footing	Area, A_z
Square	$(B+z)^2$
Rectangular	$(B+z)(L+z)$
Circular	$\pi(D+z)^2/4$
Strip or wall	$(B+z).1$

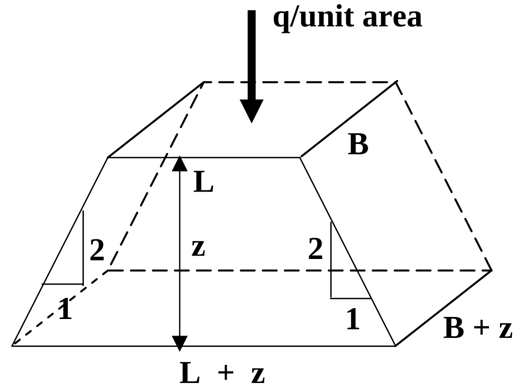


Figure (4.6): 2:1 Stress distribution method.

(3) UNIFORMLY LOADED LINE OF FINITE LENGTH:

Figure (4.7) shows a line load of equal intensity **q** applied at the surface. For an element selected at an arbitrary fixed point in the soil mass, an expression for σ_z could be derived by integrating Boussinesq's expression for point load as:

$$\sigma_z = \frac{Q}{.z^2} P_o \quad \text{......................(4.5)}$$

where:

$$P_o = \frac{1}{2\pi(m^2+1)^2}\left[\frac{3n}{\sqrt{n^2+1+m^2}} - \left(\frac{n}{\sqrt{n^2+1+m^2}}\right)^3\right]$$

$m = x/z$, and $n = y/z$

Values of P_o for various combinations of **m** and **n** are given in **Table (4.1).**

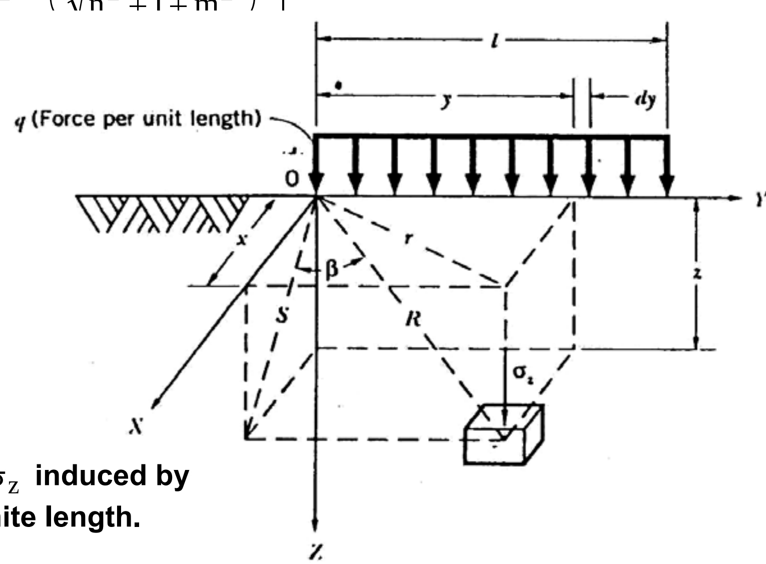


Figure (4.7): Vertical stress σ_z induced by Line loads of finite length.

Table (4.1): Influence values P_0 for case of uniform line load of finite length.

m	n											
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.0	0.04735	0.09244	0.13342	0.16917	0.19929	0.22398	0.24379	0.25947	0.27176	0.28135	0.29464	0.30277
0.1	0.04619	0.09020	0.13023	0.16520	0.19470	0.21892	0.23839	0.25382	0.26593	0.27539	0.28853	0.29659
0.2	0.04294	0.08391	0.12127	0.15403	0.18178	0.20466	0.22315	0.23787	0.24947	0.25857	0.27127	0.27911
0.3	0.03820	0.07472	0.10816	0.13764	0.16279	0.18367	0.20066	0.21429	0.22511	0.23365	0.24566	0.25315
0.4	0.03271	0.06406	0.09293	0.11855	0.14058	0.15905	0.17423	0.18651	0.19634	0.20418	0.21532	0.22235
0.5	0.02715	0.05325	0.07742	0.09904	0.11782	0.13373	0.14694	0.15775	0.16650	0.17354	0.18368	0.19018
0.6	0.02200	0.04322	0.06298	0.08081	0.09646	0.10986	0.12112	0.13045	0.13809	0.14430	0.15339	0.15931
0.7	0.01752	0.03447	0.05035	0.06481	0.07762	0.08872	0.09816	0.10608	0.11265	0.11805	0.12607	0.13140
0.8	0.01379	0.02717	0.03979	0.05136	0.06172	0.07080	0.07862	0.08525	0.09082	0.09546	0.10247	0.10722
0.9	0.01078	0.02128	0.03122	0.04041	0.04872	0.05608	0.06249	0.06800	0.07268	0.07663	0.08268	0.08687
1.0	0.00841	0.01661	0.02441	0.03169	0.03832	0.04425	0.04948	0.05402	0.05793	0.06126	0.06645	0.07012
1.2	0.00512	0.01013	0.01495	0.01949	0.02369	0.02752	0.03097	0.03403	0.03671	0.03905	0.04281	0.04558
1.4	0.00316	0.00626	0.00927	0.01213	0.01481	0.01730	0.01957	0.02162	0.02345	0.02508	0.02777	0.02983
1.6	0.00199	0.00396	0.00587	0.00770	0.00944	0.01107	0.01258	0.01396	0.01522	0.01635	0.01828	0.01979
1.8	0.00129	0.00256	0.00380	0.00500	0.00615	0.00724	0.00825	0.00920	0.01007	0.01086	0.01224	0.01336
2.0	0.00085	0.00170	0.00252	0.00333	0.00410	0.00484	0.00554	0.00619	0.00680	0.00736	0.00836	0.00918
2.5	0.00034	0.00067	0.00100	0.00133	0.00164	0.00194	0.00224	0.00252	0.00278	0.00303	0.00349	0.00389
3.0	0.00015	0.00030	0.00045	0.00060	0.00074	0.00088	0.00102	0.00115	0.00127	0.00140	0.00162	0.00183
4.0	0.00004	0.00008	0.00012	0.00016	0.00020	0.00024	0.00027	0.00031	0.00035	0.00038	0.00045	0.00051

m	n										
	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0	∞
0.0	0.30784	0.31107	0.31318	0.31593	0.31707	0.31789	0.31813	0.31822	0.31828	0.31830	0.31831
0.1	0.30161	0.30482	0.30692	0.30966	0.31080	0.31162	0.31186	0.31195	0.31201	0.31203	0.31204
0.2	0.28402	0.28716	0.28923	0.29193	0.29307	0.29388	0.29412	0.29421	0.29427	0.29428	0.29430
0.3	0.25788	0.26092	0.26293	0.26558	0.26670	0.26750	0.26774	0.26783	0.26789	0.26790	0.26792
0.4	0.22683	0.22975	0.23169	0.23426	0.23535	0.23614	0.23638	0.23647	0.23653	0.23654	0.23656
0.5	0.19438	0.19714	0.19899	0.20147	0.20253	0.20331	0.20354	0.20363	0.20369	0.20371	0.20372
0.6	0.16320	0.16578	0.16753	0.16990	0.17093	0.17169	0.17192	0.17201	0.17207	0.17208	0.17210
0.7	0.13496	0.13735	0.13899	0.14124	0.14224	0.14297	0.14320	0.14329	0.14335	0.14336	0.14338
0.8	0.11044	0.11264	0.11416	0.11628	0.11723	0.11795	0.11818	0.11826	0.11832	0.11834	0.11835
0.9	0.08977	0.09177	0.0318	0.09517	0.09608	0.09677	0.09699	0.09708	0.09713	0.9715	0.09716
1.0	0.07270	0.07452	0.07580	0.07766	0.07852	0.07919	0.07941	0.07949	0.07955	0.07957	0.07958
1.2	0.04759	0.04905	0.05012	0.05171	0.05248	0.05310	0.05330	0.05338	0.05344	0.05345	0.05347
1.4	0.03137	0.03253	0.03340	0.03474	0.03542	0.03598	0.03617	0.03625	0.03630	0.03632	0.03633
1.6	0.02097	0.02188	0.02257	0.02368	0.02427	0.02478	0.02496	0.02504	0.02509	0.02510	0.02512
1.8	0.01425	0.01496	0.01551	0.01643	0.01694	0.01739	0.01756	0.01765	0.01768	0.01769	0.01771
2.0	0.00986	0.01041	0.01085	0.01160	0.01203	0.01244	0.01259	0.01266	0.01271	0.01272	0.01273
2.5	0.00424	0.00453	0.00477	0.00523	0.00551	0.00581	0.00593	0.00599	0.00603	0.00605	0.00606
3.0	0.00201	0.00217	0.00231	0.00258	0.00277	0.00298	0.00307	0.00312	0.00316	0.00317	0.00318
4.0	0.00057	0.00063	0.00068	0.00078	0.00086	0.00096	0.00102	0.00105	0.00108	0.00109	0.00110

Problem (4.4): **Given:** $q = 100$ kN/m.
Find: The vertical stress σ_z at points 0, 1, and 2 shown in Fig. (4.8)?

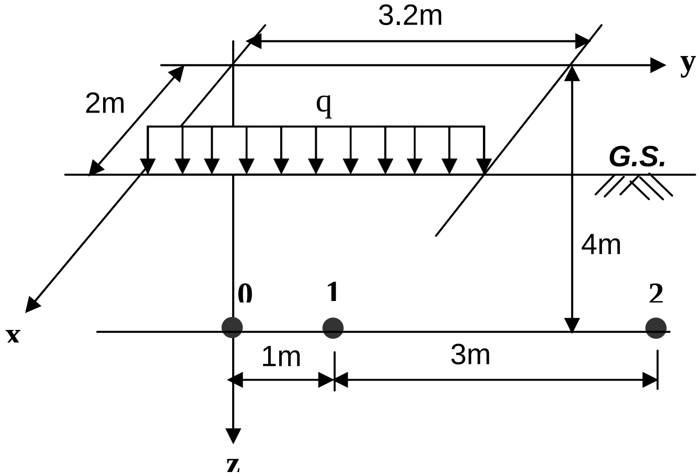


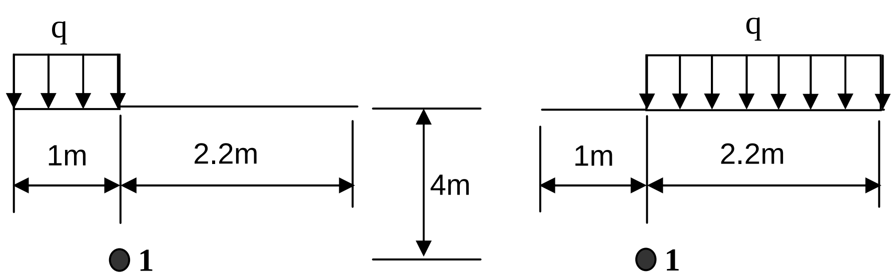
Figure (4.8): Uniformly line surface load of finite length.

Solution:

(a) σ_z at point (0):

For $m = x/z = 2/4 = 0.5$ and $n = y/z = 3.2/4 = 0.8$ from **Table (4.1)**: $P_0 = 0.15775$

$$\sigma_{z(0)} = \frac{Q}{z} P_0 = \frac{100}{4} 0.15775 = 3.944.kN/m^2$$



(a) Left load. **(b) Right load.**

Figure (4.9): σ_z **at point (1).**

(b) σ_z at point (1):

$$\sigma_{z(1)} = \sigma_{z(1L)} + \sigma_{z(1R)}$$

From Fig. (4.9-a)

For $m = x/z = 2/4 = 0.5$ and $n = y/z = 1/4 = 0.25$ from **Table (4.1)**: $P_0 = 0.06534$

$$\sigma_{z(1L)} = \frac{Q}{z} P_0 = \frac{100}{4} (0.06534) = 1.634.kN/m^2$$

From Fig. (4.9-b)

For m = x/z = 2/ 4 = 0.5 and n = y/z = 2.2/ 4 = 0.55 from **Table (4.1)**: $P_0 = 0.12578$

$$\sigma_{z(1R)} = \frac{Q}{z} P_0 = \frac{100}{4}(0.12578) = 3.144.kN/m^2$$

$$\sigma_{z(1)} = \sigma_{z(1L)} + \sigma_{z(1R)} = 1.634 + 3.144 = 4.778.kN/m^2$$

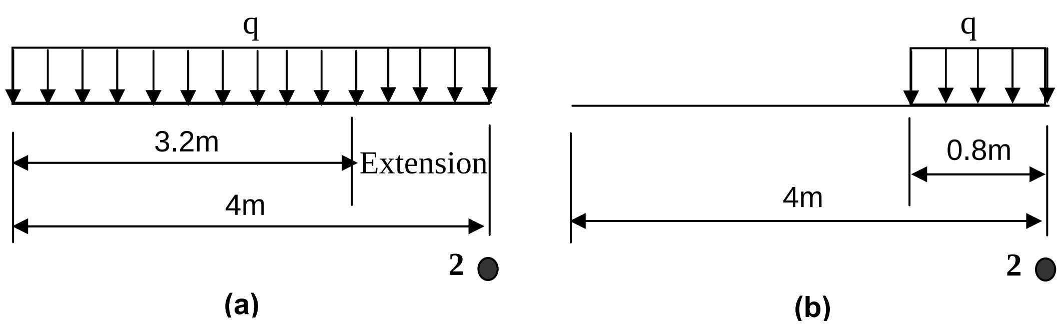


Figure (4.10): σ_z **at point (2).**

(c) σ_z at point (2):

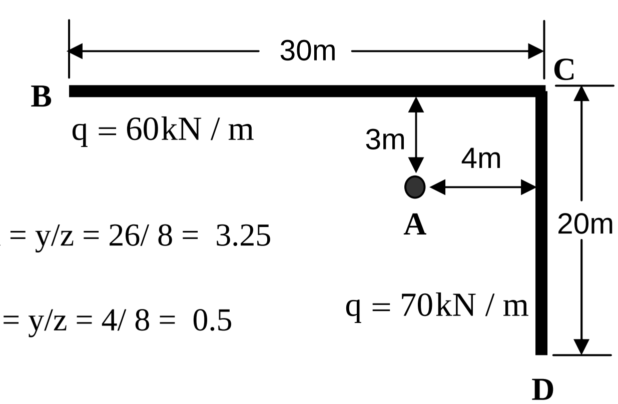
$$\sigma_{z(2)} = \sigma_{z(2L)} - \sigma_{z(2R)}$$

From Fig. (4.10-a):
For m = x/z = 2/ 4 = 0.5 and n = y/z = 4/ 4 = 1.0 from **Table (4.1)**: $P_0 = 0.1735$

From Fig. (4.10-b):
For m = x/z = 2/ 4 = 0.5 and n = y/z = 0.8/ 4 = 0.2 from **Table (4.1)**: $P_0 = 0.0532$

$$\sigma_{z(2)} = \sigma_{z(2L)} - \sigma_{z(2R)} = \frac{100}{4}(0.1735 - 0.0532) = 3.01.kN/m^2$$

Problem (4.5): Given: Two walls loaded as shown in Fig..
Find: the vertical stress σ_z at z = 8m below point A?



Solution:

Wall BC:

26m: For m = x/z = 3/ 8 = 0.375 and n = y/z = 26/ 8 = 3.25
From **Table (4.1)**: $P_0 = 0.26$

4m: For m = x/z = 3/ 8 = 0.375 and n = y/z = 4/ 8 = 0.5
From **Table (4.1)**: $P_0 = 0.15$

$$\sigma_{z(BC)} = \frac{q}{z} P_0 = \frac{60}{8}(0.26 + 0.15) = 3.10.kN/m^2$$

Wall CD:

3**m:** For $m = x/z = 4/8 = 0.5$ and $n = y/z = 3/8 = 0.375$

From **Table (4.1)**: $P_0 = 0.09$

17m: For $m = x/z = 4/8 = 0.5$ and $n = y/z = 17/8 = 2.125$

From **Table (4.1)**: $P_0 = 0.20$

$$\sigma_{z(CD)} = \frac{q}{z} P_0 = \frac{70}{8}(009 + 0.20) = 2.50.kN/m^2$$

$$\sigma_{z(A)} = 3.10 + 2.50 = 5.6.kN/m^2$$

(4) UNIFORMLY LOADED STRIP AREA:

To calculate the vertical stress under uniformly loaded strip area **(see Figure 4.11)**.

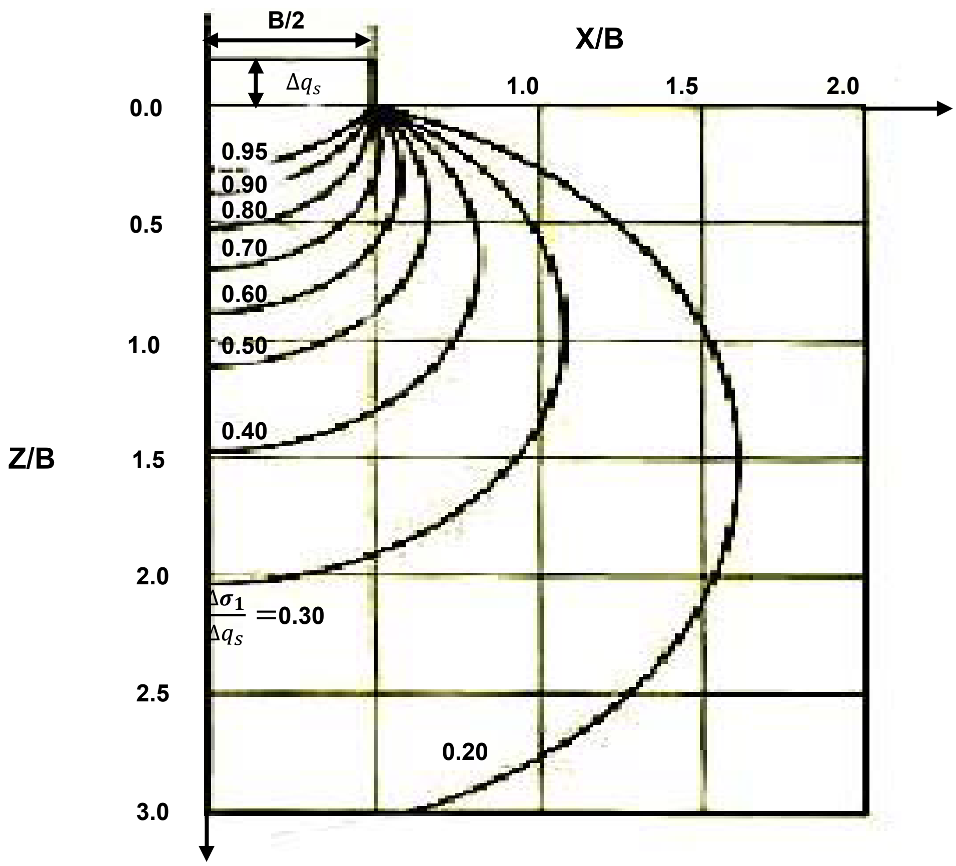


Figure (4.11): Pressure bulbs for vertical stresses under strip load.

(5) TRIANGULAR LOADED STRIP AREA:

To calculate the vertical stress under triangular loaded strip area **(see Figure 4.12**).

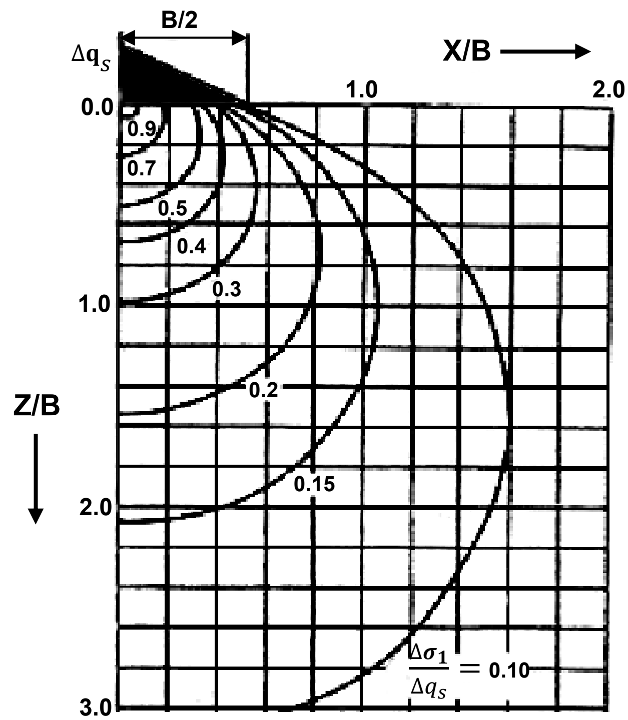
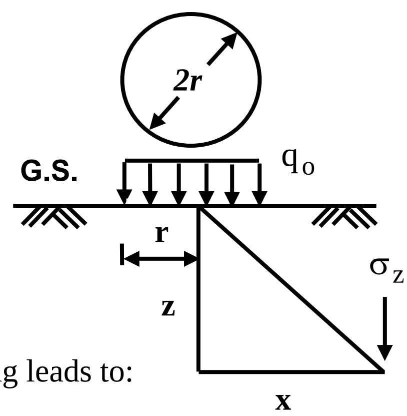


Figure (4.12): Pressure bulbs for vertical stresses under triangular strip load.

(6) UNIFORMLY LOADED CIRCULAR AREA:



$$\sigma_z = \int_{\theta=0}^{2\pi} \frac{3q}{.2\pi.z^2} \left[\frac{1}{1+(r/z)^2}\right]^{5/2} dA$$

where, $dA = \frac{1}{2} r^2 .d\theta$; which after integrating and simplifying leads to:

$$\sigma_z = \frac{I.xq_o}{100} \qquad \text{.......(4.5)}$$

where, I = Influence factor depends on (z/r and x/r); expressed in percentage of surface contact pressure, q_o, for vertical stress under uniformly loaded circular area (**see Figure 4.13**).

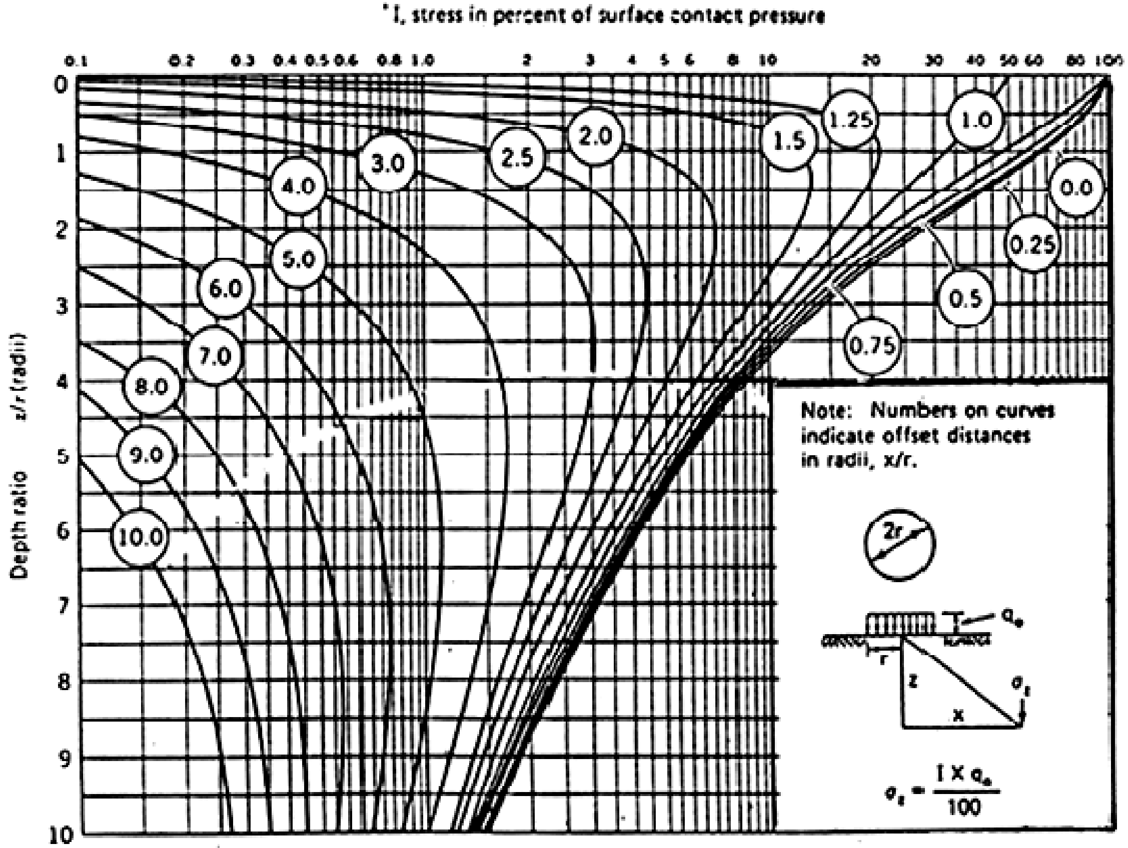


Figure (4.13): Influence values expressed in percentage of surface contact pressure for vertical stress under uniformly loaded circular area *(after Foster and Ahlvin, 1954, as cited by U.S. Navy, 1971).*

Problem (٤.6):

Given: A circular area, r = 1.6m, induces a soil pressure at the surface of 100 kN/m^2.

Find: the vertical stress σ_z at:

(a) z = 2m directly under the center of the circular area.

(b) z = 2m below and 2m away from the center of the circle.

Solution:

a. For z/r = 2/1.6 = 1.25 and x/r = 0; *from **Fig. (4.13):*** $I = 52$

$$\sigma_z = \frac{I.q_o}{100} = \frac{52.(100)}{100} = 52 \ \mathbf{kN/m^2}.$$

b. For z/r = 2/1.6 = 1.25 and x/r = 2/1.6 = 1.25; *from **Fig. (4.13):*** $I = 22$

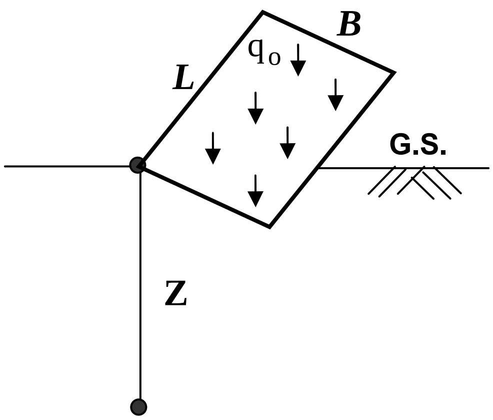
$$\sigma_z = \frac{I.q_o}{100} = \frac{22.(100)}{100} = 22 \ \text{kN/m}^2.$$

(6) UNIFORMLY LOADED RECTANGULAR OR SQUARE AREA:

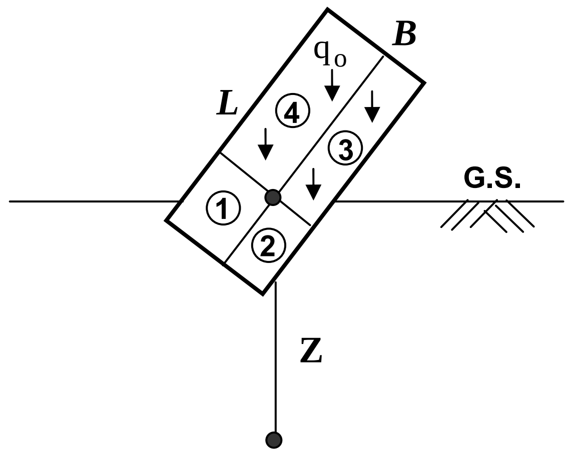
The vertical stress increase below the corner of a flexible rectangular or square loaded area is calculated as:

$$\sigma_z = I.q_o \quad \text{...(4.6)}$$

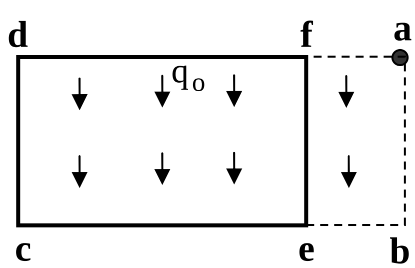
where, I = influence factor, depends on (m = B/z, and n = L/z) obtained from **(Figure 4.14).**



Example (1): $\sigma_z = I.q_o$



Example (2): $\sigma_z = q_o[I_1 + I_2 + I_3 + I_4]$



Example (3): $\sigma_{z(a)} = q_o[I_{abcd} - I_{abef}]$

Examples for Vertical stress under the corner of a uniformly loaded rectangular area.

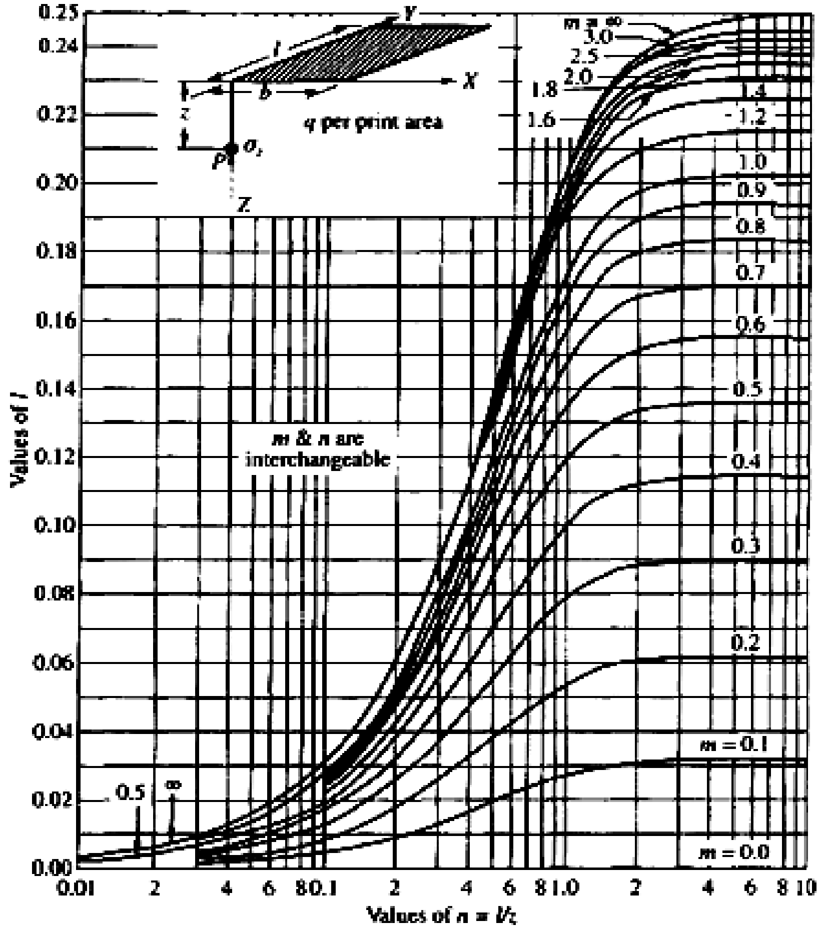
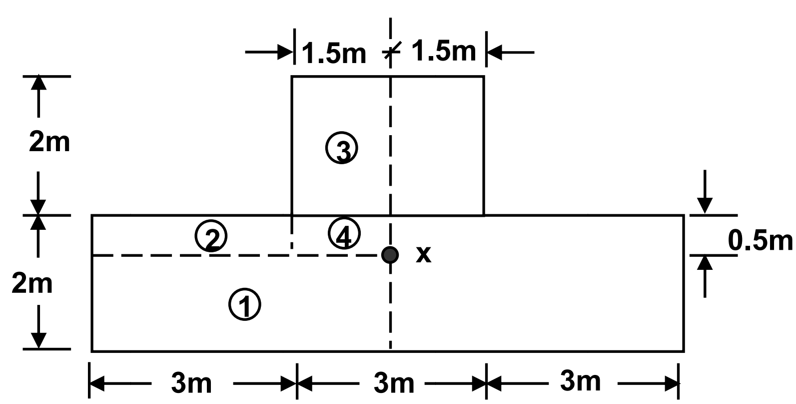


Figure (4.14): Values of I for vertical stress below the corner of a flexible rectangular area *(after Fadum, 1948).*

Problem (4.7): The plan of a foundation is given in the Fig. below. The uniform contact pressure is 40 kN/m^2. Determine the vertical stress increment due to the foundation at a depth of (5m) below the point (x).



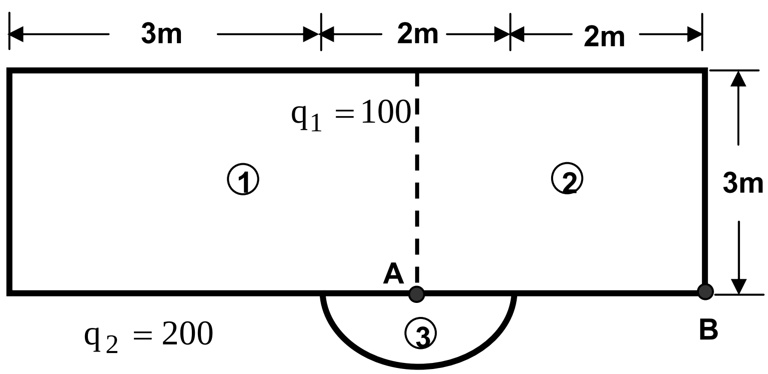
Solution:

- Using **Fig. (4.14)** the following table of results can be prepared for z = 5m

Segment	B	L	m = B/z	n = L/z	I
1	1.5	4.5	0.3	0.9	0.077
2	0.5	4.5	0.1	0.9	0.027
3	1.5	2.5	0.3	0.5	0.056
4	0.5	1.5	0.1	0.3	0.013

$\sigma_z = I.q_o = (2)(40)[\ 0.077 + 0.027 + 0.056 - 0.013] = 11.76 \text{ kN/m}^2$

Problem (4.8): Determine the vertical stress increase at points (A) and (B) due to the loaded area shown in Fig. knowing that A and B points are located at depth of (5m) below the foundation level.



Solution:

(1) for point A:

- ***For half-circular area:***
  From ***Fig. (4.13):*** for z = 5m, z/r = 5/1 = 5 and x/r = 0: $\mathbf{I_1} = 5.7$
- ***For rectangular loaded area:***
  From ***Fig. (4.14):*** for z = 5m, **m =** B/z = 3/5 = 0.6 and **n** = L/z = 4/5 = 0.8: $\mathbf{I_2} = 0.125$
  for z = 5m, **m =** B/z = 3/5 = 0.6 and **n** = L/z =3/5 = 0.6: $\mathbf{I_3} = 0.107$

$\therefore \quad \sigma_z = (0.5)(200)(5.7/100) + (100)(0.125 + 0.107) = 28.9 \text{ kN/m}^2$

(2) for point B:

- ***For half-circular area:***
  From ***Fig. (4.13):*** for z = 5m, z/r = 5/1 = 5 and x/r = 3/1 = 3: $\mathbf{I_1} = 2.7$
- ***For rectangular loaded area:***
  From ***Fig. (4.14):*** for z = 5m, **m =** B/z = 3/5 = 0.6 and **n** = L/z = 7/5 = 1.4: $\mathbf{I_2} = 0.147$

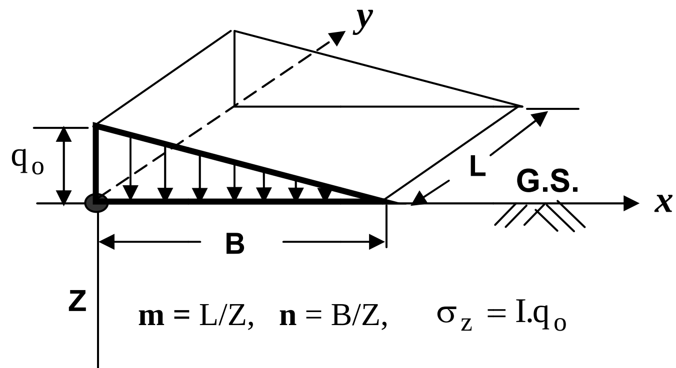
$\therefore \quad \sigma_z = (0.5)(200)(2.7/100) + (100)(0.147) = 17.4 \text{ kN/m}^2$

(7) TRIANGULAR LOAD OF LIMITED LENGTH:

The vertical stress under the corners of a triangular load of limited length is calculated as:

$$\sigma_z = I.q_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.7)$$

where, I = influence factor, depends on (m = L/z, and n = B/z) obtained from **(Figure 4.15).**



Vertical stress under the corner of a triangular load of limited length.

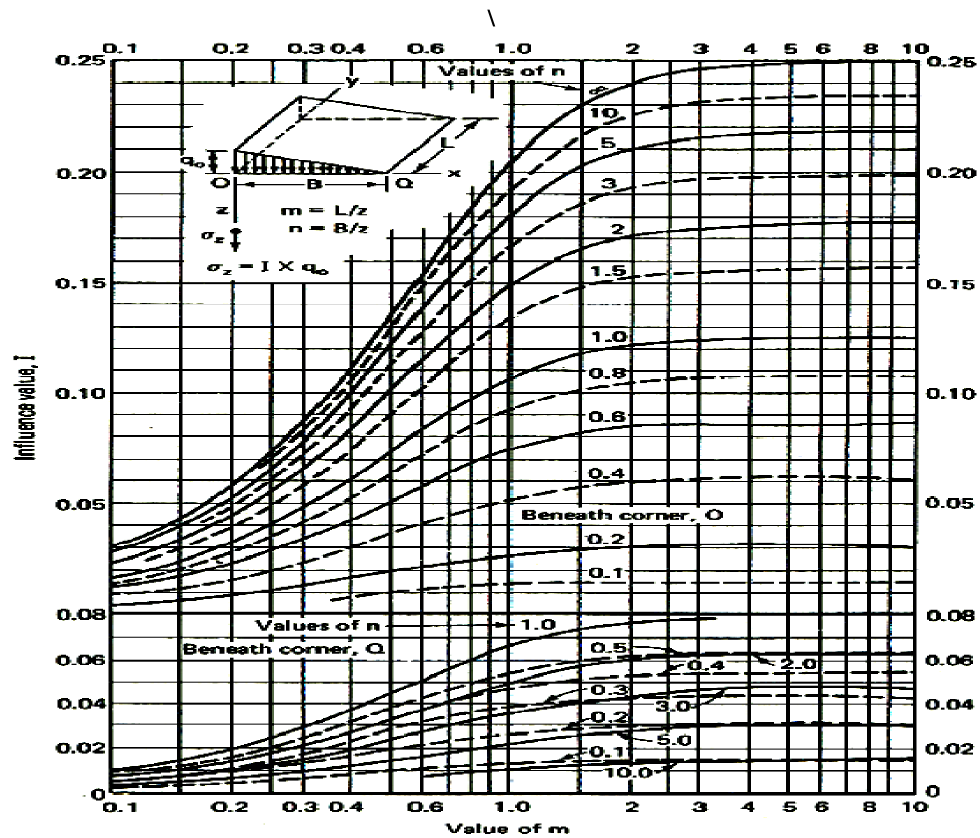


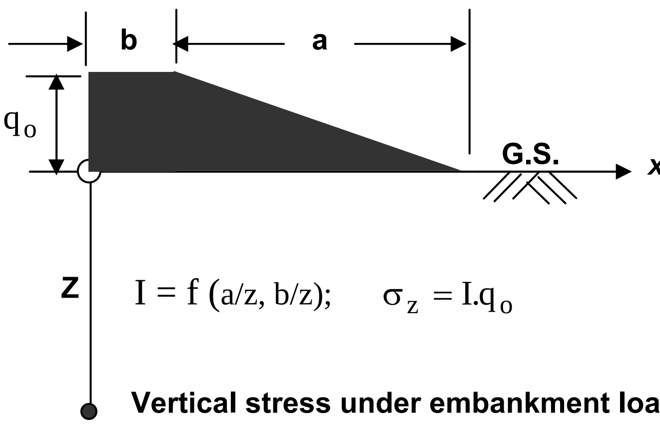
Figure (4.15): Influence values for vertical stress under the corners of a triangular load of limited length *(after U.S. Navy, 971).*

(8) EMBANKMENT LOADING:

The vertical stress under embankment loading is calculated as:

$$\sigma_z = I.q_o \quad \text{…………………………………………………………….(4.8)}$$

where, I = influence factor depends on (a/z, and b/z) determined from **(Figure 4.16).**



Vertical stress under embankment loading.

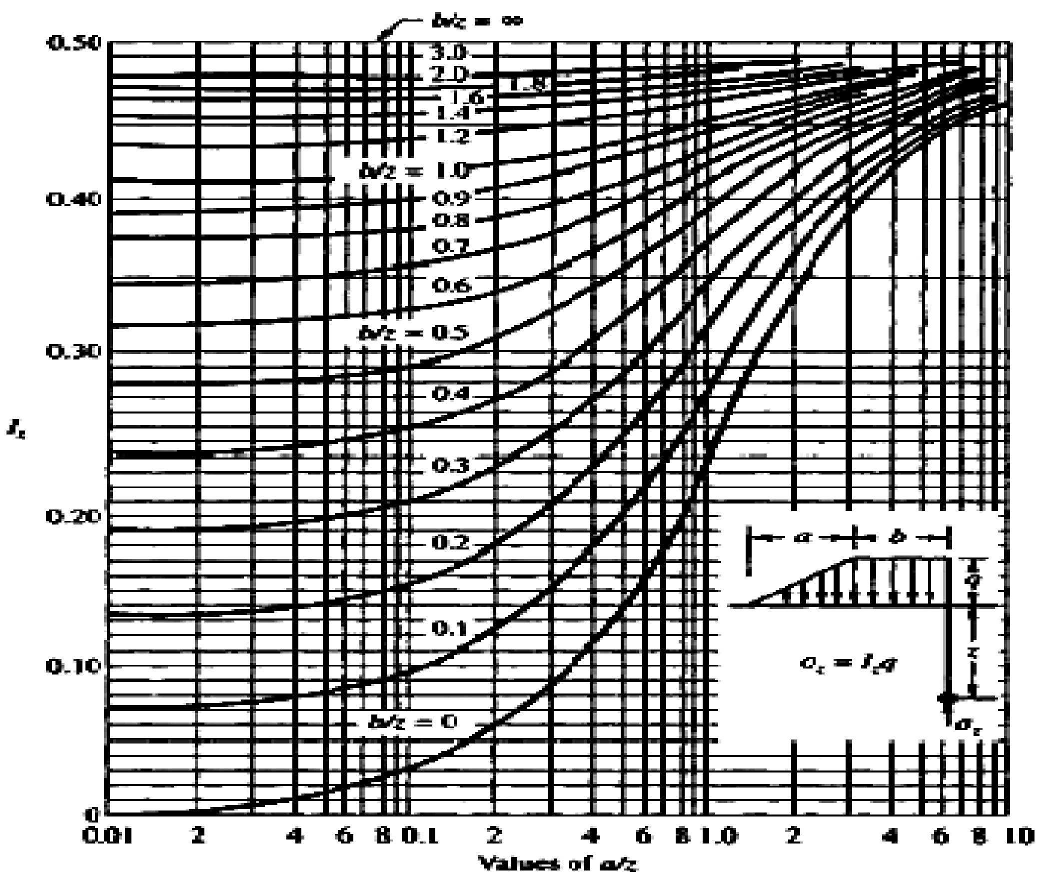
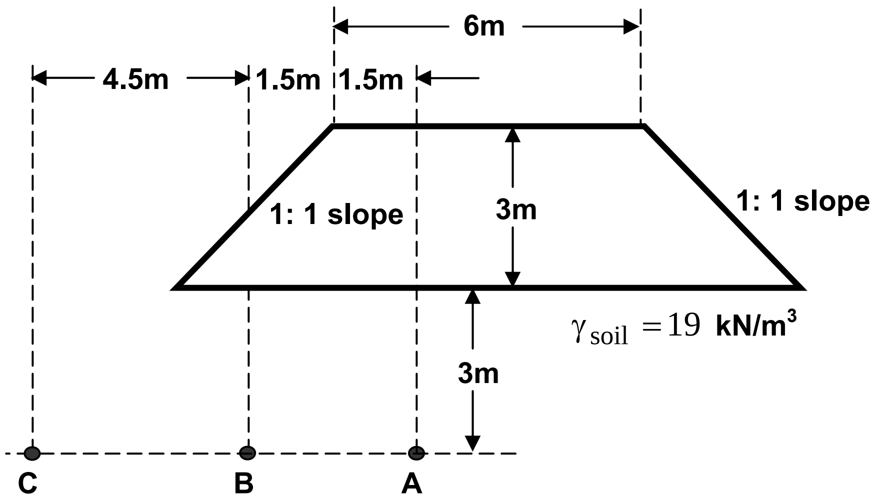


Figure (4.16): Influence factor for embankment loading *(after Osterberg, 1957).*

Problem (4.9): An embankment of (3m) high is to be constructed as shown in the figure below. If the unit weight of compacted soil is 19 kN/m^3, calculate the vertical stress due to the embankment loading at (A), (B), and (C) points.



Solution:

(1) Vertical stress at A:

From Fig. (4.17a): $\sigma_{zA} = \sigma_{z(1)} + \sigma_{z(2)}$

Left-hand section: b/z = 1.5/3 = 0.5 and a/z = 3/3 = 1.0, *from* ***Fig. (4.16);*** $I_1 = 0.396$

Right-hand section: b/z = 4.5/3 = 1.5 and a/z = 3/3 = 1.0, *from* ***Fig. (4.16);*** $I_2 = 0.477$

$\sigma_{zA} = (I_1 + I_2)q = [0.396 + 0.477](19)(3) = 49.761$ kN/m^2

(2) Vertical stress at B:

From Fig. (4.17b): $\sigma_{zB} = \sigma_{z(1)} + \sigma_{z(2)} - \sigma_{z(3)}$

Left-hand section: b/z = 0/3 = 0 and a/z = 1.5/3 = 0.5, *from* ***Fig. (4.16);*** $I_1 = 0.140$

Middle section: b/z =7.5/3 = 2.5 and a/z = 3/3 = 1.0, *from* ***Fig. (4.16);*** $I_2 = 0.493$

Right-hand section: b/z = 0/3 = 0 and a/z = 1.5/3 = 0.5, *from* ***Fig. (4.16);*** $I_3 = 0.140$

$\sigma_{zB} = (I_1.q_1) + (I_2.q_2) - (I_3.q_3)$

$= (0.14)(19)(1.5) + (0.493)(19)(3) - (0.14)(19)(1.5) = 28.101$ kN/m^2

(3) Vertical stress at C:

Using Fig. (4.17c): $\sigma_{zC} = \sigma_{z(1)} - \sigma_{z(2)}$

Left-hand section, b/z = 12/3 = 4 and a/z = 3/3 = 1.0, *from* ***Fig. (4.16);*** $I_1 = 0.498$.

Right-hand section, b/z = 3/3 = 1.0 and a/z = 3/3 = 1.0, *from* ***Fig. (4.16);*** $I_2 = 0.456$.

$\sigma_{zC} = (I_1 - I_2)q = (0.498 - 0.456)(19)(3) = 2.394$ kN/m^2

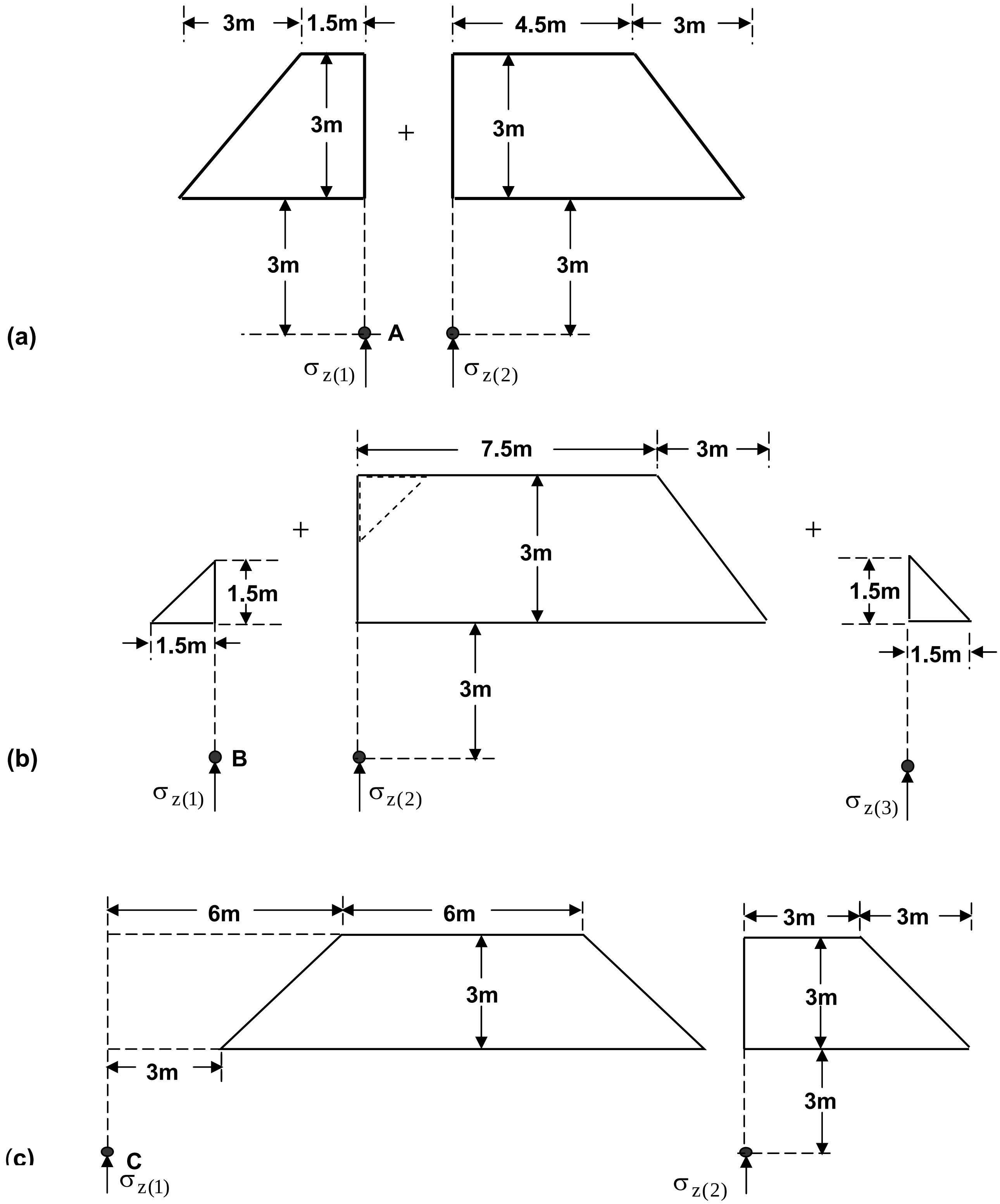


Figure (4.17): Solution of Example (4.9).

(9) ANY SHAPE LOADED AREA (NEWMARK CHART):

The stress on an elemental area dA of soil due to surface contact pressure q_o is calculated as:

$$dq = \frac{3q_o}{2\pi . z^2} \frac{1}{[1+(r/z)^2]^{5/2}} dA$$

but $dA = 2\pi.r.dr$ $\quad \therefore \quad q = \int_0^r \frac{3q_o}{2\pi.z^2} \frac{2\pi.r.dr}{[1+(r/z)^2]^{5/2}}$

or $\quad q = q_o \left\{ 1 - \frac{1}{[1+(r/z)^2]^{3/2}} \right\}$

$$(r/z) = \sqrt{(1-q/q_o)^{-2/3} - 1} \quad \text{.......................................(4.9)}$$

Prepare a chart on transparent paper with r_i circles as follows with 18° sectors:

q/q_o	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
(r/z)	0.270	0.400	0.518	0.637	0.766	0.918	1.110	1.387	1.908	∞

In this case, each circle of the chart is subdivided into 20 units, therefore the number of units for 10 circles = (20 units x 10 circles) = 200 and the influence value (I_V =1/200 = 0.005). If the scale distance (AB) is assumed = 5 cm, then:

r_i (cm)	1.35	2	2.59	3.18	3.83	4.59	5.55	6.94	9.54	∞

To estimate σ_z:

(1) Adopt a scale such that, the scale distance (AB) is equal to the required depth (z),

(2) Based on the scale adopted in (1), replot the plan of the loaded area,

(3) Place the plan plotted in (2) on the Newmark chart in such a way that the point (P) at which the vertical stress is required,

(4) Count the number of blocks, N of the chart which fall inside the plan, and

(5) calculate σ_z as:

$$\sigma_z = q.(I_V).(N) \quad \text{.......................................(4.10)}$$

where, I_v = Influence value of the chart **(see Figure 4.18).**

• Important Notes about Newmark Chart

a. If the stress is required at different depth, then the plan is drawn again to a different scale such that the new depth z is equal to the distance (AB) on the chart.

b. The use of Newmark's chart is based on a factor termed the influence value, determined from the number of units into which the chart is subdivided. For example; **Fig.(4.18)** is subdivided into 200 units (20 units x 10 circles), therefore the influence value is (1/200 = 0.005). But if the series of rings are subdivided into 400 units, then, $I_V = 1/400 = 0.0025$.

c. In making a chart, it is necessary that the sum of units between two concentric circles multiplied by I_V be equal to the change in q/q_o of the two rings. (i.e., if the change in two rings is 0.1 q/q_o, then I_V x number of units should equal to 0.1).

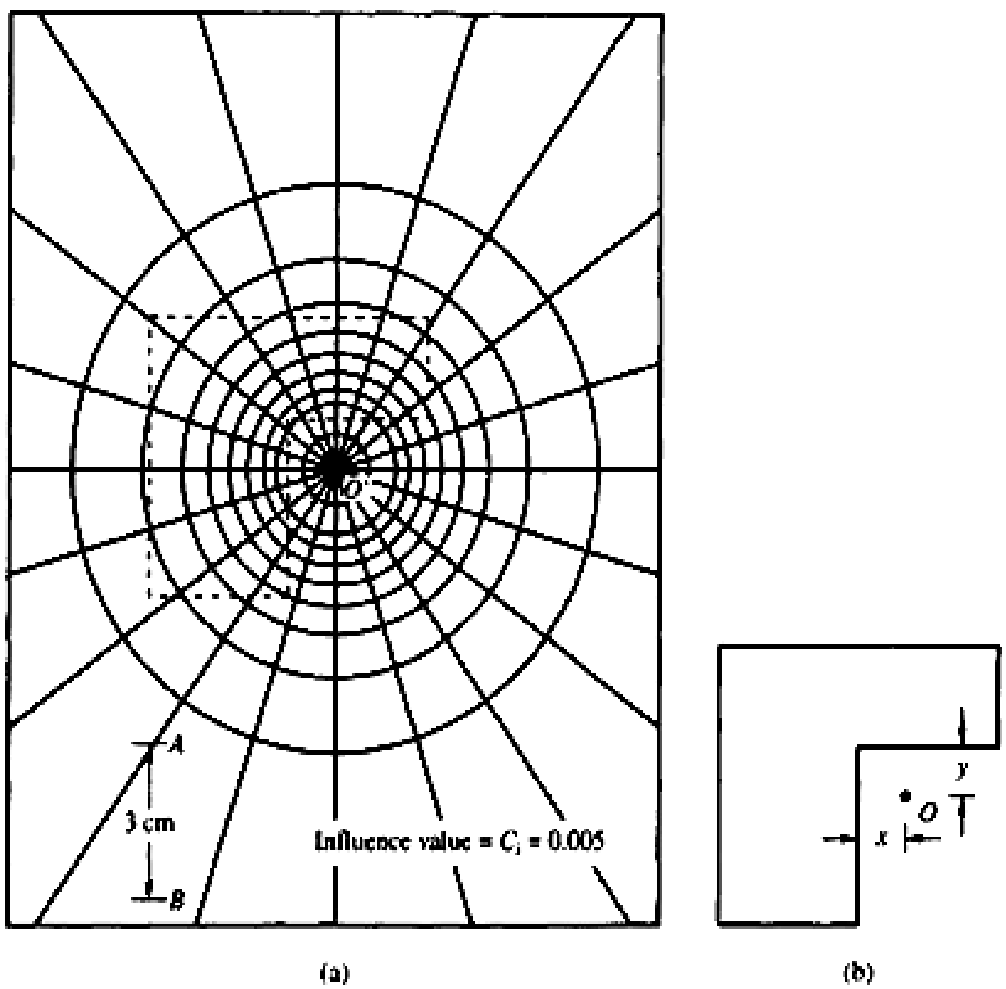
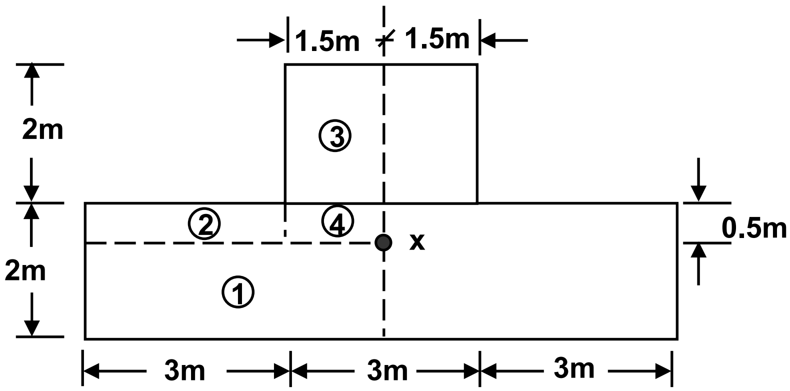


Figure (4.18): Influence chart for computation of vertical pressure *(after Newmark, 1942).*

Problem (4.10): The foundation plan shown in the figure below is subjected to a uniform contact pressure of 40 kN/m^2. Determine the vertical stress increment due to the foundation load at (5m) depth below the point (x).



Solution:

Using Fig. (4.18): N ≈ 58

$\sigma_z = q.(I_V).(N) = (40)(0.005)(58) = 11.6 \text{ kN/m}^2$