CHAPTER 5

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SETTLEMENT OF BUILDINGS

5.1 TYPES OF SETTLEMENT

Settlement is a term that describes the vertical displacement of a structure, footing, road or embankment due to the downward movement of a point.

From structural point of view, settlement of structures may be of two types:

- **Equal or uniform settlement:** This type has no serious implication on the structure or civil engineering performance of the building. But it should have a maximum limit to prevent the failure of soil under the structure.

- **Differential settlement:** It means that one point of the structure settles more or less than the others, therefore, it may lead to damage of the superstructure. Usually it occurs due to one or more of the following:

1. Variation of soil stratum (the subsoil is not homogeneous).
2. Variation in loading condition.
3. Large loaded area on flexible footing.
4. Differential difference in time of construction, and
5. Ground condition, such as slopes.

5.2 TILTING OF FOUNDATIONS

The limiting values of foundation tilting are presented in Table (5.1) and can be calculated as:

$$\tan_{\omega_L} = \frac{M_L}{L^2\, B}\,\frac{1-\mu_s^2}{E_s}\, I_m \quad \text{...}\textbf{(5.1a)}$$

$$\tan_{\omega_B} = \frac{M_B}{B^2\, L}\,\frac{1-\mu_s^2}{E_s}\, I_m \quad \text{...}\textbf{(5.1b)}$$

where,

M_L = moment in L - direction = Q. e_L

M_B = moment in B - direction = Q. e_B

ω_L and ω_B = tilting angles in L and B directions, respectively, and

I_m = moment factor that depends on the footing size as given in Table (5.2).

Table (5.1): Effect of foundation tilting on structures.

ω *(in radians)*	*Result to structure*
1/150	*Major damage*
1/250	*Tilting becomes visible*
1/300	*First cracks appear*
1/500	*No cracks (safe limit)*

Table (5.2): Values of I_m for various footing shapes.

Footing type		I_m
Circular		*6.00*
Rectangular with L/B =	*1.00 (Square)*	*3.70*
	1.50	*5.12*
	1.25	*5.00*
	2.00	*5.38*
	2.50	*5.71*
	5.00	*5.82*
	10.0	*5.93*
	∞ *(Strip)*	*5.10*

5.3 LIMITING VALUES OF SETTLEMENT PARAMETERS

Many investigators and building codes recommended the allowable values for the various parameters of total and differential settlements as presented in Tables (5.3 - 5.6).

Table (5.3): Limiting values of maximum total settlement, maximum differential settlement, and maximum angular distortion for building purposes
(Skempton and MacDonald, 1956).

Settlement parameter	*Settlement (mm)*		
	Sand		*Clay*
	Ref.1	*Ref.2*	*Rf.2*
Maximum total settlement, $S_{T(max.)}$	*20*	*32*	*45*
Maximum differential settlement, $\Delta S_{T(max.)}$			
• *Isolated foundations.*	*25*	*51*	*76*
• *Raft foundations.*	*50*	*51-76*	*76 - 127*
Maximum angular distortion, $\beta_{max.}$	*1/300*		

Ref. 1 - Terzaghi and Peck (1948), Ref. 2 - Skempton and MacDonald (1956)

Table (5.4): Limiting values of deflection ratios
(The 1955 Soviet Code of Practice).

Building type	*Deflection ratio* (Δ / L)		*Average maximum Settlement (cm)*
	Sand	*Clay*	
Steel and concrete frames	*0.0010*	*0.0013*	*10*
Multistory buildings $L/H \le 3$ $L/H \ge 5$	 *0.003* *0.005*	 *0.004* *0.007*	 8 $L/H \ge 2.5$ 10 $L/H \le 1.5$
One-story building	*0.001*	*0.001*	-----------
Water towers, Ring foundations	*0.004*	*0.004*	-----------

L = length between two adjacent points under consideration, and
H = height of wall above foundation.

Table (5.5): Limiting angular distortion for various structures
(Bjerrum, 1963).

Category of potential damage	*Angular distortion* $\beta_{max.}$
Safe limit for flexible brick walls (L/H >4)	*1/150*
Danger for structural damage of general buildings	*1/150*
Cracking in panel and brick walls	*1/150*
Visible tilting of high rigid buildings	*1/250*
First cracking in panel walls	*1/300*
Safe limit of no cracking of building	*1/500*
Danger for frames with diagonals	*1/600*

Table (5.6): Recommendation of European Committee for Standardization (1994) **on differential settlement parameters**.

Item	*Parameter*	*Magnitude*	*Comments*
Limiting values for serviceability	S_T	*25 mm* *50 mm*	*Isolated shallow foundation* *Raft foundation*
	ΔS_T	*5 mm* *10 mm* *20 mm*	*Frames with rigid cladding* *Frames with flexible cladding* *Open frames*
	β	*1/500*	-----------
Maximum acceptable foundation movement	S_T	*50mm*	*Isolated shallow foundation*
	ΔS_T	*20mm*	*Isolated shallow foundation*
	β	$\approx$ *1/500*	-----------

5.4 COMPONENTS OF TOTAL SETTLEMENT

Foundation settlement mainly consists of three components (see **Fig. (5.1)**):

(i) **Immediate settlement (S_i):** occurs due to elastic deformation of soil particles upon load application with no change in water content.

(ii) **Primary consolidation settlement (S_c):** occurs as the result of volume change in saturated fine grained soils due to expulsion of water from the void spaces of the soil mass with time.

(iii) **Secondary consolidation settlement (S_{sc}):** occurs after the completion of the primary consolidation due to plastic deformation of soil (reorientation of the soil particles). It forms the major part of settlement in highly organic soils and peats.

$$\therefore \quad S_T = S_i + S_c + S_{sc} \qquad \text{...(5.2)}$$

These components occur in different types of soils with varying circumstances:

- **For clay:** $S_T = S_i$ (**minimum**) + S_c (**major**) + S_{sc} (**small, but present to certain extent**)
  Therefore, for clay these settlements must be calculated.
- **For sand:** $S_T = S_i$ (**major**) + S_c (**present but mixed with S_i**) + S_{sc} (**undefined**)
  Since sand is permeable, therefore, Terzaghi theory cannot be applicable.

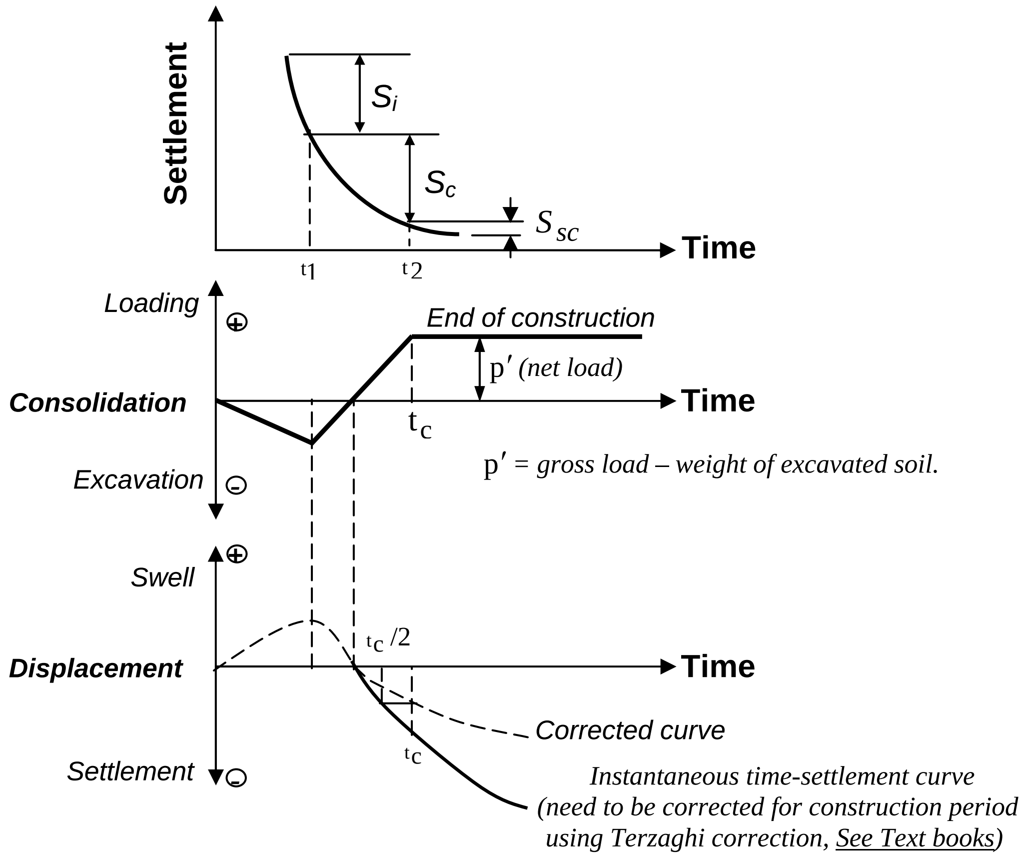


Fig.(5.1): Settlement versus time relationship.

5.5 METHODS OF COMPUTING IMMEDIATE SETTLEMENT

Many methods are available to calculate the elastic (immediate) settlement of shallow foundations. But, only those methods of practical interest are discussed herein:

1. Theory of Elasticity method *for granular soils or partially saturated clays*.
2. Schmertmann method *for granular soils.*
3. Bjerrum method *for layered clay under undrained condition*.

5.5.1 Immediate Settlement Based on the Theory of Elasticity

The elastic settlement of a footing rested on *granular soils or partially saturated clays*, can be estimated using the elastic theory as (**see Fig.(5.2)**):

$$S_{i(flexible)} = q_o.B'\frac{1-\mu_s^{\ 2}}{E_s} I_s.I_D.C_N \text{..} \textbf{(5.3)}$$

$$S_{i(rigid)} \approx 0.93.S_{i(flexible)} \text{..} \textbf{(5.4)}$$

$$S_{i(average)} = 0.85.S_{i(center)} \text{..} \textbf{(5.5)}$$

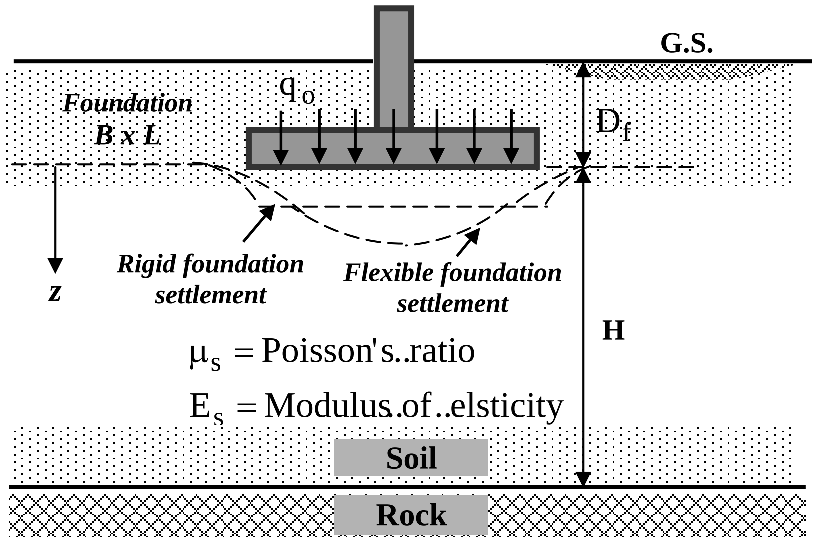


Fig.(5.2): Elastic settlement of flexible and rigid foundations.

where,

S_i = immediate or elastic,

q_o = net applied pressure on the foundation,

B' = **B/2** *for center* of foundation, and

= **B** *for corners* of foundation,

μ_s = Poisson's ratio of soil, ***(see Table (5.7) for typical values).***

E_s = weighted average modulus of elasticity of the soil over a depth of H. For a multi-layered soil stratum it is computed as:

$$E_{s(avg.)} = \frac{\sum Es_{(i)}.H_i}{\sum H_i}$$

in which, H_i and E_i are the thickness and modulus of elasticity of layer **i**, and $\sum H_i$ = **H** (the depth of hard stratum) or **5B** whichever is smaller,

(see Table (5.8) for typical values of E_s).

I_s = Shape factor (Steinbrenner, 1934) computed by:

$$I_s = I_1 + \frac{1-2\mu_s}{1-\mu_s} I_2$$

where, $I_1 ... and ... I_2$ are influence factors = $f(H/B', .. L/B)$ *obtained from **Table (5.9)***, and
H = depth of hard stratum

I_D = Depth factor (Fox, 1948) = $f(D_f / B, .. \mu_s, .. and .. L/B)$ which can be approximated by:

$$I_D = 0.66\left(\frac{D_f}{B}\right)^{(-0.19)} + 0.025\left(\frac{L}{B} + 12\mu_s - 4.6\right)$$

Note: *when* $D_f = 0$*, the value of* $I_D = 1$ *in all cases.*

C_N = Number of contributing corners = **4** *for center*, **2** *for edges*, and **1** *for corners*.

Table (5.8): Typical values of E_s for selected soils
(filed values depend on stress history, water content, density, etc.).

Type of Soil	E_s *(MPa)*
Clay	
Very soft	*2-15*
Soft	*5-25*
Medium	*15-50*
Hard	*50-100*
Sandy	*25-250*
Glacial till	
Loose	*10-153*
Dense	*144-720*
Very Dense	*478-1440*
Loess	*14-57*
Sand	
Silty	*7-21*
Loose	*10-24*
Dense	*48-81*
Sand and gravel	
Loose	*48-144*
Dense	*96-192*
Shale	*144-14400*
Silt	*2-20*

Table (5.7): Typical values of μ_s.

Type of Soil	μ_s
Clay, saturated	*0.40 – 0.50*
Clay, unsaturated	*0.10 – 0.30*
Sandy clay	*0.20 – 0.30*
Silt	*0.30 – 0.35*
Sand (dense)	*0.20 – 0.40*
Coarse (void ratio = 0.4 - 0.7)	*0.15*
Fine-grained (void ratio = 0.4 - 0.7)	*0.25*
Rock	*0.10 – 0.40*
Loess	*0.10 – 0.30*
Concrete	*0.15*

Table (5.9a): Values of I_1 to compute Steinbrenner's influence factor

$$I_s = I_1 + \frac{1-2\mu_s}{1-\mu_s} I_2 \ .$$

H / B′	L/B										
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
0.2	0.009	0.008	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007
0.4	0.033	0.032	0.031	0.030	0.029	0.028	0.028	0.027	0.027	0.027	0.027
0.6	0.066	0.064	0.063	0.061	0.060	0.059	0.058	0.057	0.056	0.056	0.055
0.8	0.104	0.102	0.100	0.098	0.096	0.095	0.093	0.092	0.091	0.090	0.089
1.0	0.142	0.140	0.138	0.136	0.134	0.132	0.130	0.129	0.127	0.126	0.125
1.5	0.224	0.224	0.224	0.223	0.222	0.220	0.219	0.217	0.216	0.214	0.213
2	0.285	0.288	0.290	0.292	0.292	0.292	0.292	0.292	0.291	0.290	0.289
3	0.363	0.372	0.378	0.384	0.389	0.393	0.396	0.398	0.400	0.401	0.402
4	0.408	0.421	0.431	0.440	0.448	0.455	0.460	0.465	0.469	0.473	0.476
5	0.437	0.452	0.465	0.477	0.487	0.496	0.503	0.510	0.516	0.522	0.526
6	0.457	0.473	0.488	0.501	0.513	0.524	0.533	0.542	0.549	0.556	0.562
7	0.471	0.489	0.506	0.520	0.533	0.545	0.556	0.566	0.575	0.583	0.590
8	0.482	0.502	0.519	0.534	0.549	0.561	0.573	0.584	0.594	0.602	0.611
9	0.491	0.511	0.529	0.545	0.560	0.574	0.587	0.598	0.609	0.618	0.627
10	0.498	0.519	0.537	0.554	0.570	0.584	0.597	0.610	0.621	0.631	0.641
20	0.529	0.553	0.575	0.595	0.614	0.631	0.647	0.662	0.677	0.690	0.702
500	0.560	0.586	0.612	0.635	0.656	0.677	0.696	0.714	0.731	0.748	0.763

H / B′	L/B										
	2.5	4	5	6	7	8	9	10	25	50	100
0.2	0.007	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
0.4	0.026	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024
0.6	0.053	0.051	0.050	0.050	0.050	0.049	0.049	0.049	0.049	0.049	0.049
0.8	0.086	0.082	0.081	0.080	0.080	0.080	0.093	0.092	0.091	0.090	0.089
1.0	0.121	0.115	0.113	0.112	0.112	0.112	0.111	0.111	0.110	0.110	0.110
1.5	0.207	0.197	0.194	0.192	0.191	0.190	0.190	0.189	0.188	0.188	0.188
2	0.284	0.271	0.267	0.264	0.262	0.261	0.260	0.259	0.257	0.256	0.256
3	0.402	0.392	0.386	0.382	0.378	0.376	0.374	0.373	0.378	0.367	0.367
4	0.484	0.484	0.479	0.474	0.470	0.440	0.464	0.462	0.453	0.451	0.451
5	0.543	0.554	0.552	0.548	0.543	0.540	0.536	0.534	0.522	0.522	0.519
6	0.585	0.609	0.610	0.608	0.604	0.601	0.598	0.595	0.579	0.576	0.575
7	0.618	0.653	0.658	0.658	0.656	0.653	0.650	0.647	0.628	0.624	0.623
8	0.643	0.688	0.697	0.700	0.700	0.698	0.695	0.692	0.672	0.666	0.665
9	0.663	0.716	0.730	0.736	0.737	0.736	0.735	0.732	0.710	0.704	0.702
10	0.679	0.740	0.758	0.766	0.770	0.770	0.770	0.768	0.745	0.738	0.735
20	0.756	0.856	0.896	0.925	0.945	0.959	0.969	0.977	0.982	0.965	0.957
500	0.832	0.977	1.046	1.102	1.150	1.191	1.227	1.259	1.532	1.721	1.879

B′ = **B/2** *for center of foundation, and* = **B** *for corners of foundation,*
H = *depth of hard stratum (rock) under the footing.*

Table (5.9b): Values of I_2 to compute Steinbrenner's influence factor

$$I_s = I_1 + \frac{1-2\mu_s}{1-\mu_s} I_2 .$$

H/B'	L/B										
	1.0	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2**
0.2	0.041	0.042	0.042	0.042	0.042	0.042	0.043	0.043	0.043	0.043	0.043
0.4	0.066	0.068	0.069	0.070	0.070	0.071	0.071	0.072	0.072	0.073	0.073
0.6	0.079	0.081	0.083	0.085	0.087	0.088	0.089	0.090	0.091	0.091	0.092
0.8	0.083	0.087	0.090	0.093	0.095	0.097	0.098	0.100	0.101	0.102	0.103
1.0	0.083	0.088	0.091	0.095	0.098	0.100	0.102	0.104	0.106	0.108	0.109
1.5	0.075	0.080	0.084	0.089	0.093	0.096	0.099	0.102	0.105	0.108	0.110
2	0.064	0.069	0.074	0.078	0.083	0.086	0.090	0.094	0.097	0.100	0.102
3	0.048	0.052	0.056	0.060	0.064	0.068	0.071	0.075	0.078	0.081	0.084
4	0.037	0.041	0.044	0.048	0.051	0.054	0.057	0.060	0.063	0.066	0.069
5	0.031	0.034	0.036	0.039	0.042	0.045	0.048	0.050	0.053	0.055	0.058
6	0.026	0.028	0.031	0.033	0.036	0.038	0.040	0.043	0.045	0.047	0.050
7	0.022	0.024	0.027	0.029	0.031	0.033	0.035	0.037	0.039	0.041	0.043
8	0.020	0.022	0.023	0.025	0.027	0.029	0.031	0.033	0.035	0.036	0.038
9	0.017	0.019	0.021	0.023	0.024	0.026	0.028	0.029	0.031	0.033	0.034
10	0.016	0.017	0.019	0.020	0.022	0.023	0.025	0.027	0.028	0.030	0.031
20	0.008	0.099	0.010	0.010	0.011	0.012	0.013	0.013	0.014	0.015	0.016
500	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001

H/B'	L/B										
	2.5	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**25**	**50**	**100**
0.2	0.043	0.044	0.044	0.044	0.044	0.044	0.044	0.044	0.044	0.044	0.044
0.4	0.074	0.075	0.075	0.075	0.076	0.076	0.076	0.076	0.076	0.076	0.076
0.6	0.094	0.097	0.097	0.098	0.098	0.098	0.098	0.098	0.098	0.098	0.098
0.8	0.107	0.111	0.112	0.113	0.113	0.113	0.113	0.114	0.114	0.114	0.114
1.0	0.114	0.120	0.122	0.123	0.123	0.124	0.124	0.124	0.125	0.125	0.125
1.5	0.118	0.130	0.134	0.136	0.137	0.138	0.138	0.139	0.140	0.140	0.140
2	0.114	0.131	0.136	0.139	0.141	0.143	0.144	0.145	0.147	0.147	0.148
3	0.097	0.122	0.131	0.137	0.141	0.144	0.145	0.147	0.152	0.153	0.154
4	0.082	0.110	0.121	0.129	0.135	0.139	0.142	0.145	0.154	0.155	0.156
5	0.070	0.098	0.111	0.120	0.128	0.133	0.137	0.140	0.154	0.156	0.157
6	0.060	0.087	0.101	0.111	0.120	0.126	0.131	0.135	0.153	0.157	0.157
7	0.053	0.078	0.092	0.103	0.112	0.119	0.125	0.129	0.152	0.157	0.158
8	0.047	0.071	0.084	0.095	0.104	0.112	0.118	0.124	0.151	0.156	0.158
9	0.042	0.064	0.077	0.088	0.097	0.105	0.112	0.118	0.149	0.156	0.158
10	0.038	0.059	0.071	0.082	0.091	0.099	0.106	0.112	0.147	0.156	0.158
20	0.020	0.031	0.039	0.046	0.053	0.059	0.065	0.071	0.124	0.148	0.156
500	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.008	0.016	0.031

B' = **B/2** *for center of foundation, and* = **B** *for corners of foundation,*
H = *depth of hard stratum (rock) under the footing.*

5.5.2 Schmertmann's Method (1978): Use of Strain Influence Factor

This method is based on the Dutch cone penetration resistance q_c using the strain influence factor diagram. It is proposed for two cases, square foundation (L/B = 1) where axisymmetric stress and strain conditions occur and strip foundation (L/B = 10) where plane strain conditions exist.

For square foundation: $$S_i = \frac{C_1 C_2}{2.5} \Delta p \sum_0^{2B} \frac{I_z \Delta z}{q_c} \quad \textbf{......(5.6a)}$$

For strip foundation: $$S_i = \frac{C_1 C_2}{3.5} \Delta p \sum_0^{4B} \frac{I_z \Delta z}{q_c} \quad \textbf{......(5.6b)}$$

where, P = gross applied pressure,

P'_o = effective stress at the foundation level,

ΔP = net applied pressure = $P - P'_o$ (in kN/m^2),

q_c = cone end resistance, kN/m^2, for each soil layer,

Δz = thickness for each soil layer, (in meters),

C_1 = correction for depth of foundation $= 1 - 0.5\dfrac{P'_o}{\Delta p} \geq 0.5$

C_2 = correction for creep or time related settlement $= 1 + 0.2\log_{10}\dfrac{t}{0.1}$

t = time in (years) after construction,

I_z = average strain influence factor for each soil layer obtained as the value at the mid−point of each soil layer from a diagram drawn alongside the q_c − depth graph with a depth of 2B for square foundation and 4B for strip foundation as shown in **Fig.(5.3)**, and

$I_{z\,max} = 0.5 + 0.1\sqrt{\dfrac{\Delta p}{\sigma v'}}$ is the maximum value of I_z, where σ'_v = vertical effective stress at a depth of B/2 for a square foundation and B for strip foundation.

Notes:

- *Values of Δz, average q_c and average I_z for each soil layer are required for the summation term.*
- *Settlements for shapes intermediate between square and strip can be obtained by interpolation.*

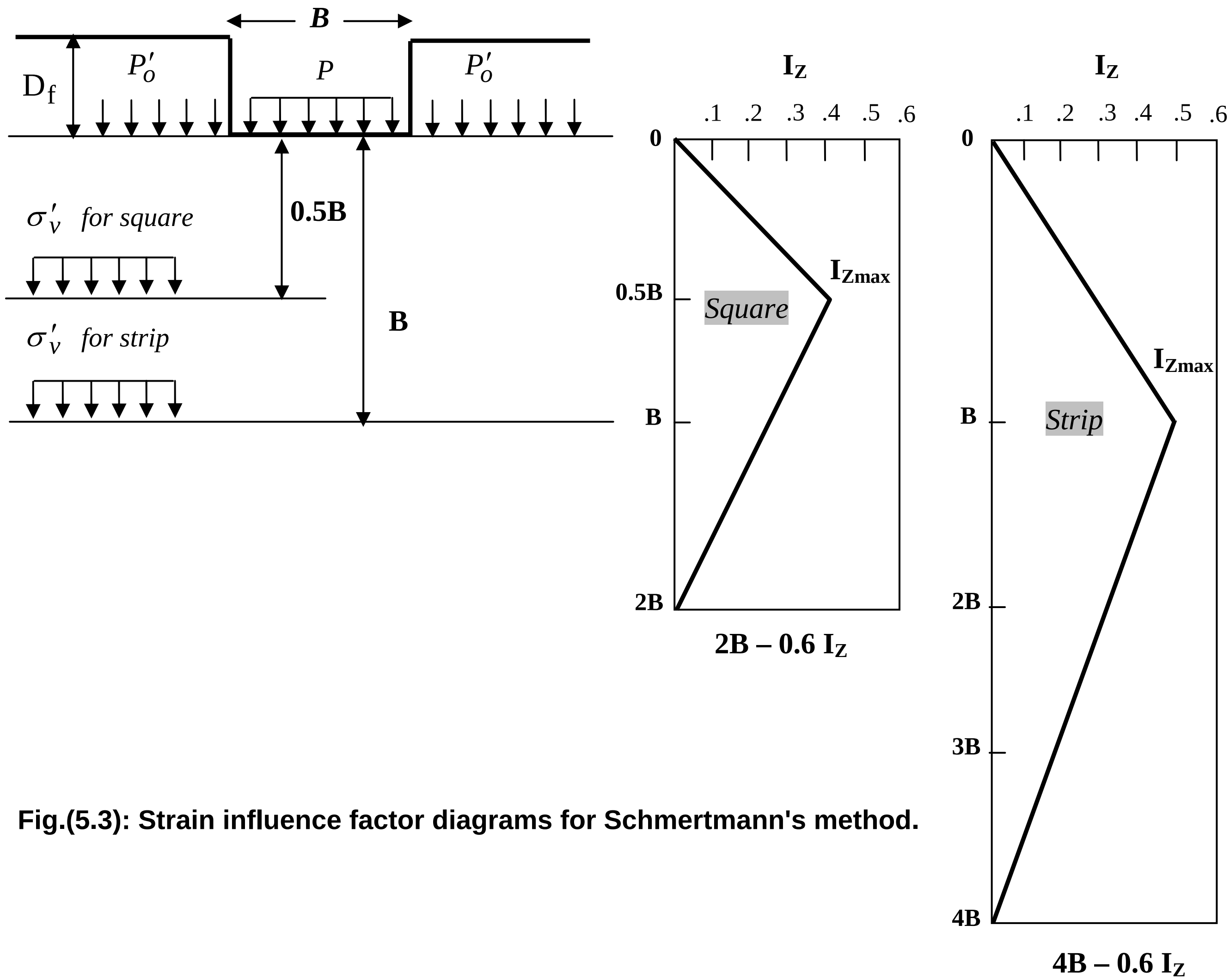


Fig.(5.3): Strain influence factor diagrams for Schmertmann's method.

5.5.3 Bjerrum's Method for Average Settlement of Layered Clay Soil

$$S_{i(\text{average})\text{flexible}} = \mu_o . \mu_1 \frac{q.B}{E_u} \quad \text{..(5.7)}$$

where, μ_o and μ_1 are factors for depth of embedment and thickness of soil layer beneath the foundation, respectively; obtained from **Fig.(5.4)**. Remember that the principle of layering could be applied with this method such that the overlapping is equal to the number of layers − 1.

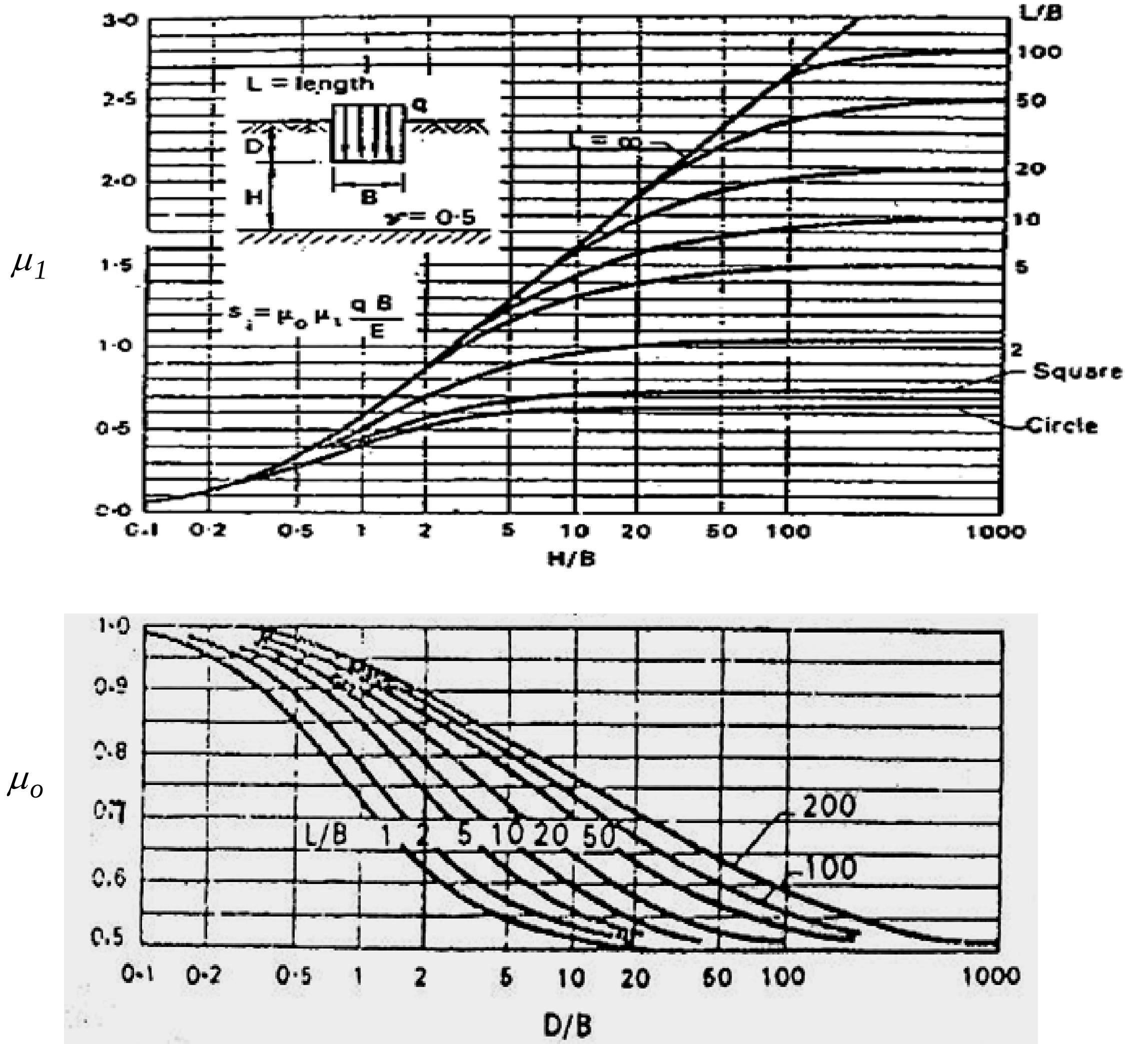
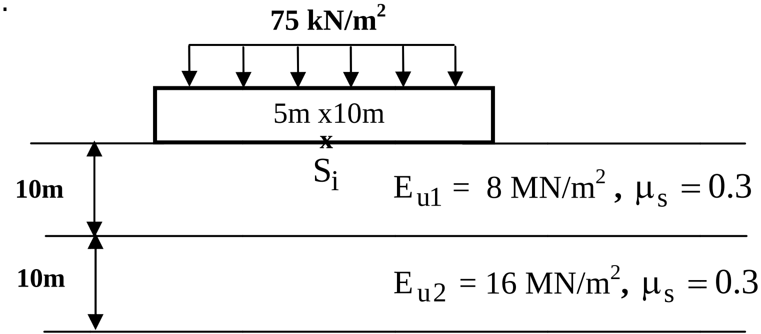


Fig.(5.4): Coefficients of vertical displacement for foundations on saturated clays
(after Janbu et al., 1956).

Problem (5.1): A (5m x 10m) rectangular flexible foundation is placed on two layers of clay, both 10m thick as shown in the figure below. The modulus of elasticity of the upper layer is 8 MN/m^2 and that of the lower layer is 16 MN/m^2. Determine the immediate settlement at the center of the foundation using:

(1) Elastic Theory Method.
(2) Bjerrum Method.



Solution:

(1) (Elastic Theory Method):

$$E_{avg.} = \frac{8(10)+16(10)}{20} = 12.MN/m^2 = 12000.kN/m^2$$

$$S_{i(flexible)} = q_o.B'\frac{1-\mu_s^2}{E_s} I_s.I_D.C_N \text{ ....(5.3)}$$

For H/B' =20/2.5 = 8, L/B = 10/5 = 2**:** $I_1 = 0.611$ and $I_2 = 0.038$; *from* ***Table (4.9)***

$$I_s = I_1 + \frac{1-2\mu_s}{1-\mu_s} I_2 = 0.611 + \frac{1-2(0.3)}{1-0.3} 0.038 = 0.633$$

I_D = 1 *(for* $D_f = 0$)**;** and C_N = 4 (*for center)*.

$$S_{i(flexible)_{(surface,center)}} = (75)(2.5)\frac{1-(0.3)^2}{12000}(0.633)(1)(4) = \underline{\mathbf{36\ mm.}}$$

(2) (Bjerrum Method):

- **Settlement of 1st. layer** (average settlement)**:**

From **Fig.(5.4):** for D_f /B = 0 and L/B = 2; μ_o = 1.00

For H/B = 10/5 = 2 and L/B = 2; μ_1 = 0.70

$$S_{i(average)flexible} = \mu_o.\mu_1 \frac{q.B}{E_u} \text{ ....(5.7)}$$

$$S_{1(average)flexible} = (1.00)(0.70)\frac{(75)(5)(1000)}{(8x1000)} = 32.81 \text{ mm}$$

- **Settlement of 2nd. layer** (average settlement)**:**

From **Fig.(5.4):** for D_f /B = 0 and L/B = 2; μ_o = 1.00

For H/B = 20/5 = 4 and L/B = 2; μ_1 = 0.85

$$S_{2(average)flexible} = (1.00)(0.85)\frac{(75)(5)(1000)}{(16x1000)} = 19.92 \text{ mm}$$

- **The interaction between the 1st. and 2nd. Layers:**

$$S_{3(average)flexible} = (1.00)(0.70)\frac{(75)(5)(1000)}{(16x1000)} = 16.41 \text{ mm}$$

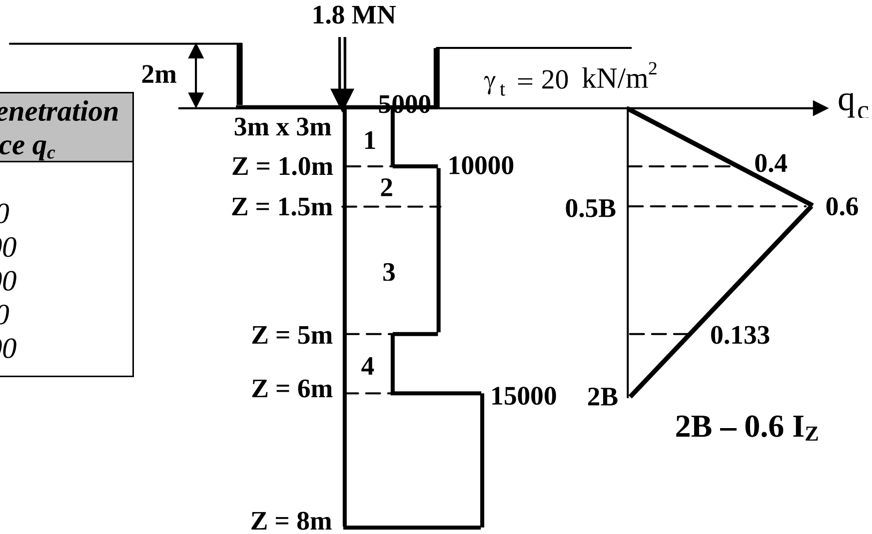
The immediate settlement at foundation center = $S_1 + S_2 - S_3$

= 32.81 + 19.92 – 16.41 = **36.32 mm**

Problem (5.2): (Schmertmann's method–settlement on sand)

A (3m x 3m) square footing rested at a depth of (2m) below the ground surface. Estimate the immediate settlement of the footing under the load and soil conditions shown in the figure below after (0.1 year) from construction.

Depth from base (m)	*Static cone penetration resistance q_c*
0-1.0	*5000*
1.0-1.5	*10000*
1.5-5.0	*10000*
5.0-6.0	*5000*
6.0-8.0	*15000*



Solution:

For square foundation:
$$S_i = \frac{C_1 C_2}{2.5}\Delta p \sum_0^{2B} \frac{I_z \Delta z}{q_c} \quad \text{..........................}\mathbf{(5.6a)}$$

- C_1 = correction for depth of foundation $= 1 - 0.5\dfrac{P'_o}{\Delta p} \ge 0.5$

  P'_o = effective stress at the foundation level = $D_f.\gamma$ = 2(20) = 40 kN/m²

  ΔP = net increase in stress at footing level = $P - P'_o = \dfrac{1.8x10^3}{3x3} - 40 = 160$ kN/m²

  $C_1 = 1 - 0.5\dfrac{40}{160} = 0.875 > 0.5$ (O.K.)

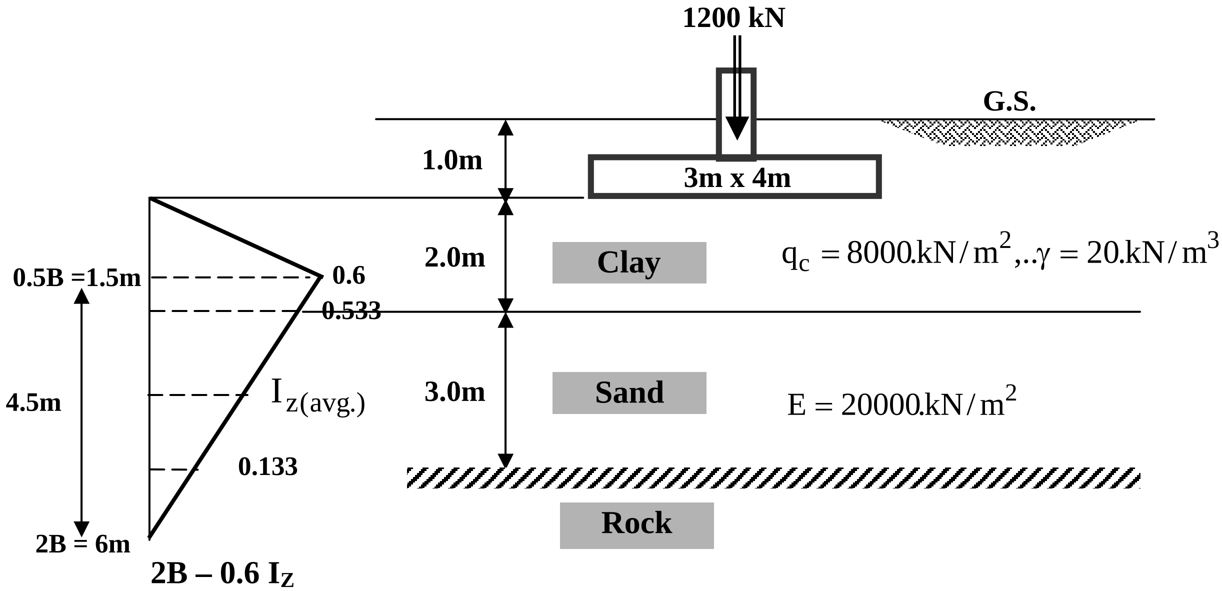
- C_2 = Time correction factor $= 1 + 0.2\log_{10}\dfrac{t}{0.1} = 1 + 0.2\log_{10}\dfrac{0.1}{0.1} = 1.0$

No.	Δ_Z **(m)**	q_c	I_Z **(average)**	$\dfrac{\Delta_Z . I_Z}{q_c}$
1	1.0	5000	(0 + 0.4)/2= 0.2	0.000040
2	0.5	10000	0.5	0.000025
3	3.5	10000	0.366	0.000128
4	1.0	5000	0.066	0.0000132
				$\sum 20.62x10^{-5}$

$$S_i = \frac{(0.875)(1.0)}{2.5}(160)(20.62x10^{-5}) = 0.01155 \text{ m} = \mathbf{11.55\ mm}$$

Problem (5.3): (Total immediate settlement)

Determine the total immediate settlement of the rectangular footing shown in figure below after 2 months.



Solution:

Since the soil profile is made up of two different soils, then the total immediate settlement will be:

$$S_{i(Total)} = S_{i(clay)} + S_{i(sand)}$$

- **Immediate Settlement of clay by Bjerrum's method:**

$$S_{i(average)flexible} = \mu_o . \mu_1 \frac{q.B}{E_u} \quad \text{..(5.7)}$$

From **Fig.(5.4):** for D_f /B = 1/3 = 0.33 and L/B = 4/3 = 1.33; μ_o = 0.93

for H/B = 2/3 = 0.66 and L/B = 1.33; μ_1 = 0.38

$$S_{1(average)flexible} = (0.93)(0.38)\frac{(1200/3x4)(3)(1000)}{(2x8x1000)} = \underline{\mathbf{6.6\ mm}}$$

- **Immediate Settlement of sand by Schmertmann's method:**

For square foundation:

$$S_i = \frac{C_1 C_2}{2.5} \Delta p \sum_0^{2B} \frac{I_z \Delta z}{q_c} \quad \text{..(5.6a)}$$

$$C_1 = 1 - 0.5 \frac{P'_o}{\Delta p} \geq 0.5$$

At foundation level:

$$P'_o = D_f . \gamma = 1(20) = 20 \text{ kN/m}^2, \quad \Delta P = P/A - P'_o = \frac{1200}{3x4} - 20 = 80 \text{ kN/m}^2.$$

On sand surface:

$$P'_o = D_f.\gamma = 3(20) = 60 \text{ kN/m}^2, \qquad \Delta P = \frac{(80)(3)(4)}{(3+2)(4+2)} = 32 \text{ kN/m}^2 \ \textit{(2:1 method)}$$

$$C_1 = 1 - 0.5\frac{60}{32} = 0.06 < 0.5 \quad \therefore \quad \text{Use } C_1 = 0.5$$

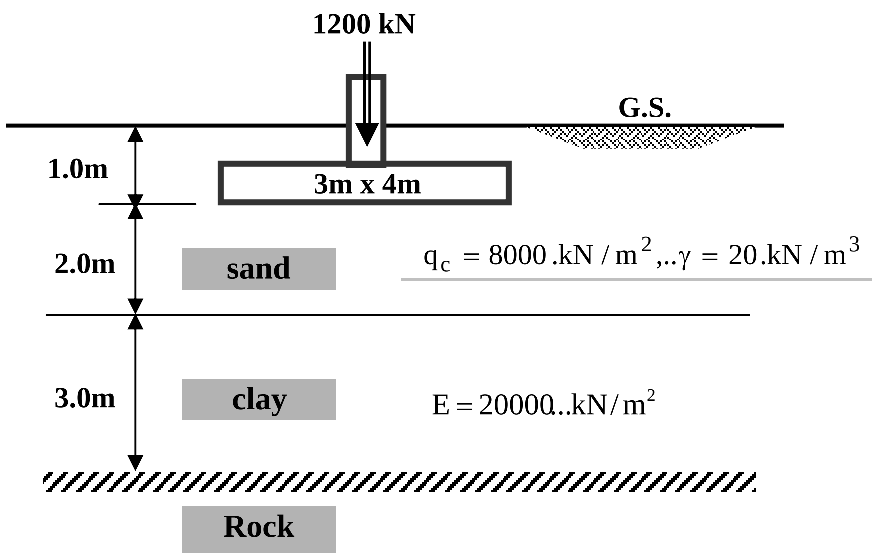
$$C_2 = 1 + 0.2\log_{10}\frac{t}{0.1} = 1 + 0.2\log_{10}\frac{2/12}{0.1} = 1.04$$

$$I_{z\,(avg.)} = \frac{0.533 + 0.133}{2} = 0.333\,, \qquad \frac{I_z.\Delta z}{E} = \frac{(0.333)(3)}{20000} = 4.9.x.10^{-5}$$

$$S_{i\,(sand)} = (0.5)(1.04)(32)(4.9.x.10^{-5}) = \underline{\mathbf{0.815\ mm}}$$

$$\therefore \quad S_{i\,(Total)} = 6.6 + 0.815 = \underline{\mathbf{7.415\ mm}}$$

Home work: ***Redo problem (5.3) but with sand instead of clay as shown in the figure below.***
*(Ans.: $S_{i\,(Total)}$ = **5.75 mm**).*



5.6 PRIMARY CONSOLIDATION SETTLEMENT

5.6.1 Compression Index C_c Method:

This method is adopted for normally and lightly overconsolidated clays. The compression index C_c is the gradient of $e - log\ P$ plot for normally consolidated clay. While for overconsolidated clay, C_c is also the slope of the $e - log\ P$ but beyond the preconsolidation pressure P'_c. C_c values obtained from oedometer tests are likely to be underestimated due to

sampling disturbance. Therefore, some correlations which relate C_c with soil composition parameter have been published and two of them are as follows:

$$C_c = 0.009(LL - 10) \text{ .... (\textbf{Terzaghi and Peck}, 1948)}$$

$$C_c \approx 0.5\rho_s \frac{PI}{100} \text{ .... (\textbf{Wroth}, 1979)}$$

where, LL = liquid limit, PI = plasticity index, and ρ_s = particle density.

Method (A):

1. Calculate the effective pressure σ'_o at center of the clay layer before the application of load.

2. Calculate the ***weighted average pressure increase at mid of clay layer using Simpson's rule***:

$$\Delta\sigma_{avg.} = \frac{1}{6}(\Delta\sigma_t + 4\Delta\sigma_m + \Delta\sigma_b)$$

where, $\Delta\sigma_t$, $\Delta\sigma_m$, and $\Delta\sigma_b$ are respectively the pressure increase due to applied load at the top, middle and bottom of clay layer.

3. Using σ'_o and $\Delta\sigma_{avg.}$ calculated above, obtain Δe from equations below, whichever is applicable.

(i) If $\sigma'_p < \sigma'_o$, the soil is **under consolidated**:

$$\Delta e = C_c \log_{10} \frac{\sigma'_o + \Delta\sigma_{avg.}}{\sigma'_p} \quad \text{......................................}\textbf{(5.8a)}$$

(ii) If $\sigma'_p = \sigma'_o$ (OCR = 1), the soil is **normally consolidated**:

$$\Delta e = C_c \log_{10} \frac{\sigma'_o + \Delta\sigma_{avg.}}{\sigma'_o} \quad \text{......................................}\textbf{(5.8b)}$$

(iii) If $\sigma'_p > \sigma'_o$ (OCR > 1), the soil is **overconsolidated,** and

(a) If $\sigma'_p \geq \sigma'_o + \Delta\sigma_{avg.}$ then;

$$\Delta e = C_s \log_{10} \frac{\sigma'_o + \Delta\sigma_{avg.}}{\sigma'_o} \quad \text{......................................}\textbf{(5.8c)}$$

(b) If $\sigma'_p < \sigma'_o + \Delta\sigma_{avg.}$ then;

$$\Delta e = C_c \log_{10} \frac{\sigma'_o + \Delta\sigma_{avg.}}{\sigma'_p} + C_s \log_{10} \frac{\sigma'_p}{\sigma'_o} \quad \text{.................}\textbf{(5.8d)}$$

5. Calculate the consolidation settlement by:

$$S_c = \frac{\Delta e}{1 + e_o} H_t \quad \text{..}\textbf{(5.8e)}$$

where, $e_o = \omega_o . G_s$

Method (B):

1. For thick clay layer, better results in settlement calculation can be obtained by dividing a given clay layer into (n) sub-layers.

2. Calculate the effective stress $\sigma'_{o(i)}$ at the middle of each clay sub-layer.

3. Calculate the increase of stress at the middle of each sub-layer $\Delta\sigma_{(i)}$ due to the applied load.

5. Calculate $\Delta e_{(i)}$ for each sub-layer from **Eqs.(5.8a to 5.8e)** mentioned before in method (A) −step 3, whichever is applicable.

5. Calculate the total consolidation settlement of the entire clay layer from:

$$S_c = \sum_{i=1}^{n} \Delta S_c = \sum_{i=1}^{n} \frac{\Delta e_i}{1+e_o} \Delta H_i \quad \text{where} \quad e_o = \omega_o . G_s \quad \textbf{.....................(5.9)}$$

Layer	Values at mid-point of each sub-layer						
	$\sigma'_{o(i)}$	$\Delta\sigma_{(i)}$	$\Delta e_{(i)}$	ω_o	e_o	ΔH_i	$\frac{\Delta e_{(i)}}{1+e_o}\Delta H_i$
1							
2							
3							

$S_c = \sum$ ▬

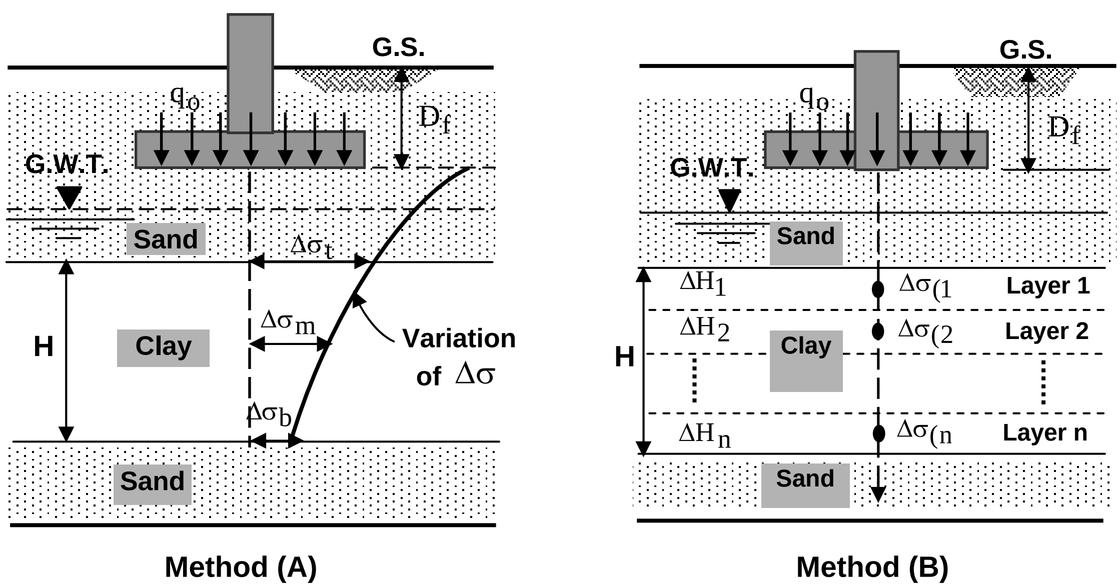


Fig.(5.5): Calculation of consolidation settlement Methods.

4.6.2 Oedometer or m_v Method:

From oedometer test, the values of volume change for each pressure increment is obtained as:

$$m_v = \frac{a_v}{1+e_o} \quad \text{but} \quad a_v = \frac{\Delta e}{\Delta P} \quad \text{and} \quad \Delta H = \frac{\Delta e}{1+e_o} H_t \quad \text{therefore;} \quad m_v = \frac{1}{\Delta p}\frac{\Delta H}{H_t}$$

$$\Delta H = S_c = m_v . H_t . \Delta p \quad \text{...} \quad \textbf{(5.10)}$$

where,

a_v = coefficient of compressibility of soil sample.

e_o = initial void ratio of soil sample.

Δe = the change in void ratio corresponding to a pressure change Δp.

$\Delta p = \Delta\sigma$ = change in stress.

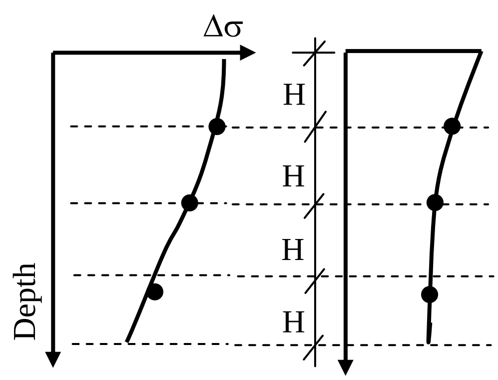
H_t = total thickness of the clay soil layer.

ΔH = change in thickness, and

m_v = coefficient of volume compressibility of soil sample determined during an oedometer test for each pressure increment applied above the vertical effective stress or overburden pressure P'_o at the depth from which the sample was taken. *If the applied stress or m_v values vary with depth, then the soil deposit must be divided into layers and the change in thickness determined for each layer*. Typical values of m_v for different clay types are given in **Table (5.10).**

Table (5.10): Typical values of m_v.

Type of clay	m_v **m²/MN**
Very stiff heavily	*< 0.05*
Overconsolidated clay	*0.05 - 0.1*
Firm overconsolidated clay, Laminated clay, weathered clay	*0.1 - 0.3*
Soft normally consolidated clay	*0.3 – 1.0*
Soft organic clay, sensitive clay	*0.5 – 2.0*
Peat	*> 1.5*



$\Delta\sigma$	m_v	ΔH
-----	-----	-----
-----	-----	-----
-----	-----	-----
-----	-----	-----
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Oedometer or m_v method.

5.7 SKEMPTON - BJERRUM MODIFICATION FOR 3-DIMENTIONAL CONSOLIDATION

In one-dimensional consolidation tests, there is no lateral yield of the soil specimen and the ratio of minor to major principal effective stresses, K_o, remains constant. In that case, the increase of pore water pressure due to an increase of vertical stress is equal in magnitude, (i.e., $\Delta u = \Delta\sigma$) where Δu is the increase in pore water pressure and $\Delta\sigma$ is the increase of vertical stress. While for actual simulation of field condition, in 3-dimensions, any point in a clay layer due to a given load suffers from lateral yield and therefore, K_o does not remain constant.

$$\therefore \quad S_c = \rho \; S_{c\,(Oed)} \quad \text{.......} \quad \textbf{(5.11)}$$

where, ρ = correction factor depends on pore-pressure parameter (A); obtained from **Fig.(4.6)**.

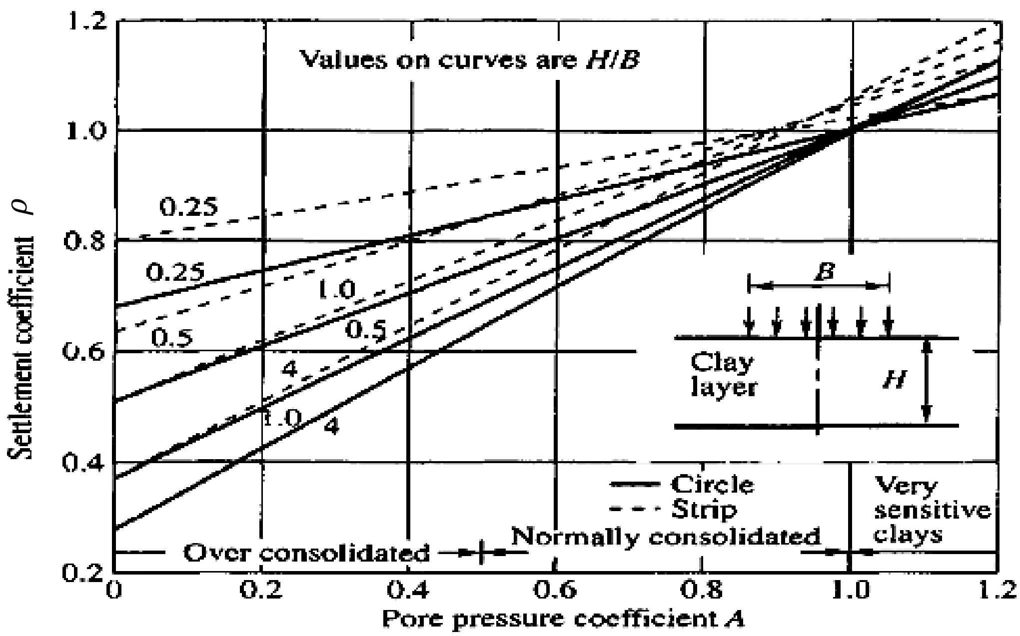
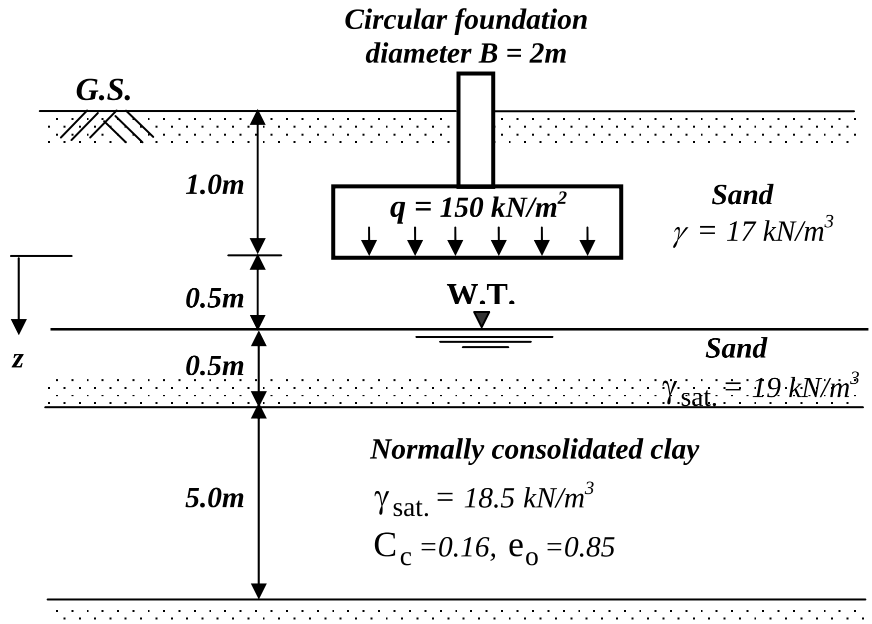


Fig.(5.6): Settlement correction factor versus pore-pressure coefficient for circular and strip footings *(after Skempton and Bjerrum, 1957).*

Problem (5.4): *(consolidation settlement$-C_c$ method)*

A circular foundation 2m in diameter is shown in the figure below. A normally consolidated clay layer 5m thick is located below the foundation. Determine the consolidation settlement of the clay.



Solution:

(1) As one layer of clay of 5m thick:

At the center of clay: $\sigma'_o = 1.5(17) + 0.5(19\text{-}9.81) + 2.5(18.5\text{-}9.81) = 51.82 \text{ kN/m}^2$

For circular loaded area, the increase of stress below the center is given by:

$$\Delta\sigma = q\left\{1 - \frac{1}{[(b/z)^2 + 1]^{3/2}}\right\}$$ where: b = the radius of the circular foundation,

At mid-depth of the clay layer: z = 3.5m; $\Delta\sigma = 150\left\{1 - \frac{1}{[(1/3.5)^2 + 1]^{3/2}}\right\} = 16.66 \text{ kN/m}^2$

$$\Delta e = C_c \log_{10}\frac{\sigma'_o + \Delta\sigma}{\sigma'_o} = 0.16.\log_{10}\frac{51.82 + 16.66}{51.82} = 0.0194$$

$$S_c = \frac{\Delta e}{1 + e_o} H_t = \frac{0.0194}{1 + 0.85}(5)(1000) = \underline{\mathbf{52.4\ mm}}$$

(2) Divide the clay layer into (5) sub-layers each of 1m thick:

- ***Calculation of effective stress at the middle of each sub-layer*** $\sigma'_{o(i)}$ **:**

For 1st. Layer: $\sigma'_{o(1)} = 1.5(17) + 0.5(19\text{-}9.81) + 0.5(18.5\text{-}9.81) = 35.44 \text{ kN/m}^2$

For 2nd. Layer: $\sigma'_{o(2)} = 35.44 + 1.0(18.5\text{-}9.81) = 35.44 + 8.69 = 43.13 \text{ kN/m}^2$

For 3rd. Layer: $\sigma'_{o(3)} = 43.13 + 8.69 = 51.81 \text{ kN/m}^2$

For 4th. Layer: $\sigma'_{o(4)} = 51.81 + 8.69 = 60.51 \text{ kN/m}^2$

For 5th. Layer: $\sigma'_{o(5)} = 60.51 + 8.69 = 69.20$ kN/m^2

• ***Calculation of increase of stress below the center of each sub-layer*** $\Delta\sigma_{(i)}$ **:**

For 1st. Layer: $\Delta\sigma_{(1)} = 150\left\{1 - \dfrac{1}{[(1/1.5)^2 + 1]^{3/2}}\right\} = 63.59$ kN/m^2

For 2nd. Layer: $\Delta\sigma_{(2)} = 150\left\{1 - \dfrac{1}{[(1/2.5)^2 + 1]^{3/2}}\right\} = 29.93$ kN/m^2

For 3rd. Layer: $\Delta\sigma_{(3)} = 150\left\{1 - \dfrac{1}{[(1/3.5)^2 + 1]^{3/2}}\right\} = 16.66$ kN/m^2

For 4th. Layer: $\Delta\sigma_{(4)} = 150\left\{1 - \dfrac{1}{[(1/4.5)^2 + 1]^{3/2}}\right\} = 10.46$ kN/m^2

For 5th. Layer: $\Delta\sigma_{(5)} = 150\left\{1 - \dfrac{1}{[(1/5.5)^2 + 1]^{3/2}}\right\} = 7.14$ kN/m^2

Layer no.	ΔH_i **m**	$\sigma'_{o(i)}$ **kN/m^2**	$\Delta\sigma_{(i)}$ **kN/m^2**	$\Delta e^*_{(i)}$	$\dfrac{\Delta e_{(i)}}{1+e_o}\Delta H_i$ **m**
1	1	35.44	63.59	0.0727	0.0393
2	1	43.13	29.93	0.0366	0.0198
3	1	51.82	16.66	0.0194	0.0105
4	1	60.51	10.46	0.0111	0.0060
5	1	69.20	7.14	0.00682	0.0037
					$\sum$ = 0.0793

$\Delta e^*_{(i)} = C_c \log_{10} \dfrac{\sigma'_{o(i)} + \Delta\sigma_{(i)}}{\sigma'_{o(i)}}$; $C_c = 0.16$, $e_o = 0.85$, $S_c = 0.0793$ m = **79.3 mm.**

(3) Weighted average pressure increase (Simpson's rule):

At the center of clay: $\sigma'_o = 1.5(17) + 0.5(19-9.81) + 2.5(18.5-9.81) = 51.82$ kN/m^2

At z = 1.0m from the base of foundation: $\Delta\sigma = 150\left\{1 - \dfrac{1}{[(1/1)^2 + 1]^{3/2}}\right\} = 75$ kN/m^2

At z = 3.5m from the base of foundation: $\Delta\sigma = 150\left\{1 - \frac{1}{[(1/3.5)^2 + 1]^{3/2}}\right\} = 16.66 \text{ kN/m}^2$

At z = 6.0m from the base of foundation: $\Delta\sigma = 150\left\{1 - \frac{1}{[(1/6)^2 + 1]^{3/2}}\right\} = 6.04 \text{ kN/m}^2$

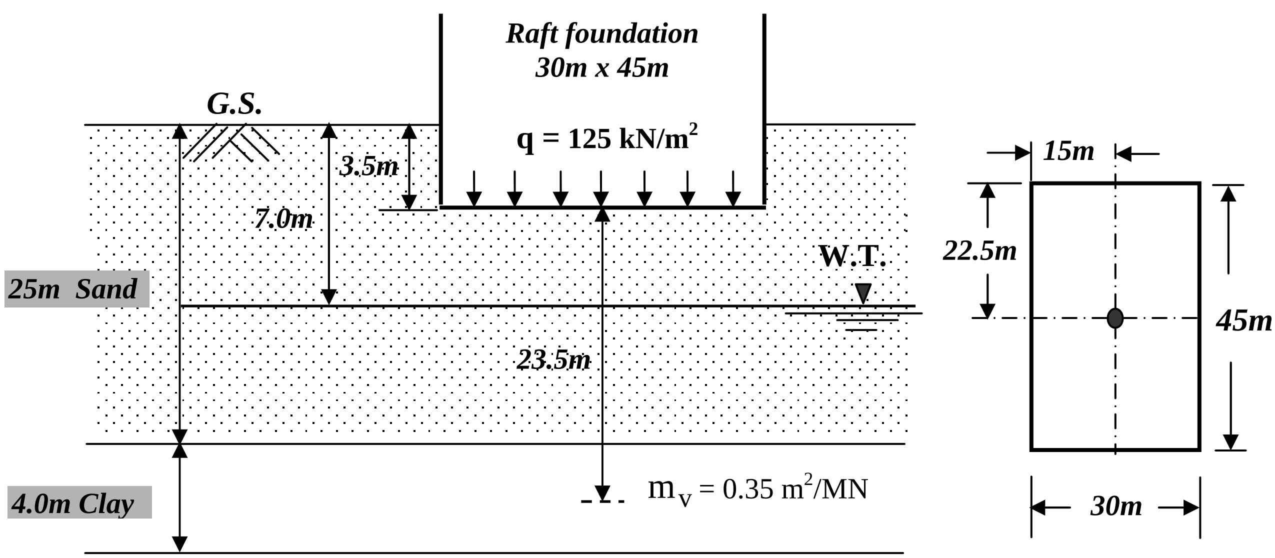
$$\therefore \ \Delta\sigma_{avg.} = \frac{1}{6}(\Delta\sigma_t + 4\Delta\sigma_m + \Delta\sigma_b) = \frac{1}{6}[75 + 4(16.66) + 6.04] = 24.61 \text{ kN/m}^2$$

$$\Delta e = C_c \log_{10}\frac{\sigma'_o + \Delta\sigma}{\sigma'_o} = 0.16.\log_{10}\frac{51.82 + 24.61}{51.82} = 0.027$$

$$S_c = \frac{\Delta e}{1 + e_o} H_t = \frac{0.027}{1 + 0.85}(5)(1000) = \underline{\mathbf{72.9 \ mm}}$$

Problem (5.5): *(consolidation settlement – m_v method)*

A building is supported on a raft of (30m x 45m), the net pressure being 125 kN/m^2 as shown in the figure below. Determine the settlement under the center of the raft due to consolidation of the clay.



Solution:

From Ch.(4) the vertical stress below the corner of flexible rectangular or square loaded area

$$\Delta\sigma_z = I.q_o$$

At mid-depth of the layer , z = 23.5m below the center of the raft:

m/z = 22.5/23.5 = 0.96 and n/z = 15/23.5 = 0.64 therefore; I = 0.140

$\Delta\sigma_z$ = (4)(0.140)(125) = 70 kN/m^2

$$S_c = m_v . H_t . \Delta\sigma \quad \text{..........(5.10)}$$

S_c = (0.35)(70)(4)(1000) = **98 mm.**

5.8 SECONDARY CONSOLIDATION SETTLEMENT

It occurs after the primary consolidation settlement has finished when all pore water pressures have dissipated (see **Fig.(5.7)**). Secondary consolidation can be ignored for hard or overconsolidated soils. But, it is highly increased for organic soil such as peat. This can explained due to the redistribution of forces between particles after large structural rearrangements that occurred during the normal consolidation stage of the soil.

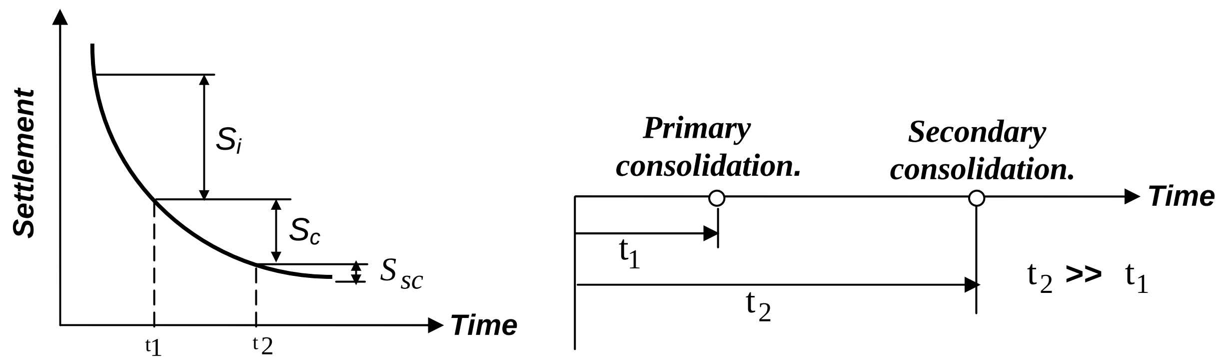


Fig. (5.7): Definition of secondary compression.

$$S_{cs} = C_\alpha . H . \log_{10} \frac{t_2}{t_1} \quad \text{..........(5.12)}$$

where, S_{cs} = secondary consolidation settlement.

C_α = coefficient of secondary consolidation; obtained from table below.

H = thickness of clay layer.

t_1 = time of primary consolidation settlement, and

t_2 = time of secondary consolidation settlement.

To determine t_1: from $T_v = \frac{C_v . t}{H^2}$ take T_v = 1.0 and t = t_1; then $1.0 = \frac{C_v . t_1}{H^2}$ or $t_1 = \frac{H^2}{C_v}$

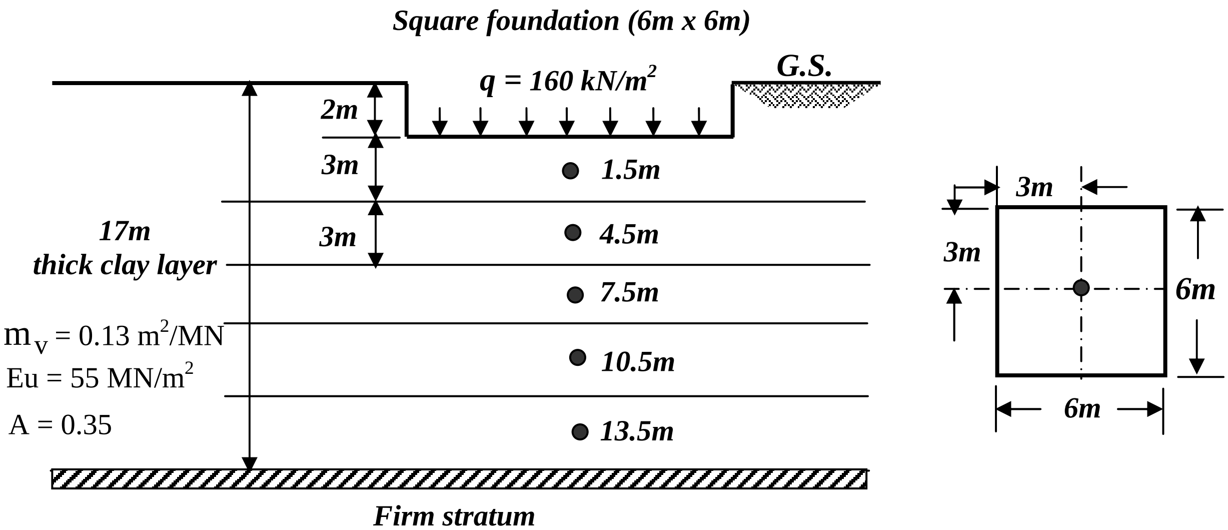
Table (5.7): Values of C_α for some typical soils.

Type of clay	C_α
Normally consolidated clay	*0.005-0.02*
Plastic or organic soil	*≥ 0.03*

Hard clay or overconsolidated clay with O.C.R > 2	*0.001 - 0.0001*

Problem (5.6): *(Total settlement)*

As shown in the figure below, a footing 6m square, carrying a net pressure of 160 kN/m^2 is located at a depth of 2m in a deposit of stiff clay 17m thick; a firm stratum lies immediately below the clay. Form Oedometer tests on specimens of the clay, the value of m_v was found to be 0.13 m^2/MN and from Triaxial tests the value of A was found to be 0.35. The undrained Young's modulus for the clay is estimated to be 55 MN/m^2. Determine the total settlement under the center of the footing.



Solution:

(1) Immediate settlement (Using Bjerrum method):

From **Fig.(5.4):** for H/B = 15/6 = 2.5, L/B = 1 and D_f /B = 2/6 = 0.33

$\mu_o = 0.91$ and $\mu_1 = 0.60$

$$S_{i(average)flexible} = \mu_o . \mu_1 \frac{q.B}{E_u} \quad \text{..(5.7)}$$

$$S_{i(average)flexible} = (0.91)(0.60)\frac{(160)(6)(1000)}{(55x1000)} = 9.5 \text{ mm}$$

(2) Consolidation settlement (m_v - method):

From Ch.(4), the vertical stress below the corner of flexible rectangular or square loaded area

$$\Delta\sigma_z = I.q_o$$

At mid-depth of each 3 m depth as shown in the table below:

Layer no.	z **(m)**	$m/z,\ n/z$	I **From Ch.(4) Fig.(4.15)**	$\Delta\sigma'$ **(kN/m²)**	$S_{c(oed)} = m_v.H_t.\Delta\sigma'$ **(mm)**
1	1.5	2.00	0.233	149	58.1
2	5.5	0.67	0.121	78	30.4
3	7.5	0.40	0.060	38	15.8
4	10.5	0.285	0.033	21	8.2
5	13.5	0.222	0.021	13	5.1
					$\sum$ = 116.6

For 1st. Layer: m/z = 3/1.5 = 2.00 and n/z = 3/1.5 = 2.00 therefore; I = 0.233

$\Delta\sigma'_z$ = (4)(160)(I)..........................(kN/m²)

$$S_{c(oed)} = m_v.H_t.\Delta P \quad \textbf{...(5.10)}$$

$S_{c(oed)}$ = (0.13)($\Delta\sigma'_z$)(3)(1000) .... (mm).

(3) Correction for A pore water pressure:

From **Fig.(5.6):** for H/B = 15/6.77 = 2.2 (equivalent diameter = 6.77 m) and A = 0.35; ρ_{circle}=0.55 then, $S_{c(oed)}$ = (0.55)(116.6) = 64 mm.

$\therefore$ Total settlement = $S_T = S_i + S_c$ = 9.5 + 64 = **73.5 mm**

5.9 DEGREE OR RATE OF SETTLEMENT

It is the ratio of consolidation at time (t) to that of 100% consolidation when the pore water pressure was diminishes. It is calculated as follows:

(1) First, from Oedometer tests, the coefficient of consolidation (C_v) is calculated as:

$$C_v = \frac{k}{m_v.\gamma_w} \quad \textbf{...(5.13)}$$

where, m_v = volume.change.coefficient $= \frac{a_v}{1+e_o}$, $a_v = \frac{\Delta e}{\Delta p} =$ compressibility coefficient and k = permeability of soil.

(2) Second, the time factor (T_v) is calculated from:

$$T_v = \frac{C_v.t}{(H_d)^2} \quad \text{...}\textbf{(5.14)}$$

where, H_d (drainage path) = H *(for one-way drainage)* and
= H/2 *(for two-way drainage).*

(3) Third, with (T_v) value obtained from Eq. (5.14), the degree of consolidation $U\%$ at any time (t) is calculated from **Fig.(5.8)** depending on the distribution of the excess pore water pressure; or one of the following equations:

$$T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2 \quad \text{for } U \le 60\% \text{ ....}\textbf{(5.15a)}$$

$$T_v = 1.781 - 0.933.\log_{10}(100 - U\%) \quad \text{for } U > 60\% \text{ ....}\textbf{(5.15b)}$$

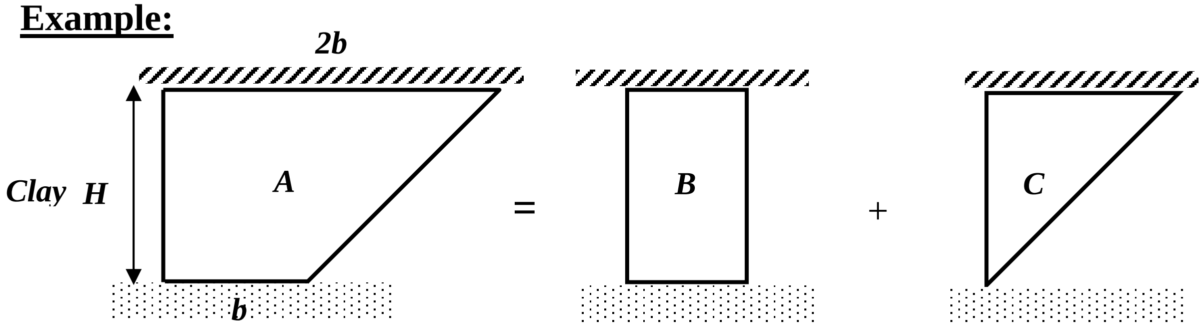
(4) From the degree of consolidation $U\%$ at any time (t), the settlement at any time is calculated from the following relation if the total settlement is known:

$$U_t = \frac{S_t}{S_\infty} = \frac{\text{Settlement.at.any.time(t)}}{\text{Total.settlement}} \quad \text{.....................................}\textbf{(5.16)}$$

where, $S_\infty = S_T = S_i + S_c + S_{sc}$.

Note: $U\%$ for any layer depends on pore water pressure distribution using Figs.(5.20a and 5.20b) to find U_t at any time. But, for other shapes use division to suit with figures above as shown in the following example.

Example:



Curve (1+3) ***Curve (1)*** ***Curve (3)***

$$U_A = \frac{U_B.A_B + U_c.A_C}{\sum A}$$

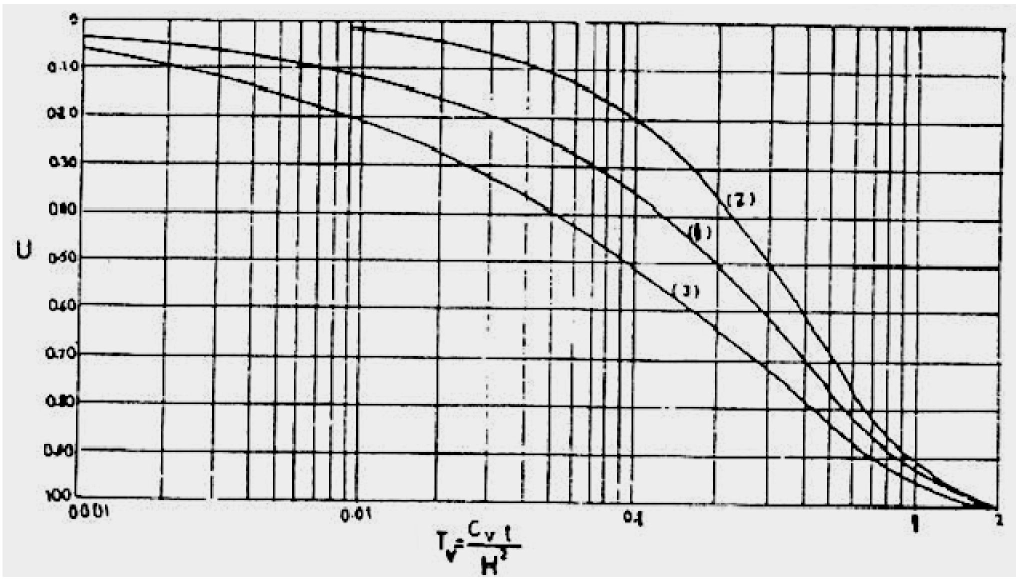
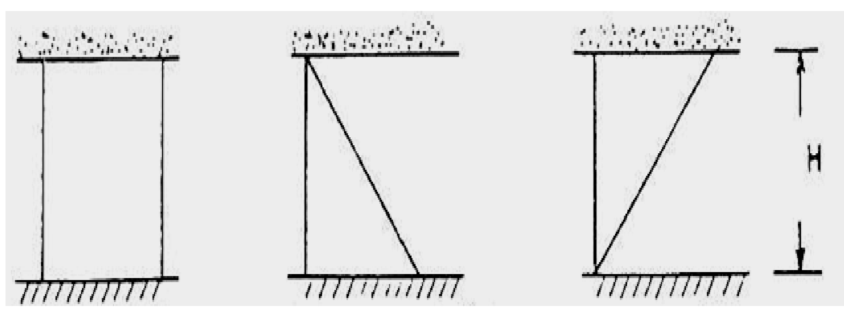


Fig.(5.8): Variation of average degree of consolidation and time factor
(for EPWP conditions given in Figs. a, and b) .

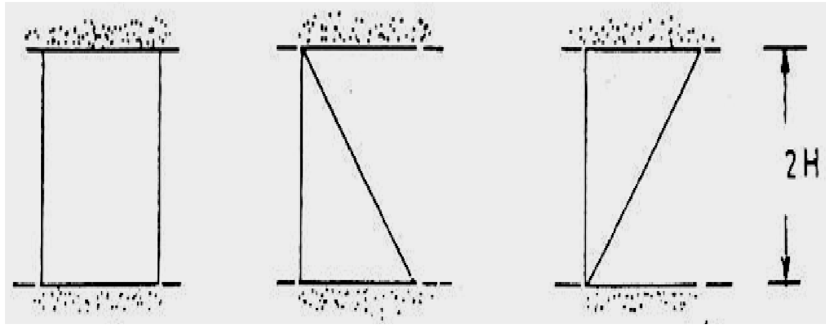
(a) 1-D drainage



Curve (1) *Curve (2)* *Curve (3)*

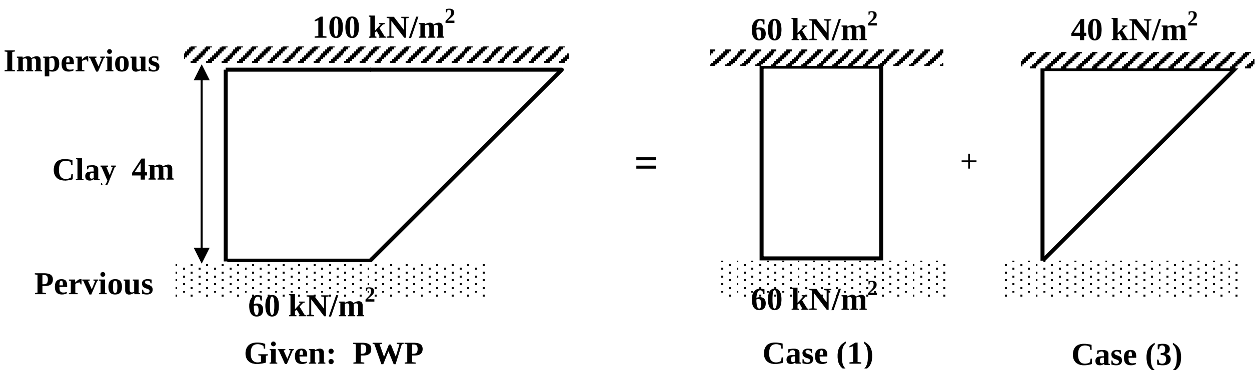
(b) 2-D drainage

Curve (1) *Curve (2)* *Curve (3)*



Problem (5.7): *(degree of consolidation)*

For pore water pressure distribution across a clay soil layer shown below, find the average degree of consolidation after (15) years.



$C_v = 0.4$ **m^2/year**

Solution:

$$T_v = \frac{C_v.t}{(H_d)^2} = \frac{(0.4)(15)}{(4)^2} = 0.375$$

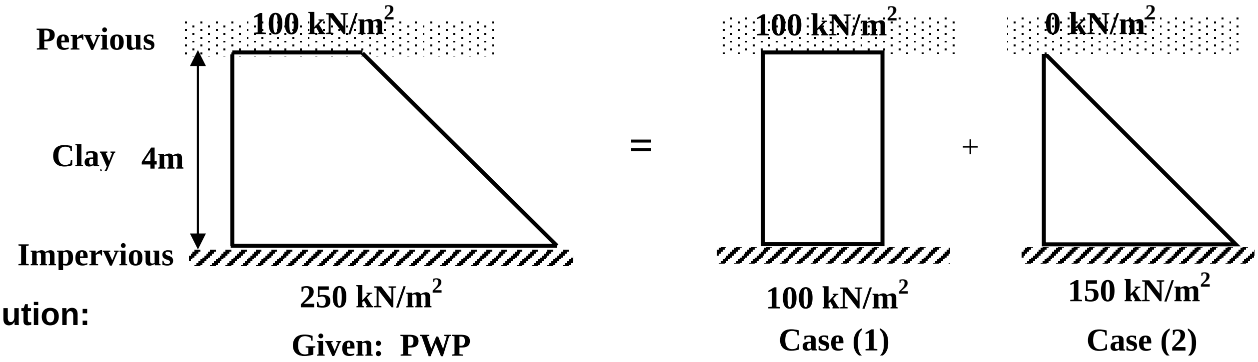
From **Fig.(5.8)** for $T_v = 0.375$; $U_1 = 65\%$ **(curve 1)** and

for $T_v = 0.375$; $U_2 = 75\%$ **(curve 3)**

$$U_{avg.} = \frac{U_1.A_1 + U_2.A_2}{\sum A} = \frac{0.65(4)(60) + 0.75(4)(40)/2}{(\frac{100+60}{2})(4)} = 0.675 = 67.5\%$$

Problem (5.8): *(degree of consolidation)*

For a layer of clay of 4m thick, if the coefficient of consolidation C_v = 0.4 m^2/year, and PWP distribution is given as below. Calculate: (1) the average degree of consolidation after 20 years, and (2) the time required for 62 % consolidation.



Solution:

Given: PWP

$C_v = 0.4\ m^3/year$

(1) $$T_v = \frac{C_v \cdot t}{(H_d)^2} = \frac{(0.4)(20)}{(4)^2} = 0.50$$

From **Fig.(5.8)** for $T_v = 0.50$; $U_1 = 76\%$ **(curve 1)** and
for $T_v = 0.50$; $U_2 = 69\%$ **(curve 2)**

$$U_{avg.} = \frac{U_1.A_1 + U_2.A_2}{\sum A} = \frac{0.76(4)(100) + 0.69(4)(150)/2}{(\frac{100+250}{2})(4)} = 0.73 = 73\%$$

(2) To calculate the time required for any degree % of consolidation, take several times $t_{i\,(year)}$ and find the corresponding $U_{i\,(avg.)}$ as follows:

t_i ***(year)***	T_v	U_1	U_2	$U_{avg.} = \frac{U_1.A_1 + U_2.A_2}{\sum A}$ ***(%)***
10	0.25	0.55	0.45	50.7 < 62
12	0.30	0.62	0.50	56.8 < 62
15	0.375	0.67	0.57	62.7 ≈ 62
18	0.45	0.72	0.64	68.5 > 62

Sample of Calculation:

- For t = 10 (years): $$T_v = \frac{C_v.t}{(H_d)^2} = \frac{(0.4)(10)}{(4)^2} = 0.25$$

From **Fig. (5.8)** for $T_v = 0.25$; $U_1 = 55\%$ **(curve 1)** and

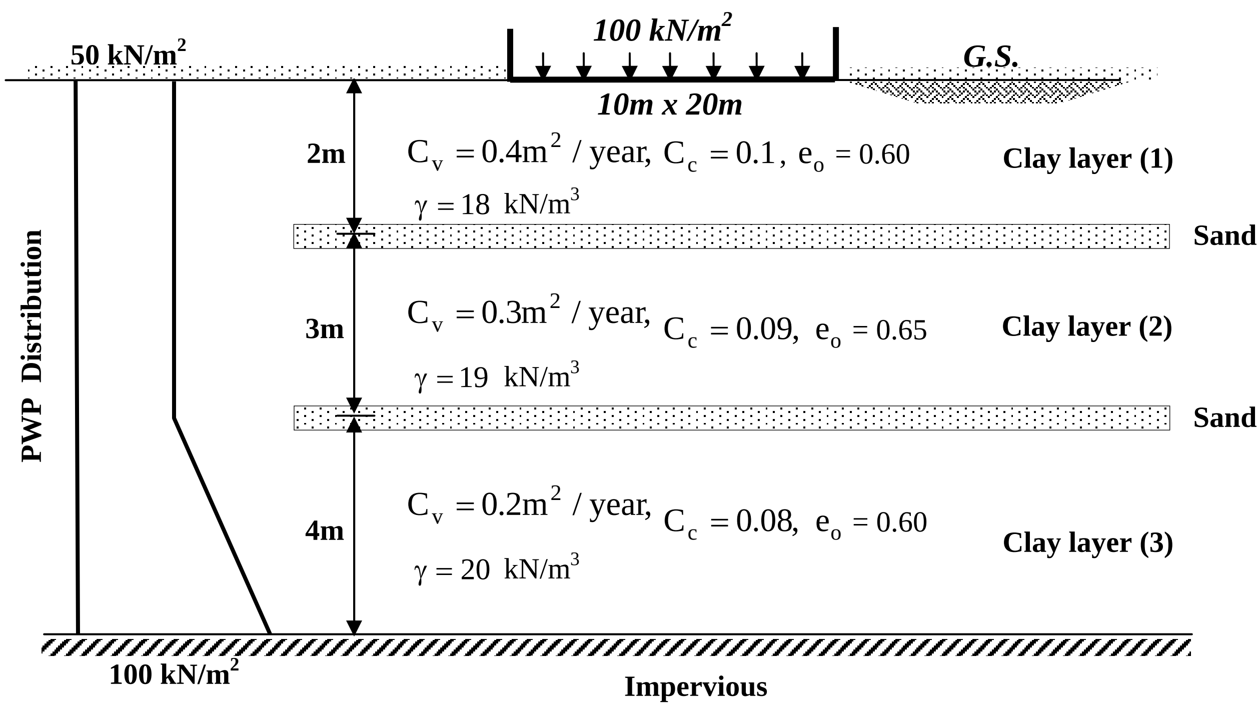
for $T_v = 0.25$; $U_2 = 45\%$ **(curve 2)**

$$U_{avg.} = \frac{U_1.A_1 + U_2.A_2}{\sum A} = \frac{0.55(4)(100) + 0.45(4)(150)/2}{(\frac{100+250}{2})(4)} = 0.507 = 50.7\%$$

- After drawing $U_{i(avg.)}$ versus $t_{i(year)}$ as obtained from table above; it can be seen that ***the required time for 62 % consolidation = 15 (years).***

Problem (5.9): *(consolidation for layered soils)*

A raft foundation is placed at surface of a normally consolidated clay layers with internal sand drain layers as shown in the figure below. Determine the % degree of consolidation after 10 years if the PWP distribution is as given in the same figure.



Solution:

(1) Calculate (T_v) for each clay layer; 1, 2, 3:

$$T_{v1} = \left[\frac{C_{v1}.t}{(H_1)^2}\right] = \frac{0.4(10)}{(2/2)^2} = 4,$$

$$T_{v2} = \left[\frac{C_{v2}.t}{(H_2)^2}\right] = \frac{0.3(10)}{(3/2)^2} = 1.333, \text{ and}$$

$$T_{v3} = \left[\frac{C_{v3}.t}{(H_3)^2}\right] = \frac{0.2(10)}{(4)^2} = 0.125$$

(2) Calculate (S_{C_i}) for each clay layer; 1, 2, 3:

<u>*for normally consolidated clay*</u>: $S_{Ci} = \frac{C_c}{1+e_o} H_t \log_{10} \frac{\sigma'_o + \Delta\sigma}{\sigma'_o}$

<u>for clay layer (1):</u>

$\sigma'_o = \gamma.H = 18(1) = 18$ kN/m^2, $\Delta\sigma$ =100(10)(20)/(10+1)(20+1) = 86.580 kN/m^2

$\therefore$ $$S_{C1} = \frac{0.1}{1+0.60}(200)\log_{10}\frac{18+86.580}{18} = 9.55\text{cm}$$

<u>for clay layer (2):</u>

$\sigma'_o = \gamma.H = 18(2) + 19(1.5) = 65.5$ kN/m^2, $\Delta\sigma$ =100(10)(20)/(10+3.5)(23.5) = 63.042 kN/m^2

$\therefore$ $$S_{C2} = \frac{0.09}{1+0.65}(300)\log_{10}\frac{64.5+63.042}{64.5} = 4.85\text{cm}$$

<u>for clay layer (3):</u>

$\sigma'_o = \gamma.H = 18(2) + 19(3) + 20(2) = 133$ kN/m^2, $\Delta\sigma$ =100(10)(20)/(10+7)(27) = 43.573 kN/m^2

$\therefore$ $$S_{C3} = \frac{0.08}{1+0.60}(400)\log_{10}\frac{133+43.573}{133} = 2.46\text{cm}$$

(3) Calculate (U %) for each clay layer; 1, 2, 3 after (10) years:

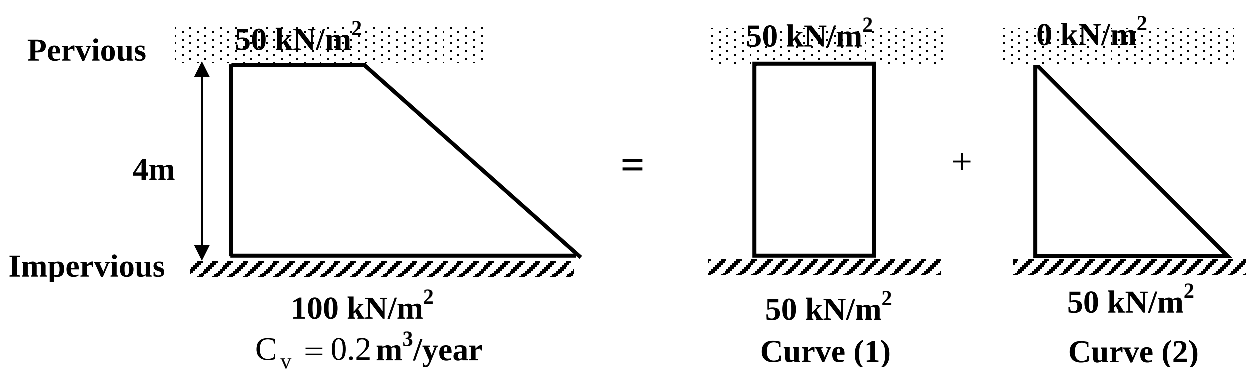
Total settlement for all layers:

$$S_c = S_{c1} + S_{c2} + S_{c3} = 9.55 + 5.85 + 2.46 = 16.86 \text{ cm}$$

From Fig.(5.8): ***for clay layer (1):*** for $T_{v1} = 4$; $U_1 = 100\%$ **(curve 1)**

for clay layer (2): for $T_{v2} = 1.33$; $U_2 = 95\%$ **(curve 1)**

for clay layer (3): the PWP distribution consists of cases (1 + 2) and calculated as:



for $T_{v3} = 0.125$; $U_1 = 0.38$ **(curve 1)** and $U_2 = 0.22$ **(curve 2)**

$$U_3 = \frac{U_1.A_1 + U_2.A_2}{\sum A} = \frac{0.38(4)(50) + 0.22(4)(50)/2}{(\frac{50+100}{2})(4)} = 0.326 = 33\%$$

The average degree of consolidation after (10) years for all layers is calculated from:

$$U_{i\,(avg.)} = U_{(t)} = \frac{1}{S_c}(S_{c1}U_1 + S_{c2}U_2 + S_{c3}U_3 + \dots\dots\dots)$$

$$U_{i(avg.)} = \frac{1}{16.86}\left[(9.55)(1.00) + (4.85)(0.95) + (2.46)(0.33)\right] = 0.89 = \mathbf{89\ \%}$$