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# CHAPTER 6

## STRUCTURAL DESIGN OF FOOTINGS

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### 6.1 TYPES OF FOOTINGS

Footings are foundation components that transmit the load from the superstructure to soil or rock. Their shapes usually vary with specific requirements and design needs. For spread footings, square shapes are common and usually most economical, but rectangular shapes are used if space is limited in one direction, or when loads are eccentric in one direction. The typically desired case is to select the footing shape that makes the soil pressure as uniform as possible. Furthermore, footings may be of uniform thickness or may be sloped or stepped. Fig.(6.1) shows typical configurations of various types of footings.

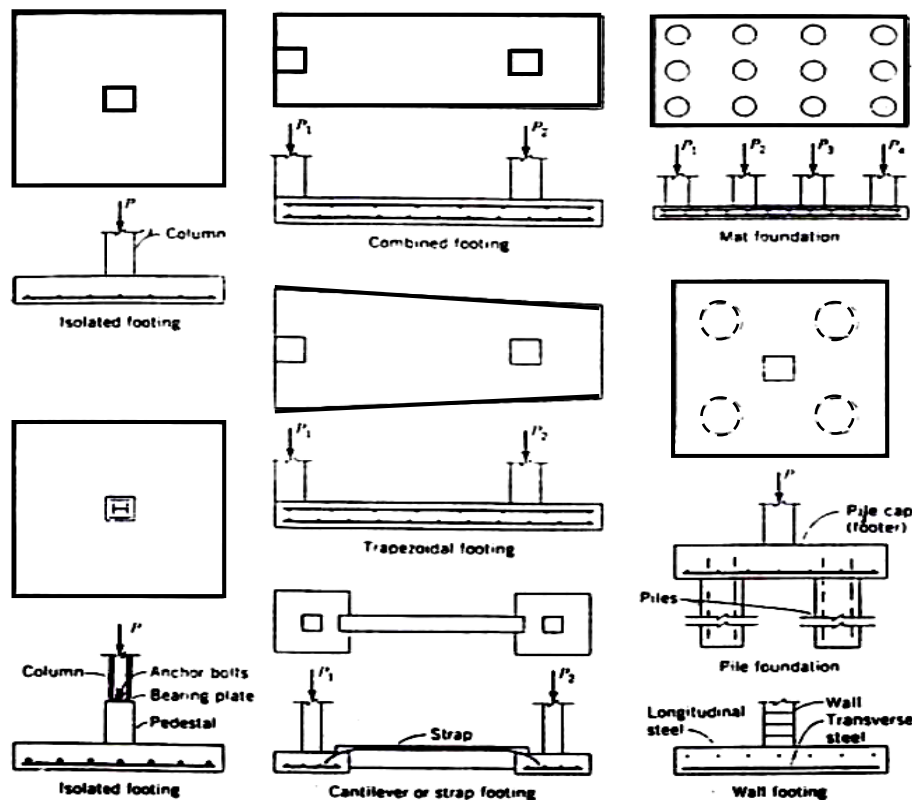


Fig.(6.1): Typical configurations of various footings types.

## 6.2 ASSUMPTIONS

1. The footing is assumed to be rigid (the thickness is sufficient enough) for easy calculation.
2. Actually, the soil pressure distribution under a footing is not uniform and depends upon footing rigidity, shape, and depth. However for simplicity, the distribution of the soil reactions is considered uniform as shown in Fig.(6.2).

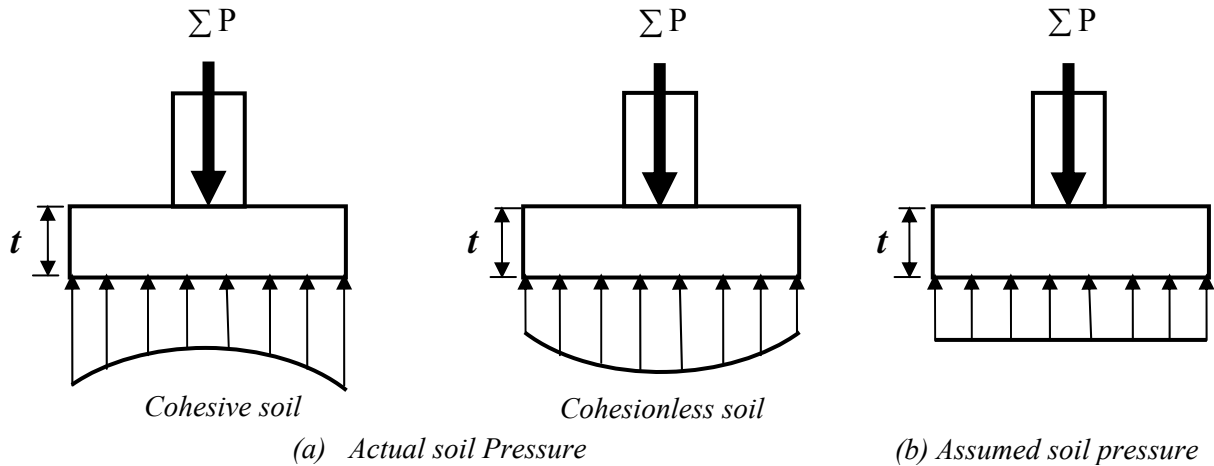


Fig.(6.2): Soil pressure distribution under rigid footings.

## 6.3 LOAD COMBINATIONS

The ACI 318–14 code for reinforced concrete foundation requirements specifies that the service loads should be converted to ultimate through several load combinations as:

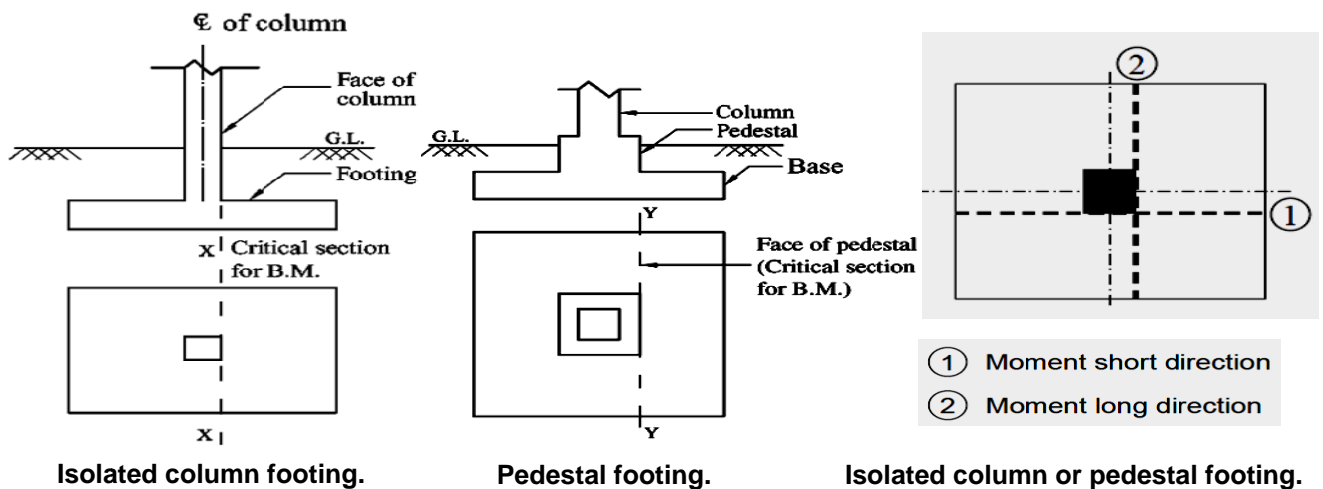
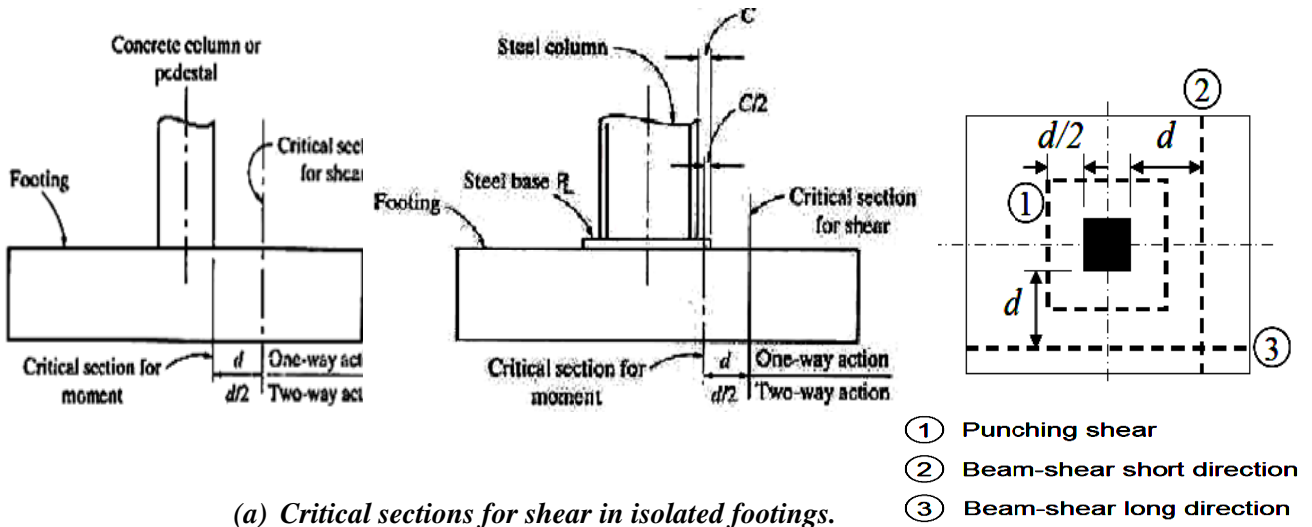
$$\begin{aligned}
 P_{ult.} &= 1.4 \text{ DL} \\
 P_{ult.} &= 1.2 \text{ DL} + 1.6 \text{ LL} \\
 P_{ult.} &= 1.2 \text{ DL} + 1.6 \text{ WL} + 1.0 \text{ LL} \\
 P_{ult.} &= 0.9 \text{ DL} + 1.6 \text{ WL} \\
 P_{ult.} &= 1.2 \text{ DL} + 1.0 \text{ EL} + 1.0 \text{ LL} \\
 P_{ult.} &= 0.9 \text{ DL} + 1.0 \text{ EL}
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} P_{ult.} &= 1.4 \text{ DL} \\ P_{ult.} &= 1.2 \text{ DL} + 1.6 \text{ LL} \\ P_{ult.} &= 1.2 \text{ DL} + 1.6 \text{ WL} + 1.0 \text{ LL} \\ P_{ult.} &= 0.9 \text{ DL} + 1.6 \text{ WL} \\ P_{ult.} &= 1.2 \text{ DL} + 1.0 \text{ EL} + 1.0 \text{ LL} \\ P_{ult.} &= 0.9 \text{ DL} + 1.0 \text{ EL} \end{aligned}} \right\} \dots\dots\dots (\text{ACI 318–14 Section 9.2})$$

where, DL, LL, WL and EL are service dead, live, wind, and earthquake loads, respectively.

From the above various combinations of the load cases, the load case control is considered as a design load. Note that in general as a rule, actual applied load ( $\Sigma P$ ) and actual net soil pressure ( $q_{all\ net.}$ ) are used to find the area of footing ( $A_f$ ). Whereas, factored loads and soil pressure are used to determine the steel area ( $A_s$ ) and footing thickness ( $t$ ).

## 6.4 CRITICAL SECTIONS FOR FOOTINGS

The ACI 318–14 code designates the critical locations for shear and moments depending on type of column or wall (e.g., concrete, steel, or masonry) as shown in Fig.(6.3). Notice that *circular columns are treated as square columns* with an equivalent width for location of critical sections for shear and moment.



(b) Critical sections for moment in isolated footings.

Fig.(6.3): Critical footing sections for shear and moment.

## 6.5 ACI 318–14 CODE REQUIREMENTS

A summary of strength design principles that apply to foundation design are presented in Table (6.1). The table is divided into two parts: (1) general design principles that apply for strength design and (2) principles that are specifically applicable to foundation design.

**Table (6.1): Summary of ACI 318 –14 code requirements.**

Principle	Design item	Code requirement	Code section
<b>General</b>	Load Combinations	$P_{ult.} = 1.4 \text{ DL}$ $P_{ult.} = 1.2 \text{ DL} + 1.6 \text{ LL}$ $P_{ult.} = 1.2 \text{ DL} + 1.6 \text{ WL} + 1.0 \text{ LL}$ $P_{ult.} = 0.9 \text{ DL} + 1.6 \text{ WL}$ $P_{ult.} = 1.2 \text{ DL} + 1.0 \text{ EL} + 1.0 \text{ LL}$ $P_{ult.} = 0.9 \text{ DL} + 1.0 \text{ EL}$ where, DL = dead load, LL= live load, WL= wind load, and EL = earthquake load.	9.2
	Load factor, $\phi$	Flexure: 0.9 Shear and torsion: 0.75 Flexure in plain concrete: 0.65	9.3
	Minimum flexure reinforcement	$A_{Smin.} = \frac{0.25\sqrt{f_c'}}{f_y} b.d \geq \frac{1.4}{f_y} b.d$	10.3
	Temperature and shrinkage reinforcement	$A_{Smin.} = 0.0020 b t$ ----- for $f_y < 420 \text{ MPa}$ $= 0.0018 b t$ ----- for $f_y = 420 \text{ MPa}$ $= \frac{0.0018 \times 420}{f_y} b t$ --- for $f_y > 420 \text{ MPa}$	7.12
	Shear reinforcement and minimum required steel	$V_c = \left( 0.16 \phi \sqrt{f_c'} + 17 \rho_w \frac{V_u d}{M_u} \right) b_w d$ $\leq 0.29 \phi \sqrt{f_c'} b_w d$ $A_{V,min.} = 0.062 \sqrt{f_c'} \frac{b_w S}{f_{Yt}} \geq 0.35 \frac{b_w S}{f_{Yt}}$	11.5  11.13

	<p>Development length, <math>l_d</math> , for deformed bars and deformed wires:</p> <p><b>(a) Tension:</b></p> <p>Either 12.1 or 12.2.2 equations is applicable provided that <math>l_d \geq 300</math> mm.</p>	$l_d = \frac{f_y}{1.1 \lambda \sqrt{f'_c}} \frac{\Psi_l \Psi_e \Psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)} d_b$ <p>in which the confinement term:</p> $\frac{c_b + K_{tr}}{d_b} \leq 2.5, \quad \text{and} \quad K_{tr} = \frac{40 A_{tr}}{s n}$ <p>where, <math>n</math> is the number of bars or wires being spliced or developed along the plane of splitting. However, it shall be permitted to use <math>K_{tr} = 0</math> as a design simplification even if transverse reinforcement is present.</p>	12.1																		
	<table> <tr> <th>Spacing and cover</th> <th>No. 19 and smaller bars</th> <th>No. 22 and larger bars</th> </tr> <tr> <td>Clear spacing of bars or wires being developed or spliced not less than <math>d_b</math>, clear cover not less than <math>d_b</math>, and stirrups or ties throughout <math>l_d</math> not less than the Code minimum.</td> <td> <math display="block">l_d = \frac{f_y \Psi_l \Psi_e}{2.1 \lambda \sqrt{f'_c}} d_b</math> </td> <td> <math display="block">l_d = \frac{f_y \Psi_l \Psi_e}{1.7 \lambda \sqrt{f'_c}} d_b</math> </td> </tr> <tr> <td>or Clear spacing of bars or wires being developed or spliced not less than <math>2d_b</math> and clear cover not less than <math>d_b</math>.</td> <td> <math display="block">l_d = \frac{f_y \Psi_l \Psi_e}{1.4 \lambda \sqrt{f'_c}} d_b</math> </td> <td> <math display="block">l_d = \frac{f_y \Psi_l \Psi_e}{1.1 \lambda \sqrt{f'_c}} d_b</math> </td> </tr> </table>	Spacing and cover	No. 19 and smaller bars	No. 22 and larger bars	Clear spacing of bars or wires being developed or spliced not less than $d_b$ , clear cover not less than $d_b$ , and stirrups or ties throughout $l_d$ not less than the Code minimum.	$l_d = \frac{f_y \Psi_l \Psi_e}{2.1 \lambda \sqrt{f'_c}} d_b$	$l_d = \frac{f_y \Psi_l \Psi_e}{1.7 \lambda \sqrt{f'_c}} d_b$	or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than $d_b$ .	$l_d = \frac{f_y \Psi_l \Psi_e}{1.4 \lambda \sqrt{f'_c}} d_b$	$l_d = \frac{f_y \Psi_l \Psi_e}{1.1 \lambda \sqrt{f'_c}} d_b$	<table> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table>										12.2.2
Spacing and cover	No. 19 and smaller bars	No. 22 and larger bars																			
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	<p>where,</p> <p><math>\Psi_l</math> = <b>bar location factor</b>; <math>\Psi_l = 1.3</math> for top reinforcement, <math>\Psi_l = 1.0</math> for other reinforcement.</p> <p><math>\Psi_e</math> = <b>coating factor</b>; <math>\Psi_e = 1.5</math> for epoxy-coated bars or wires with cover less than <math>3d_b</math> or clear spacing less than <math>6d_b</math>; <math>\Psi_e = 1.2</math> for other epoxy-coated bars or wires; <math>\Psi_e = 1.0</math> for uncoated reinforcement; provided that <math>\Psi_l \Psi_e \leq 1.7</math>.</p> <p><math>\Psi_s</math> = <b>bar size factor</b>, <math>\Psi_s = 0.8</math> for <math>\emptyset 19</math> mm and smaller bars and deformed wires; and <math>\Psi_s = 1.0</math> for <math>\emptyset 22</math> mm and larger bars.</p>																				

	<p><math>c_b</math> = <b>spacing or cover dimension</b>: Use the smaller of either the distance from the center of the bar to the nearest concrete surface or one-half the center-to-center spacing of the bars being developed.</p> <p><math>K_{tr}</math> = <b>transverse reinforcement index</b>, which is equal to <math>(1.6 A_{tr}/sn)</math>, where <math>A_{tr}</math> = total cross-sectional area of all transverse reinforcement within <math>l_d</math> that crosses the potential plane of splitting adjacent to the reinforcement being developed, <math>s</math> is the maximum spacing of transverse reinforcement within <math>l_d</math>, center-to-center (mm); and <math>n</math> is the number of bars or wires being developed along the plane of splitting.</p> <p><math>\lambda</math> = <b>lightweight-aggregate concrete factor</b>, <math>\lambda = 1.0</math> for normal weight concrete. <math>\lambda = 0.75</math> for lightweight aggregate concrete; however, when <math>f_{ct}</math> is specified, use <math>\lambda = \sqrt{f'_c}/1.8 f_{ct}</math> (in SI-units).</p>		
	<p><b><u>(b) Compression:</u></b></p> <p>The ACI code 318-14 permits a reduction multiplier of <math>l_{dc}</math> given by: <math>\lambda_s</math> = <b>excess reinforcement factor</b>; if the longitudinal flexural reinforcement is in excess of that required by analysis except where anchorage or development for <math>f_y</math> is specifically required or the reinforcement is designed for seismic effects. The reduction multiplier is: <math>\lambda_s = (A_s \text{ required})/(A_s \text{ provided})</math>, or <math>\lambda_{s1} = 0.75</math> for spirally enclosed reinforcement not less than 6 mm diameter and not more than 100 mm pitch or within No.13 ties and spaced at not more than 100 mm on center, and <math>\lambda_{s2} = f_y/415</math> for cases where <math>f_y &gt; 415 \text{ MPa}</math>.</p>	$l_{dc} = \frac{0.24 f_y}{\lambda \sqrt{f'_c}} d_b \geq (0.043 f_y) d_b$ <p>But not less than 200 mm.</p>	12.3.2
	Reinforcement spacing	<p>Clear distance not less than diameter of bar or 1.0 inch (25 mm).</p> <p>Walls and slabs: not to be spaced farther apart than 3 times the wall or slab thickness or 18 in. (450 mm).</p>	<p>7.6.1</p> <p>7.6.5</p>

	Minimum reinforcement cover	3.0 inch (75 mm) for cast-in place concrete and permanently exposed to earth.	7.7.1
	Modulus of elasticity of concrete, $E_c$ for normal weight concrete.	$E_c = w_c^{1.5} 0.043 \sqrt{f'_c} \text{ (MPa)}$ $E_c \text{ (MPa)} = 4700 \sqrt{f'_c} \text{ (MPa)}$	8.5.1
<b>Footings</b>	General considerations	See ACI code	15
	Round columns	Use equivalent square of same area for location of critical sections for moment, shear, and development of reinforcement in footings.	15.3
	Maximum moment	See ACI code	15.4
	Minimum footing depth	(a) Not less than 6 in. (152 mm) above the bottom of reinforcement for footing on soil. (b) Not less than 12 in. (305 mm) for footing on piles.	15.7
	Shear: One-way action (Wide-beam shear):  Two-way action (Punching shear): <i>Take smaller value from a, b and c.</i>  <i>The units of both shear are in (MPa).</i>	$v_{c \text{ all.}} = 0.17 \phi \sqrt{f'_c}$  $v_{c \text{ all.}} = 0.17 \left(1 + \frac{2}{\beta}\right) \phi \sqrt{f'_c}$ -----(a) $v_{c \text{ all.}} = 0.083 \left(\frac{\alpha_s d}{b_o} + 2\right) \phi \sqrt{f'_c}$ ---(b) $v_{c \text{ all.}} = 0.33 \phi \sqrt{f'_c}$ -----(c)  <i>where,</i> $\phi = 0.75, \quad \beta = \frac{\text{length of column}}{\text{width of column}}$ $\alpha_s = 50 \text{ cm for corner columns,}$ $\quad = 75 \text{ cm for edge columns,}$ $\quad = 100 \text{ cm for interior columns.}$	11.3  11.31 11.32 11.33

	Bearing strength (column on footing): <i>Take the smaller value from (a) or (b).</i>	$f_{c_{all.}} = 0.85 \phi f'_c \sqrt{A_2/A_1}$ ----- (a) $f_{c_{all.}} = 0.85 \phi f'_c$ ----- (b)  where, $\phi = 0.65$ and $\sqrt{A_2/A_1} \leq 2.0$ $A_1$ = area of contact (or column), $A_2$ = projection area.	10.15
	Minimum steel reinforcement for bearing strength (dowels).	$A_{s,min.} = 0.005 A_1$ where, $A_1$ = area of column.	15.8.2.1
<b><i>Walls</i></b>	General considerations	See ACI code	14
	Reinforcement	Vertical: $A_s \geq 0.0012 A_g$ of wall Horizontal: $A_s \geq 0.002 A_g$ of wall where, $A_s$ = area of reinforcement, and $A_g$ = gross area of the wall.	14.3.2 14.3.3
	Maximum thickness	Not less than 1/25 the supported height or length, whichever is shorter; not less than 4 in. (102 mm).  Exterior basement walls and foundation walls not less than 7.5 in. (191 mm).	14.5.3.1   14.5.3.2
	Grade beams	See ACI code	14.7



## 6.6 SOIL PRESSURE DISTRIBUTIONS UNDER FOOTINGS

As shown in Fig.(6.4a), if the moments about both x and y axes are zero, then, the soil pressure distribution under the footing is simply equal to the total vertical load divided by the footing's area. While in case of moment or (moments), the contact pressure below the footing is non-uniform (see Fig.6.4b).

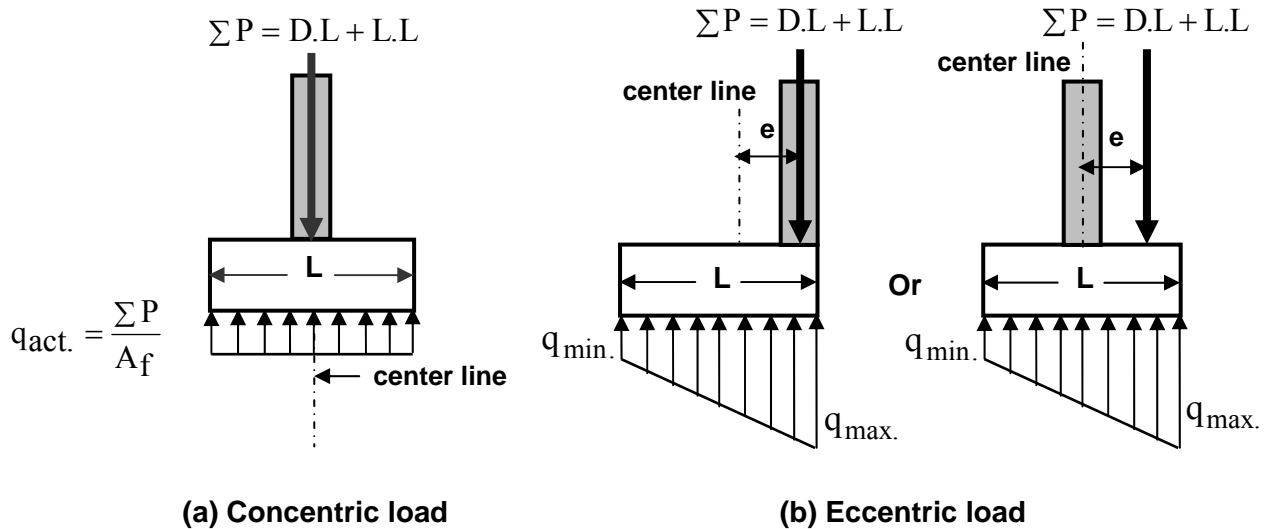


Fig.(6.4): Contact pressure distribution under footings.

In general, when the load is centric according to footing center, the contact soil pressure is uniform. But when the load is eccentric (i.e., there is a moment), the contact pressure below the footing is non-uniform as shown under the following three cases:

**Case (1):** When  $e_x < L/6$ , the resultant of load passes within the middle third of the footing.

Here, there is compression under the footing with maximum pressure on one side and minimum pressure on the other side.

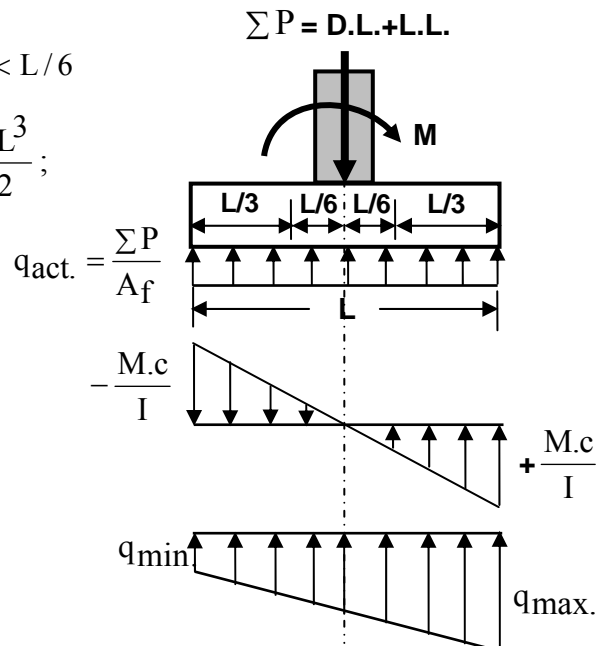
- Moment in (L-direction only) and  $e_x < L/6$**

$$e_x = \text{eccentricity} = \frac{M}{\Sigma P}; \quad c = \frac{L}{2}; \quad I = \frac{B.L^3}{12};$$

$$\frac{M.c}{I} = \frac{6M}{B.L^2}; \quad M = \Sigma P \cdot e_x$$

$$q_{\min.}^{\max.} = \frac{\Sigma P}{B.L} \pm \frac{6\Sigma P \cdot e_x}{B.L^2}$$

$$\text{or } q_{\min.}^{\max.} = \frac{\Sigma P}{B.L} \left[ 1 \pm \frac{6.e_x}{L} \right]$$

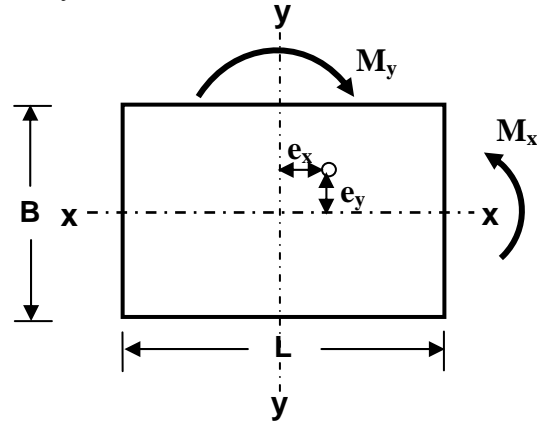


- **Moments in (both directions) and**  $e_x < L/6$  ;  $e_y < B/6$

$$e_x = \frac{M_y}{\sum P}; \quad e_y = \frac{M_x}{\sum P}$$

$$\text{or } q_{\min.}^{\max.} = \frac{\sum P}{B.L} \pm \frac{6\sum P.e_x}{B.L^2} \pm \frac{6\sum P.e_y}{B^2.L}$$

$$\text{or } q_{\min.}^{\max.} = \frac{\sum P}{B.L} \left[ 1 \pm \frac{6.e_x}{L} \pm \frac{6.e_y}{B} \right]$$

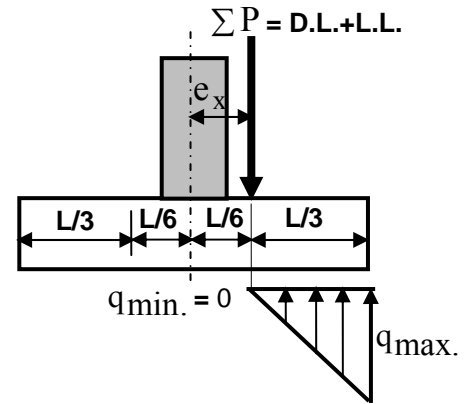


**Case (2):** When  $e_x = L/6$ , the resultant of load passes on edge of the middle third of the footing.

- **Moment in (L-direction only) and**  $e_x = L/6$

$$q_{\max.} = \frac{\sum P}{B.L} \left[ 1 + \frac{6.e_x}{L} \right] = \frac{\sum P}{B.L} \left[ 1 + \frac{L}{L} \right] = \frac{2\sum P}{B.L}$$

$$q_{\min.} = \frac{\sum P}{B.L} \left[ 1 - \frac{6.e_x}{L} \right] = \frac{\sum P}{B.L} \left[ 1 - \frac{L}{L} \right] = 0$$



**Case (3):** When  $e_x > L/6$ , the resultant of load is outside the middle third of the footing. Here, there is a tension under the footing.

- **Moment in (L-direction only) and**  $e_x > L/6$

$$\sum P = \frac{1}{2} q_{\max.} . L_1 . B \dots\dots\dots(a)$$

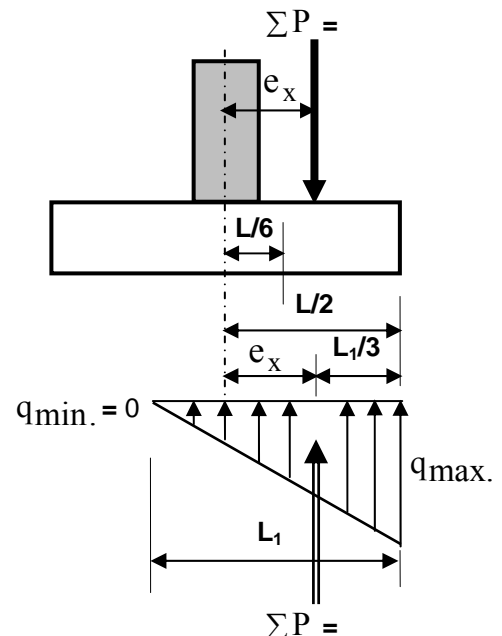
$$e_x + \frac{L_1}{3} = \frac{L}{2} \dots\dots\dots(b)$$

$$\text{From Eq.(a): } \dots\dots\dots(c)$$

$$\text{From Eq.(b): } L_1 = 3 \left[ \frac{L}{2} - e_x \right] \dots\dots\dots(d)$$

Substituting Eq.(d) into Eq.(c) gives:

$$q_{\max.} = \frac{2.\sum P}{3.B \left[ \frac{L}{2} - e_x \right]}$$



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# CHAPTER 6

## STRUCTURAL DESIGN OF FOOTINGS

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### 6.1 DESIGN OF CONCENTRIC SPREAD FOOTINGS

#### 6.1.1 Design Steps of Square Spread Footings

**(1) Find the area of footing:**

*Assume footing thickness ( $t$ ).*

*Calculate  $q_{all (net)}$  of soil from allowable bearing capacities due to DL, (DL+ LL), (DL + LL+ WL) as follows:*

$$q_{all (net) DL} = q_{all (DL)} - t_f \gamma_{conc.} - (D_f - t_f) \gamma_{soil} ; \quad A_f = \frac{DL}{q_{all (net) DL}}$$

$$q_{all (net) DL+LL} = q_{all (DL+LL)} - t_f \gamma_{conc.} - (D_f - t_f) \gamma_{soil} ; \quad A_f = \frac{DL + LL}{q_{all (net) DL+LL}}$$

$$q_{all (net) DL+LL+WL} = q_{all (DL+LL+WL)} - t_f \gamma_{conc.} - (D_f - t_f) \gamma_{soil} ; \quad A_f = \frac{DL + LL + WL}{q_{all (net) DL+LL+WL}}$$

Then, for square footing  $B = \sqrt{A_{f (control)}}$

**(2) Convert the loads into ultimate and according to control ultimate loads calculate the ultimate soil pressure:**

$$\left. \begin{array}{l} P_{ult.} = 1.4 DL \\ P_{ult.} = 1.2 DL + 1.6 LL \\ P_{ult.} = 1.2 DL + 1.6 WL + 1.0 LL \\ P_{ult.} = 0.9 DL + 1.6 WL \\ P_{ult.} = 1.2 DL + 1.0 EL + 1.0 LL \\ P_{ult.} = 0.9 DL + 1.0 EL \end{array} \right\} \dots\dots\dots (ACI 318-14 \text{ Section 9.2})$$
$$q_{ult.} = \frac{P_{ult.(control)}}{A_f}$$

(3) Check: (a) bearing capacity [S.F.  $\geq 3.0$ ]; (b) settlement [ $S \leq S_{\text{allowable}}$ ].

(4) Determine the thickness of footing:

**(i) Check two-way action or punching shear at (d/2) from column face:**

$$v_{c \text{ all.}} = 0.17 \left(1 + \frac{2}{\beta}\right) \phi \sqrt{f'_c} \dots\dots\dots (\text{ACI 318-14 section 11.31}) \dots\dots\dots (a)$$

$$v_{c \text{ all.}} = 0.083 \left(\frac{\alpha_s d}{b_o} + 2\right) \phi \sqrt{f'_c} \dots\dots\dots (\text{ACI 318-14 section 11.32}) \dots\dots\dots (b)$$

$$v_{c \text{ all.}} = 0.33 \phi \sqrt{f'_c} \dots\dots\dots (\text{ACI 318-14 section 11.33}) \dots\dots\dots (c)$$

Take the smaller value from (a), (b) and (c).

where,

$$\beta = \frac{\text{length of column}}{\text{width of column}} = \frac{b_1}{b_2} \text{ or } \frac{b_2}{b_1},$$

$$\phi = 0.75$$

$$\begin{aligned} \alpha_s &= 50 \text{ cm for corner columns,} \\ &= 75 \text{ cm for edge columns,} \\ &= 100 \text{ cm for interior columns.} \end{aligned}$$

$$\text{Punching area} = A_f - (b + d)^2;$$

$$\text{Punching force} = q_{ult.} [A_f - (b + d)^2]$$

$$\text{Punching face} = b_o d; \quad \text{and} \quad b_o = 4(b + d) \dots\dots\dots \text{for square column}$$

$$v_{c \text{ actual}} = \frac{q_{ult.} [A_f - (b + d)^2]}{b_o d}; \quad \text{Put} \quad v_{c \text{ actual}} = v_{c \text{ all.}} \quad \text{and solve for (d).}$$

**(ii) Check one-way or wide-beam shear at distance (d) from face of column:**

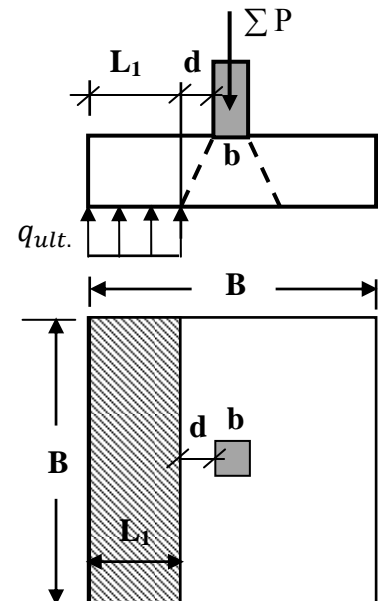
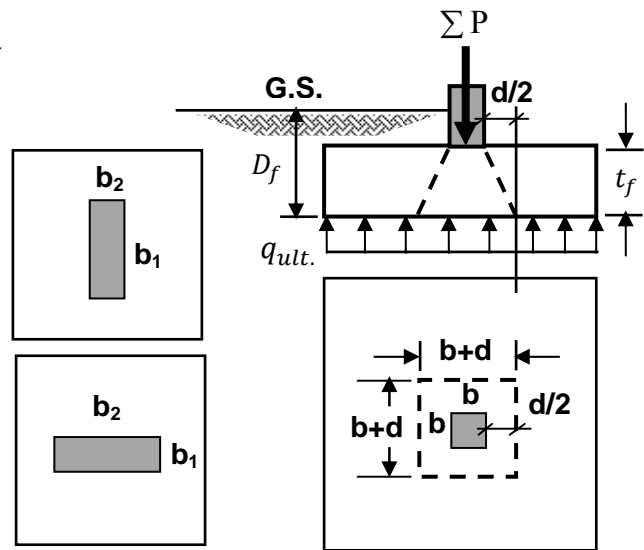
$$v_{c \text{ all.}} = 0.17 \phi \sqrt{f'_c} \dots\dots\dots (\text{ACI 318-14 section 11.3})$$

$$\phi = 0.75$$

$$L_1 = \frac{B - b}{2} - d; \quad \text{Punching area} = L_1 B$$

$$\text{Punching force} = q_{ult.} L_1 B$$

$$v_{c \text{ actual}} = \frac{q_{ult.} L_1 B}{B \cdot d} = \frac{q_{ult.} L_1}{d}$$



Put  $v_{c \text{ actual}} = v_{c \text{ all}}$  and solve for (d); then take the larger value of (d) obtained from (i) or (ii).

**(5) Determine the required steel (Calculate moment at face of column):**

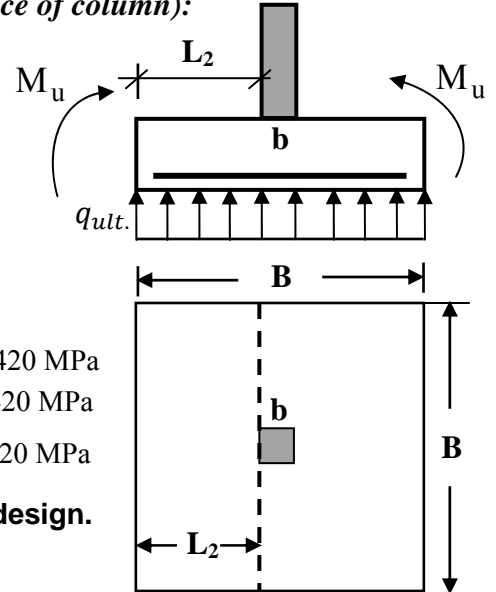
$$M_u = \frac{q_{ult.} L_2^2}{2} \quad \text{where, } L_2 = \frac{B-b}{2}$$

$$A_s = \frac{M_u}{0.9 f_y 0.9 d} ; \quad A_{s \min.} = \rho_{\min.} b \cdot d$$

where,  $\rho_{\min.}$  is the larger of:  $1.4/f_y$  or  $0.25 \sqrt{f'_c}/f_y$

$$\begin{aligned} A_{s \min.} (\text{Temp. \& shrinkage}) &= 0.0020 b t \text{ ----- for } f_y < 420 \text{ MPa} \\ &= 0.0018 b t \text{ ----- for } f_y = 420 \text{ MPa} \\ &= \frac{0.0018 \times 420}{f_y} b t \text{ ---- for } f_y > 420 \text{ MPa} \end{aligned}$$

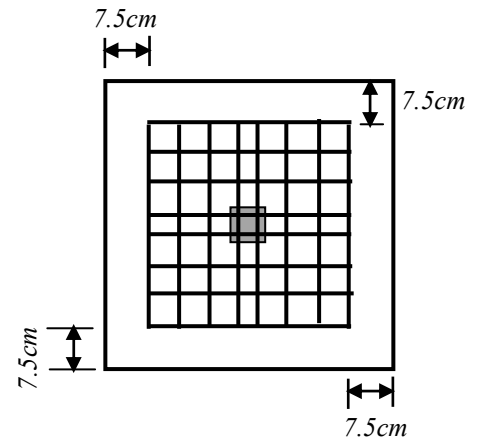
Compare  $A_s$  with  $A_{s \min.}$  and take the larger value for design.



**(6) Spacing and steel distribution:**

$$\text{Number of bars: } N = \frac{A_{s \text{ total}}}{\text{Area of bar}}$$

$$\text{Spacing (c/c)} = \frac{B - 15 \text{ cm (concrete cover)}}{N-1}$$



**(7) Check the bond:**

**(a) Steel in tension:**

$$l_{d(\text{available})} = L_2 - 7.5 \text{ (concrete cover)}$$

$l_{d(\text{required})}$  either ACI 318–14 code (12.1 or 12.2.2) Eq. is used provided that  $l_d \geq 300 \text{ mm}$

$$l_{d(\text{required})} = \frac{f_y}{1.1 \lambda \sqrt{f'_c}} \left( \frac{\Psi_l \Psi_e \Psi_s}{\left( \frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \text{ ..... (ACI 318–14 section 12.1)}$$

where,

$f_y$  = yield strength of steel reinforcement,

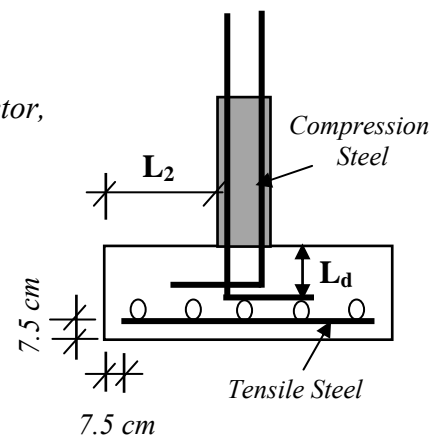
$\Psi_l$  = bar location factor,  $\Psi_e$  = coating factor,  $\Psi_s$  = bar size factor,

$\lambda$  = lightweight–aggregate concrete factor,

$f'_c$  = compressive strength of concrete,

$c_b$  = spacing or cover dimension,

$K_{tr}$  = transverse reinforcement index, and  $d_b$  = bar diameter.



**(b) Steel in compression:**

$l_{d(available)} = (d) \text{ of footing}$

$$l_{dc(required)} = \frac{0.24 f_y}{\lambda \sqrt{f'_c}} d_b \geq (0.043 f_y) d_b \dots\dots\dots(\text{ACI 318-14 section 12.3.2})$$

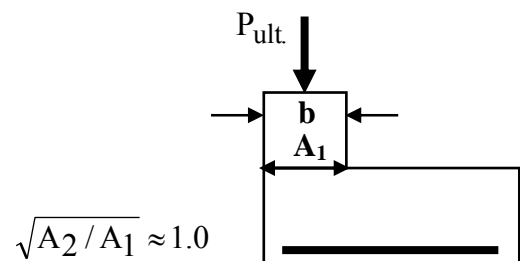
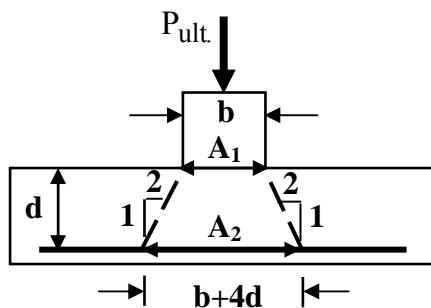
provided that  $l_d \geq 200 \text{ mm}$ .

**(8) Check contact pressure between the column and footing:**

$$f_c(available) = \frac{P_{ult.}}{A_1}$$

$$f_c(allowable) = 0.85 \phi f'_c \sqrt{A_2/A_1} \leq 0.85 \phi f'_c$$

where,  $\phi = 0.65$  and  $\sqrt{A_2/A_1} \leq 2.0$



$$\sqrt{A_2/A_1} \approx 1.0$$

For this case:  $f_{c(all.)} = 0.85 \phi f'_c$

$A_1 = \text{area of contact (or column)} ; \quad A_2 = \text{projection area} = (b + 4d)^2$

**Note:** If  $f_c(available) > f_c(allowable)$ : (1) Increase the section of column, or (2) Design dowels for excess load.

$$A_s \text{ of dowels} = \frac{[f_c(available) - f_c(allowable)] A_1}{0.9 f_y}$$

**(9) Design the dowels:**

Minimum number of dowels = 4 for square or rectangular columns.

Minimum number of dowels = 6 for circular columns.

Minimum  $A_s$  of dowels =  $0.005 A_1$  .....ACI 318-14 sec.(15.8.2.1)

where,  $A_1 = \text{area of column}$ .

**(10) Draw sketches (plan and cross sections) showing all details required for construction.**

### 6.1.2 Design Steps of Rectangular Spread Footings

**(1) Find the area of footing:**

Assume footing thickness ( $t$ ).

Calculate  $q_{all(net)}$  of soil from allowable bearing capacities due to DL, (DL+ LL), (DL + LL+ WL) as follows:

$$q_{all(net) DL} = q_{all(DL)} - t_f \gamma_{conc.} - (D_f - t_f) \gamma_{soil} ; \quad A_f = \frac{DL}{q_{all(net) DL}}$$

$$q_{all(net) DL+LL} = q_{all(DL+LL)} - t_f \gamma_{conc.} - (D_f - t_f) \gamma_{soil} ; \quad A_f = \frac{DL + LL}{q_{all(net) DL+LL}}$$

$$q_{all(net) DL+LL+WL} = q_{all(DL+LL+WL)} - t_f \gamma_{conc.} - (D_f - t_f) \gamma_{soil} ; \quad A_f = \frac{DL + LL + WL}{q_{all(net) DL+LL+WL}}$$

For rectangular footing  $B \times L = A_{f(control)}$ . Then, choose  $B$  and  $L$  such that  $(L/B < 2.0)$ .

For example if the required area =  $6 \text{ m}^2$ , then:

B	L	Area	L/B
1.0	6	6	6.00 > 2.0
1.5	4	6	2.67 > 2.0
2.0	3	6	1.50 < 2.0 $\therefore$ Take <b>L = 3.0</b> and <b>B = 2.0</b>
2.5	2.4	6	0.96 < 2.0

**(2) Convert the loads into ultimate and according to control ultimate loads calculate the ultimate soil pressure:**

$$\begin{aligned}
 P_{ult.} &= 1.4 \text{ DL} \\
 P_{ult.} &= 1.2 \text{ DL} + 1.6 \text{ LL} \\
 P_{ult.} &= 1.2 \text{ DL} + 1.6 \text{ WL} + 1.0 \text{ LL} \\
 P_{ult.} &= 0.9 \text{ DL} + 1.6 \text{ WL} \\
 P_{ult.} &= 1.2 \text{ DL} + 1.0 \text{ EL} + 1.0 \text{ LL} \\
 P_{ult.} &= 0.9 \text{ DL} + 1.0 \text{ EL}
 \end{aligned}
 \left. \vphantom{\begin{aligned} P_{ult.} &= 1.4 \text{ DL} \\ P_{ult.} &= 1.2 \text{ DL} + 1.6 \text{ LL} \\ P_{ult.} &= 1.2 \text{ DL} + 1.6 \text{ WL} + 1.0 \text{ LL} \\ P_{ult.} &= 0.9 \text{ DL} + 1.6 \text{ WL} \\ P_{ult.} &= 1.2 \text{ DL} + 1.0 \text{ EL} + 1.0 \text{ LL} \\ P_{ult.} &= 0.9 \text{ DL} + 1.0 \text{ EL} \end{aligned}} \right\} \dots\dots\dots (\text{ACI 318-14 Section 9.2})$$

$$q_{ult.} = \frac{P_{ult.(control)}}{A_f}$$

**(3) Check: (a) bearing capacity [S.F.  $\geq 3.0$ ]; (b) settlement [ $S \leq S_{allowable}$ ].**

**(4) Determine the thickness of footing:**

**(i) Check wide-beam shear at (d) from column face:**

$$v_{c \text{ all.}} = 0.17 \phi \sqrt{f'_c} \dots\dots\dots (\text{ACI 318-14 section 11.3})$$

where,  $\phi = 0.75$

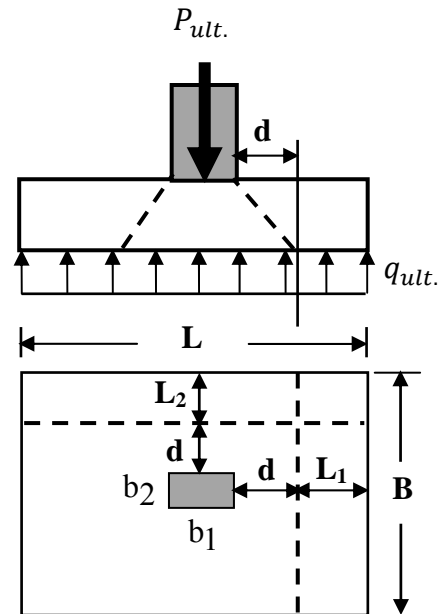
**In long-direction:**

$$v_{c \text{ actual}} = \frac{q_{ult.} L_1}{b d} \text{ where, } L_1 = \frac{L - b_1}{2} - d$$

**In short-direction:**

$$v_{c \text{ actual}} = \frac{q_{ult.} L_2}{b d} \text{ where, } L_2 = \frac{B - b_2}{2} - d$$

Put  $v_{c \text{ actual}} = v_{c \text{ all.}}$  and solve for (d).

**(ii) Check two-way action or punching shear at (d/2) from column face:**

$$v_{c \text{ all.}} = 0.17 \left(1 + \frac{2}{\beta}\right) \phi \sqrt{f'_c} \dots\dots\dots (\text{ACI 318-14 section 11.31}) \dots\dots\dots (a)$$

$$v_{c \text{ all.}} = 0.083 \left(\frac{\alpha_s d}{b_o} + 2\right) \phi \sqrt{f'_c} \dots\dots\dots (\text{ACI 318-14 section 11.32}) \dots\dots\dots (b)$$

$$v_{c \text{ all.}} = 0.33 \phi \sqrt{f'_c} \dots\dots\dots (\text{ACI 318-14 section 11.33}) \dots\dots\dots (c)$$

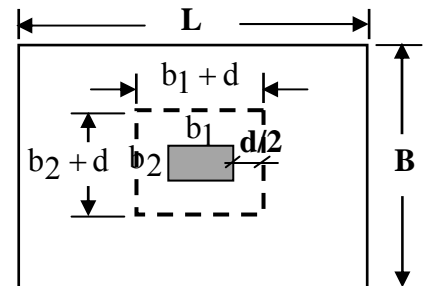
Take the smaller value from (a), (b) and (c).

where,

$$\beta = \frac{\text{length of column}}{\text{width of column}} = \frac{b_1}{b_2} \text{ or } \frac{b_2}{b_1}, \phi = 0.75$$

$\alpha_s = 50 \text{ cm}$  for corner columns,  
 $= 75 \text{ cm}$  for edge columns,  
 $= 100 \text{ cm}$  for interior columns.

$$v_{c \text{ actual}} = \frac{q_{ult.} [(L)(B) - (b_1 + d)(b_2 + d)]}{[2(b_1 + d) + 2(b_2 + d)] d}$$



Put  $v_{c \text{ actual}} = v_{c \text{ all.}}$  and solve for (d), then take the larger (d) obtained from (i) or (ii).



**(5) Determine the required steel for each direction:**

Calculate moments at column faces in both directions.

**(a) Steel in Long-Direction:**

$$M_{u(1-1)} = \frac{q_{ult.} \cdot X_1^2}{2} \quad \text{where, } X_1 = \frac{L - b_1}{2}$$

$$A_{s(1-1)} = \frac{M_{u(1-1)}}{0.9 \cdot f_y \cdot 0.9d}$$

$$A_{smin.} = \rho_{min.} \cdot b \cdot d \quad \text{where, } \rho_{min.} \text{ is the larger of: } 1.4/f_y \text{ or } 0.25\sqrt{f'_c}/f_y$$

$$A_{smin. (Temp. \& shrinkage)} = 0.0020 \cdot b \cdot t \quad \text{----- for } f_y < 420 \text{ MPa}$$

$$= 0.0018 \cdot b \cdot t \quad \text{----- for } f_y = 420 \text{ MPa}$$

$$= \frac{0.0018 \times 420}{f_y} \cdot b \cdot t \quad \text{----- for } f_y > 420 \text{ MPa}$$

**Compare  $A_{s(1-1)}$  with  $A_{smin.}$  and take the larger value for design.**

$$A_{s \text{ total (long.dir.)}} = A_{s(1-1)} \cdot B$$

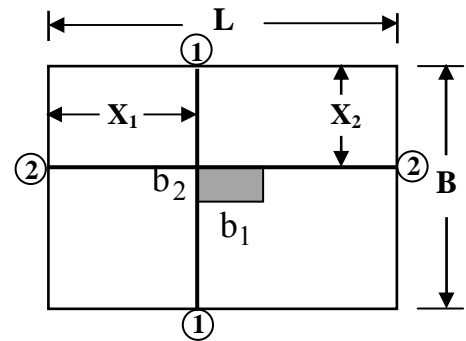
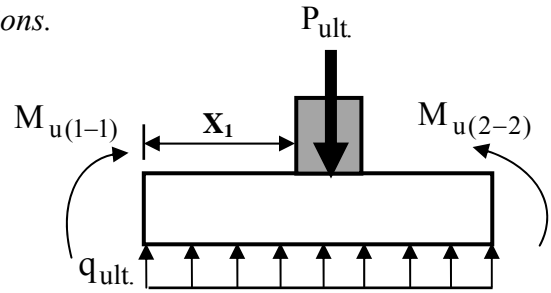
**(b) Steel in Short-Direction:**

$$M_{u(2-2)} = \frac{q_{ult.} \cdot X_2^2}{2} \quad \text{where, } X_2 = \frac{B - b_2}{2}$$

$$A_{s(2-2)} = \frac{M_{u(2-2)}}{0.9 \cdot f_y \cdot 0.9d}$$

**Compare  $A_{s(2-2)}$  with  $A_{smin.}$  and take the larger value for design.**

$$A_{s \text{ total (short.dir.)}} = A_{s(2-2)} \cdot L$$

**(6) Spacing and steel distribution:****(a) Steel in long – direction:**

Steel in long direction (*measured as that for square footing*). So it shall be distributed uniformly across entire width of footing.

$$\text{Number of bars: } N = \frac{A_{s \text{ total (long dir.)}}}{\text{Area of bar}} \quad ; \quad \text{Spacing (c/c)} = \frac{B - 15 \text{ cm (concrete cover)}}{N - 1}$$

**(b) Steel in short – direction:**

In short direction a portion of total steel (more steel) must be distributed uniformly under the column (centered on centerline of column or pedestal) within a distance (B) at S %:

$S\% = \frac{2}{(L/B) + 1}$  .....(% of steel located within B distance) and the remainder of steel is distributed uniformly between  $(\frac{L - B - 15\text{cm}}{2})$  each direction.

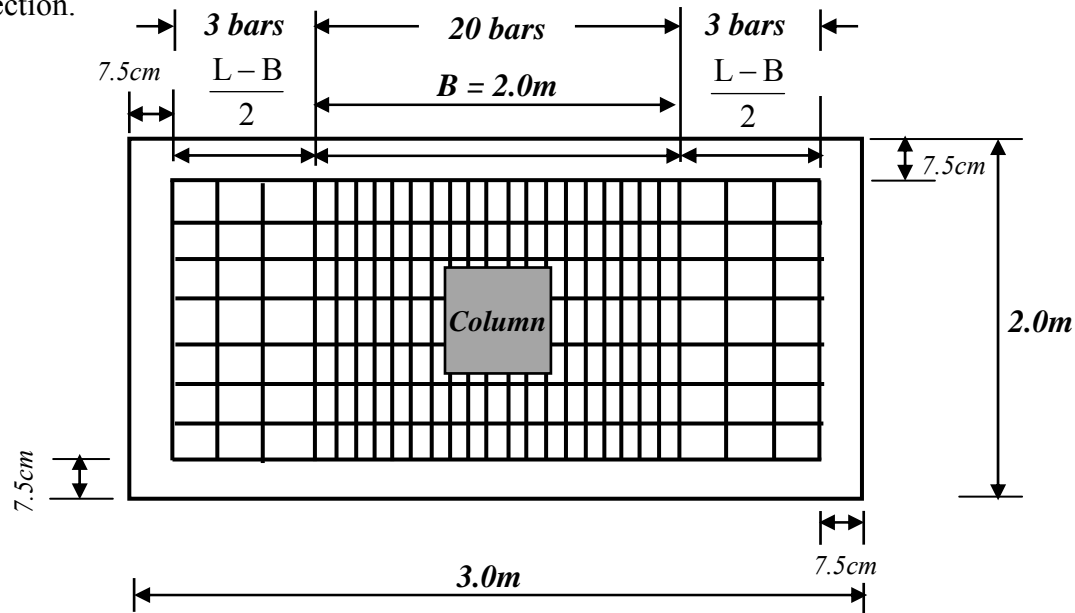
**Example:**

Assume total number of bars required (in short direction) = 25 bars with  $L = 3\text{m}$  and  $B = 2\text{m}$

$$\% \text{ of steel located within } (B) = S\% = \frac{2}{(3/2) + 1} = 0.8\%$$

Number of steel bars within (B) =  $0.8 (25 \text{ bars}) = 20 \text{ bars}$ .

$25 - 20 = 5 \text{ bars}$ ; use 6 bars divided between  $(\frac{L - B - 15\text{cm}}{2}) = \frac{300 - 200 - 15}{2} = 42.5 \text{ cm}$  each direction.

**(7) Check the bond:****(a) Steel in tension:**

$$l_{d(\text{available}) \text{ long direction}} = X_1 - 7.5 \text{ (concrete cover)}$$

$$l_{d(\text{available}) \text{ short direction}} = X_2 - 7.5 \text{ (concrete cover)}$$

$l_{d(\text{required})}$  either ACI Code 318–14 (12.1 or 12.2.2) Eq. is used provided that  $l_d \geq 300 \text{ mm}$ .

$$l_{d(\text{required})} = \frac{f_y}{1.1 \lambda \sqrt{f'_c}} \frac{\Psi_l \Psi_e \Psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)} d_b \dots\dots\dots (\text{ACI 318-14 section 12.1})$$

where,

$f_y$  = yield strength of steel reinforcement,

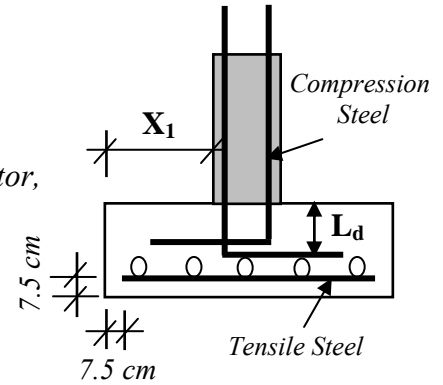
$\Psi_l$  = bar location factor,  $\Psi_e$  = coating factor,  $\Psi_s$  = bar size factor,

$\lambda$  = lightweight-aggregate concrete factor,

$f'_c$  = compressive strength of concrete,

$c_b$  = spacing or cover dimension,

$K_{tr}$  = transverse reinforcement index, and  $d_b$  = bar diameter.



**Note:** If  $l_{d(\text{available})} < l_{d(\text{required})}$  short direction is unsafe ; use Hooks.

### (b) Steel in compression:

$l_{d(\text{available})} = (d)$  of footing

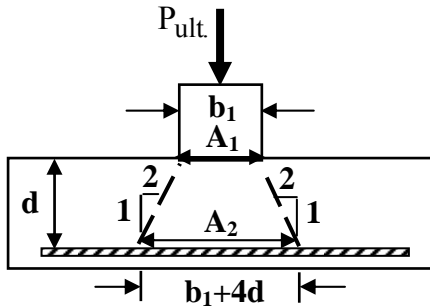
$$l_{dc(\text{required})} = \frac{0.24 f_y}{\lambda \sqrt{f'_c}} d_b \geq (0.043 f_y) d_b \dots\dots\dots (\text{ACI 318-14 section 12.3.2})$$

provided that  $l_d \geq 200$  mm.

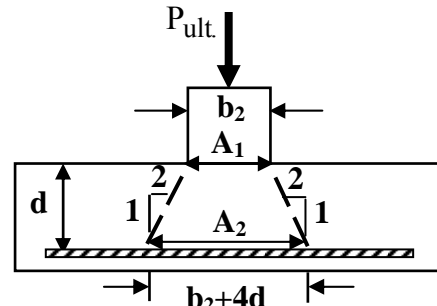
### (8) Check contact pressure between the column and footing:

$$f_c(\text{actual}) = \frac{P_{\text{ult.}}}{A_1}$$

$$f_c(\text{allowable}) = 0.85 \phi f'_c \sqrt{A_2/A_1} \leq 0.85 \phi f'_c ; \text{ where, } \phi = 0.65 \text{ and } \sqrt{A_2/A_1} \leq 2.0$$



Long direction



Short direction

$$A_1 = \text{area of contact (or column)} = b_1 \times b_2. \quad A_2 = \text{projection area} = (b_1 + 4d)(b_2 + 4d)$$

**Note:** If  $f_c(\text{actual}) > f_c(\text{allowable})$ : (1) Increase the section of column, or (2) Design dowels for excess load.

$$A_s \text{ of dowels} = \frac{[f_c(\text{actual}) - f_c(\text{allowable})] A_1}{0.9 f_y}$$

**(9) Design the dowels:**

Minimum number of dowels = **4** for square or rectangular columns.

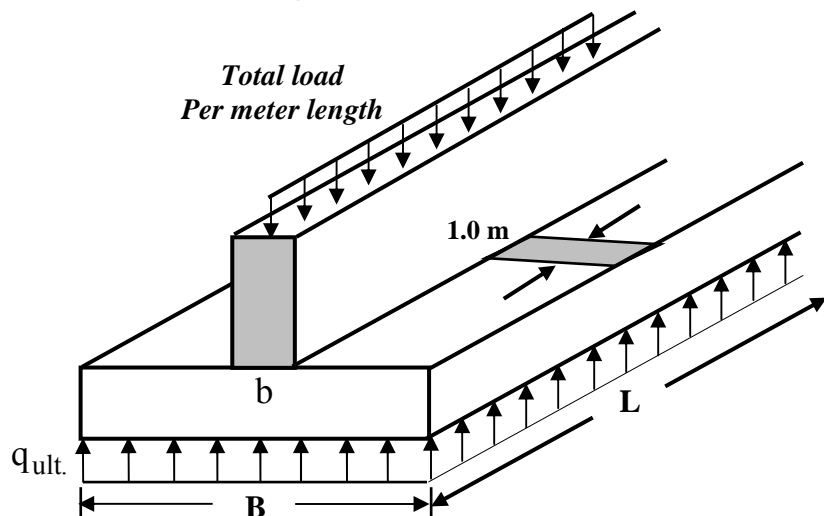
Minimum number of dowels = **6** for circular columns.

Minimum  $A_s$  of dowels =  $0.005 A_1$  .....ACI 318-14 sec.(15.8.2.1)

where,  $A_1$  = area of column.

**(10) Draw sketches (plan and cross sections) showing all details required for construction.****6.1.3 Design Steps of Wall Spread Footings****(1) Width of footing:**

$$B = \frac{\sum P_{actual} / \text{meter length}}{q_{all (net)}}$$

**(2) Convert the loads into ultimate per meter length:**

$$P_{ult.} = 1.4 \text{ DL}$$

$$P_{ult.} = 1.2 \text{ DL} + 1.6 \text{ LL}$$

$$P_{ult.} = 1.2 \text{ DL} + 1.6 \text{ WL} + 1.0 \text{ LL}$$

$$P_{ult.} = 0.9 \text{ DL} + 1.6 \text{ WL}$$

$$P_{ult.} = 1.2 \text{ DL} + 1.0 \text{ EL} + 1.0 \text{ LL}$$

$$P_{ult.} = 0.9 \text{ DL} + 1.0 \text{ EL}$$

..... (ACI 318–14 Section 9.2)

$$q_{ult./m} = \frac{P_{ult./m (control)}}{B}$$

- (3) Check: (a) bearing capacity [ $S.F. \geq 3.0$ ]; (b) settlement [ $S \leq S_{allowable}$ ].  
 (4) Determine the thickness of footing:

**Check wide-beam shear at (d) from face of wall:**

$$v_{c all.} = 0.17 \phi \sqrt{f'_c} \dots \dots \dots (\text{ACI 318-14 section 11.3})$$

where,  $\phi = 0.75$

$$v_{c actual} = \frac{q_{ult.} L_1}{d} \text{ where, } L_1 = \frac{B-b}{2} - d$$

Put  $v_{c actual} = v_{c all.}$  and solve for (d).

Minimum (d) for reinforced concrete wall = 15 cm.

- (5) Determine the required steel for each direction:

**(a) Steel in short- direction (Main steel):**

$$M_u = \frac{q_{ult.} \cdot X_1^2}{2}$$

where,  $X_1 = \frac{B-b}{2} \dots \dots \dots \text{for concrete wall.}$

or  $X_1 = \frac{B-b}{2} + \frac{b}{4} \dots \dots \dots \text{for masonry wall.}$

$$A_s = \frac{M_u}{0.9 \cdot f_y \cdot 0.9d} \text{ (per meter length)}$$

$$A_{s min.} = \rho_{min.} \cdot b \cdot d \text{ (per meter length)}$$

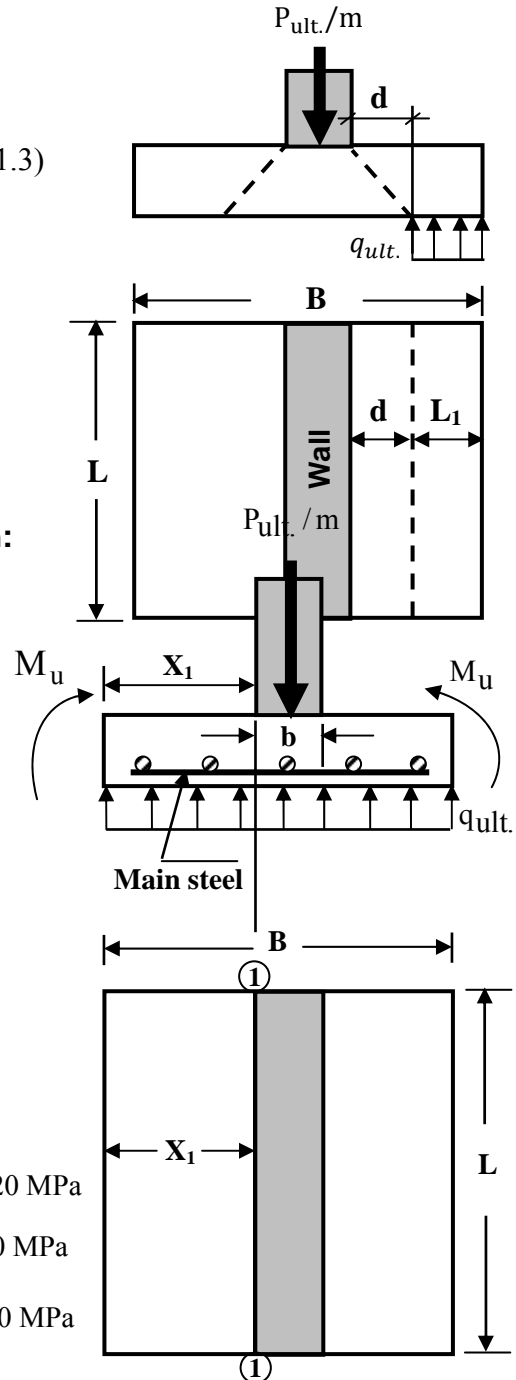
where,  $\rho_{min.}$  is the larger of:  $1.4/f_y$  or  $0.25\sqrt{f'_c}/f_y$

$$\begin{aligned} A_{s min. (Temp. \& shrinkage)} &= 0.0020 b t \text{ ----- for } f_y < 420 \text{ MPa} \\ &= 0.0018 b t \text{ ----- for } f_y = 420 \text{ MPa} \\ &= \frac{0.0018 \times 420}{f_y} b t \text{ - for } f_y > 420 \text{ MPa} \end{aligned}$$

where,  $t = d + \frac{d_b}{2} + \text{concrete cover}$

Compare  $A_s$  with  $A_{s min.}$  and take the larger value for design.

$$A_{s total} = A_s (\text{per meter}) \times L$$



**(b) Steel in long– direction:**

Use  $A_{s\min.} = \rho_{\min.} \cdot b \cdot d$  (Per meter length)

$$A_{s\text{ total}} = A_{s\text{ min. (per meter)}} \times B$$

**(6) Spacing and steel distribution:****(a) Steel in short – direction:**

$$\text{Number of bars: } N = \frac{A_{s\text{ total}}}{\text{Area of bar}} ; \quad \text{Spacing (c/c)} = \frac{L - 15 \text{ cm (concrete cover)}}{N-1}$$

**(b) Steel in long – direction:**

$$\text{Number of bars: } N = \frac{A_{s\text{ total}}}{\text{Area of bar}} ; \quad \text{Spacing (c/c)} = \frac{B - 15 \text{ cm (concrete cover)}}{N-1}$$

**(7) Check the bond:****(a) Steel in tension:**

$$l_{d(\text{available}) \text{ short direction}} = X_1 - 7.5 \text{ (concrete cover)}$$

$l_{d(\text{required})}$  either ACI 318–14 code (12.1 or 12.2.2) Eq. is used provided that  $l_d \geq 300 \text{ mm}$

$$l_{d(\text{required})} = \frac{f_y}{1.1 \lambda \sqrt{f'_c}} \left( \frac{\Psi_l \Psi_e \Psi_s}{\left( \frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \dots\dots\dots (\text{ACI 318–14 section 12.1})$$

where,

$f_y$  = yield strength of steel reinforcement,

$\Psi_l$  = bar location factor,  $\Psi_e$  = coating factor,  $\Psi_s$  = bar size factor,

$\lambda$  = lightweight–aggregate concrete factor,

$f'_c$  = compressive strength of concrete,

$c_b$  = spacing or cover dimension,

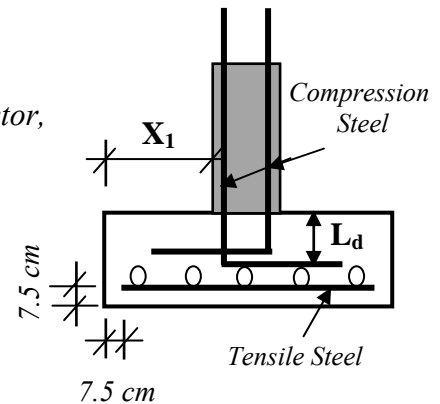
$K_{tr}$  = transverse reinforcement index, and  $d_b$  = bar diameter.

**(b) Steel in compression:**

$$l_{d(\text{available})} = (d) \text{ of footing}$$

$$l_{dc(\text{required})} = \frac{0.24 f_y}{\lambda \sqrt{f'_c}} d_b \geq (0.043 f_y) d_b \dots\dots\dots (\text{ACI 318–14 section 12.3.2})$$

provided that  $l_d \geq 200 \text{ mm}$ .

**(8) Draw sketches (plan and cross sections).**

## 6.2 DESIGN STEPS OF ECCENTRIC SPREAD FOOTINGS

### (1) Find the area of footing:

- Assume footing width ( $B$ ) and footing thickness ( $t$ ).
- Calculate  $q_{all(net)} \text{ DL+LL+WL} = q_{all(DL+LL+WL)} - t_f \gamma_{conc.} - (D_f - t_f) \gamma_{soil}$
- Find  $e = \frac{M}{\Sigma P}$ ; where,  $M$  = Overturning moment measured at base of footing,  
 $\Sigma P = \text{D.L.} + \text{L.L.} + \text{any other vertical loads.}$
- Assume  $e < \frac{L}{6}$  and obtain  $q_{max} = \frac{\Sigma P}{B L} \left[ 1 + \frac{6e}{L} \right]$ .
- Set  $q_{max} = q_{all(net)}$  and solve for  $L$  or  $B$  by trial and error. For square footing;  $L=B$ , for rectangular footing; choose  $B$  and  $L$  such that  $(L/B \leq 2.0)$ , and for wall footing; area =  $B \times 1.0m$ .
- Calculate  $q_{min.}^{max.} = \frac{\Sigma P}{B L} \left[ 1 \pm \frac{6e}{L} \right]$  and check that  $q_{max} \leq q_{all(net)}$  and  $q_{min} \geq 0$ .

### (2) Determine the factored loads and stresses:

$$\begin{array}{l}
 P_{ult.} = 1.4 \text{ DL} \\
 P_{ult.} = 1.2 \text{ DL} + 1.6 \text{ LL} \\
 P_{ult.} = 1.2 \text{ DL} + 1.6 \text{ WL} + 1.0 \text{ LL} \\
 P_{ult.} = 0.9 \text{ DL} + 1.6 \text{ WL} \\
 P_{ult.} = 1.2 \text{ DL} + 1.0 \text{ EL} + 1.0 \text{ LL} \\
 P_{ult.} = 0.9 \text{ DL} + 1.0 \text{ EL}
 \end{array}
 \left. \vphantom{\begin{array}{l} P_{ult.} = 1.4 \text{ DL} \\ P_{ult.} = 1.2 \text{ DL} + 1.6 \text{ LL} \\ P_{ult.} = 1.2 \text{ DL} + 1.6 \text{ WL} + 1.0 \text{ LL} \\ P_{ult.} = 0.9 \text{ DL} + 1.6 \text{ WL} \\ P_{ult.} = 1.2 \text{ DL} + 1.0 \text{ EL} + 1.0 \text{ LL} \\ P_{ult.} = 0.9 \text{ DL} + 1.0 \text{ EL} \end{array}} \right\} \dots\dots\dots (\text{ACI 318-14 Section 9.2})$$

$$q_{ult.(max.)} = q_{max.} \left( \frac{P_{ult.(control)}}{\Sigma P} \right); \quad q_{ult.(min.)} = q_{min.} \left( \frac{P_{ult.(control)}}{\Sigma P} \right)$$

### (3) Check: Bearing capacity [ $S.F. \geq 3.0$ ] and Settlement [ $S \leq S_{allowable}$ ].

**(4) Determine the thickness of footing:****(i) Check one-way or wide-beam shear at distance (d) from face of column:**

$$v_{c \text{ all.}} = 0.17 \phi \sqrt{f'_c} \dots\dots\dots (\text{ACI 318-14 section 11.3})$$

$$\text{where, } \phi = 0.75; \quad v_{c \text{ actual}} = \frac{V}{d}$$

**From right side:**  $V = q_{\text{av.}} L_1$

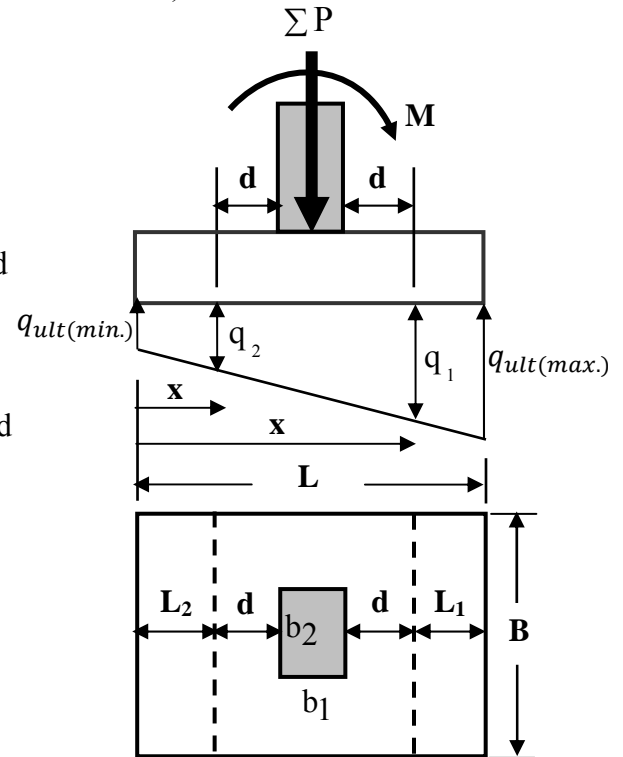
$$\text{where, } q_{\text{av.}} = \frac{q_{\text{max.}} + q_1}{2} \quad \text{and} \quad L_1 = \frac{L - b_1}{2} - d$$

**From left side:**  $V = q_{\text{av.}} L_2$

$$\text{where, } q_{\text{av.}} = \frac{q_{\text{min.}} + q_2}{2} \quad \text{and} \quad L_2 = \frac{L - b_1}{2} - d$$

$$\text{Here; } \begin{aligned} q_1 &= q_{\text{min.}} + (q_{\text{max.}} - q_{\text{min.}}) \frac{x}{L} \\ q_2 &= q_{\text{min.}} + (q_{\text{max.}} - q_{\text{min.}}) \frac{x}{L} \end{aligned}$$

**Set  $v_{c \text{ actual}} = v_{c \text{ all.}}$  and solve for (d).**

**(ii) Check two-way action or punching shear at (d/2) from column face:**

$$v_{c \text{ all.}} = 0.17 \left(1 + \frac{2}{\beta}\right) \phi \sqrt{f'_c} \dots\dots\dots \text{ACI 318-14 section 11.31..... (a)}$$

$$v_{c \text{ all.}} = 0.083 \left(\frac{\alpha_s d}{b_o} + 2\right) \phi \sqrt{f'_c} \dots\dots\dots \text{ACI 318-14 section 11.32..... (b)}$$

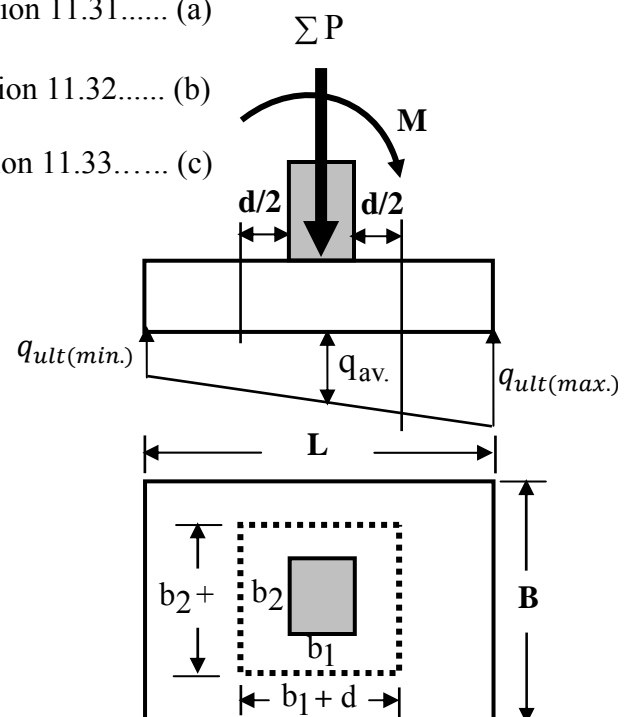
$$v_{c \text{ all.}} = 0.33 \phi \sqrt{f'_c} \dots\dots\dots \text{ACI 318-14 section 11.33..... (c)}$$

Take the smaller value from (a), (b) and (c).

where,

$$\beta = \frac{\text{length of column}}{\text{width of column}} = \frac{b_1}{b_2} \text{ or } \frac{b_2}{b_1}, \quad \phi = 0.75$$

$\alpha_s = 50 \text{ cm}$  for corner columns,





= 75 cm for edge columns,  
 = 100 cm for interior columns.

- **for square column:**  $v_{c \text{ actual}} = \frac{q_{av.} [A_f - (b+d)^2]}{4(b+d)d}$
- **for rectangular column:**  $v_{c \text{ actual}} = \frac{q_{av.} [A_f - (b_1+d)(b_2+d)]}{[2(b_1+d) + 2(b_2+d)]d}$
- **for circular column:**  $v_{c \text{ actual}} = \frac{q_{av.} [A_f - \pi \left(\frac{b+d}{2}\right)^2]}{\pi(b+d)d}$

where,  $q_{av.} = \frac{q_{max.} + q_{min.}}{2}$

Put  $v_{c \text{ actual}} = v_{c \text{ all}}$  and solve for (d), then take the larger value of (d) obtained from (i) or (ii).

**(5) Determine the required steel for each direction:**

• **Steel in long – direction:**

Calculate moments at column faces from right and left sides. Then take the maximum moment for design.

**(a) Moment from right side:**

$$M_{u(1-1)R} = M_1 + M_2$$

where,

$$M_1 = \frac{q_1 \cdot X_1^2}{2}; \quad M_2 = \frac{(q_{max.} - q_1)}{2} X_1 \frac{2}{3} X_1$$

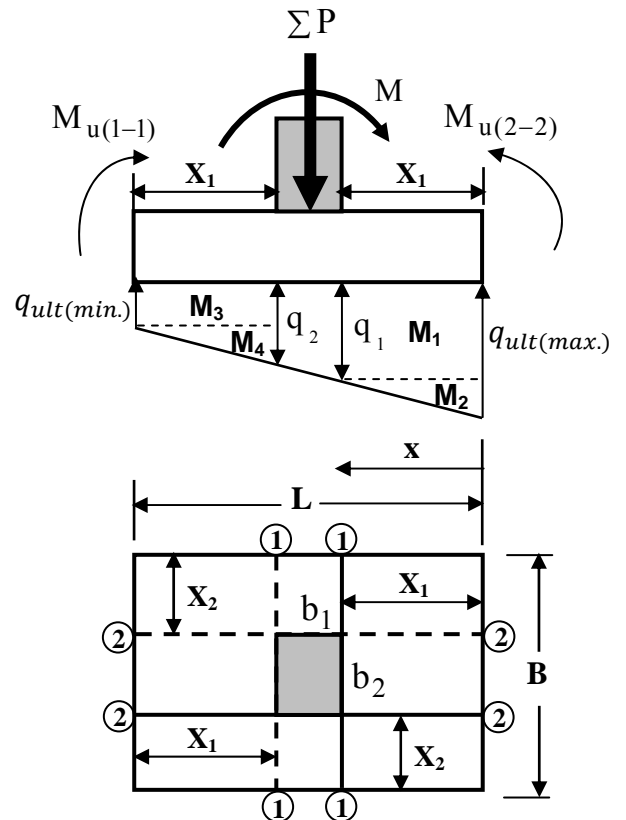
$$\text{and } X_1 = \frac{L - b_1}{2}$$

**(b) Moment from left side:**

$$M_{u(1-1)L} = M_3 + M_4$$

where,

$$M_3 = \frac{q_{min.} \cdot X_1^2}{2}; \quad M_4 = \frac{(q_2 - q_{min.})}{2} X_1 \frac{X_1}{3}$$



$$A_{s(1-1)} = \frac{\text{Max. of } (M_{u(1-1)R} \text{ or } M_{u(1-1)L})}{0.9 f_y 0.9 d}$$

$$A_{s \min.} = \rho_{\min.} b d \quad \text{where, } \rho_{\min.} \text{ is the larger of: } 1.4/f_y \text{ or } 0.25\sqrt{f'_c}/f_y$$

$$\begin{aligned} A_{s \min.}(\text{Temp. \& shrinkage}) &= 0.0020 b t \quad \text{----- for } f_y < 420 \text{ MPa} \\ &= 0.0018 b t \quad \text{----- for } f_y = 420 \text{ MPa} \\ &= \frac{0.0018 \times 420}{f_y} b t \text{ - for } f_y > 420 \text{ MPa} \end{aligned}$$

$$\text{where, } t = d + \frac{d_b}{2} + \text{concrete cover}$$

**Compare  $A_{s(1-1)}$  with  $A_{s \min.}$  and take the larger value for design.**

$$A_{s \text{ total (Long direction)}} = (A_{s(1-1)} \text{ or } A_{s \min.}) \times B$$

• **Steel in short – direction:**

$$M_{u(2-2)} = \frac{q_{\text{av.}} X_2^2}{2}; \quad \text{where, } q_{\text{av.}} = \frac{q_{\text{max.}} + q_{\text{min.}}}{2}, \quad X_2 = \frac{B - b_2}{2}$$

$$A_{s(2-2)} = \frac{M_{u(2-2)}}{0.9 f_y 0.9 d}$$

**Compare  $A_{s(2-2)}$  with  $A_{s \min.}$  and take the larger value for design.**

$$A_{s \text{ total (Short direction)}} = (A_{s(2-2)} \text{ or } A_{s \min.}) \times L$$

**(6) Spacing and steel distribution:**

• **Steel in long – direction:**

*For square or rectangular footings, steel reinforcement in long direction is measured as:*

$$\text{Number of bars: } N = \frac{A_{s \text{ total (Long direction)}}}{\text{Area of bar}}$$

$$\text{Spacing (c/c)} = \frac{B - 15 \text{ cm (concrete cover)}}{N - 1}$$

• **Steel in short – direction:**

For square footing: Use same steel as that for long direction (same steel in both directions).

For rectangular footing: % of  $A_{s \text{ total}}$  (Short direction) must be located within  $B$  distance at

$$S\% = \frac{2}{(L/B) + 1} \text{ and the remainder of steel is distributed uniformly between } \left(\frac{L - B - 15\text{cm}}{2}\right)$$

each direction.

### (7) Check the bond:

#### (a) Steel in tension:

$$l_{d(\text{available})} \text{ Long direction} = X_1 - 7.5 \text{ (concrete cover)}$$

$$l_{d(\text{available})} \text{ Short direction} = X_2 - 7.5 \text{ (concrete cover)}$$

$$l_{d(\text{required})} = \frac{f_y}{1.1 \lambda \sqrt{f'_c}} \frac{\Psi_l \Psi_e \Psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)} d_b \dots \dots \dots \text{ (ACI 318-14 section 12.1)}$$

provided that  $l_d \geq 300 \text{ mm}$ .

where,

$f_y$  = yield strength of steel reinforcement,

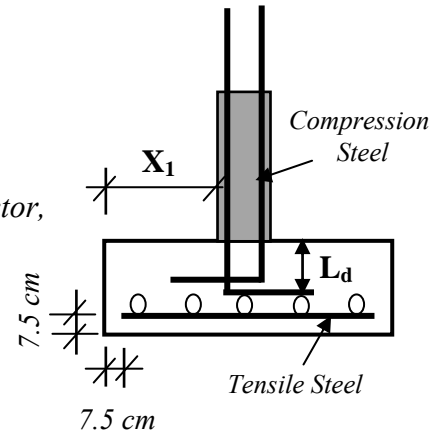
$\Psi_l$  = bar location factor,  $\Psi_e$  = coating factor,  $\Psi_s$  = bar size factor,

$\lambda$  = lightweight-aggregate concrete factor,

$f'_c$  = compressive strength of concrete,

$c_b$  = spacing or cover dimension,

$K_{tr}$  = transverse reinforcement index, and  $d_b$  = bar diameter.



#### (b) Steel in compression:

$$l_{d(\text{available})} = (d) \text{ of footing}$$

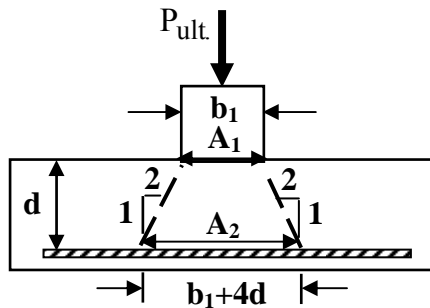
$$l_{dc(\text{required})} = \frac{0.24 f_y}{\lambda \sqrt{f'_c}} d_b \geq (0.043 f_y) d_b \dots \dots \dots \text{ (ACI 318-14 section 12.3.2)}$$

provided that  $l_d \geq 200 \text{ mm}$ .

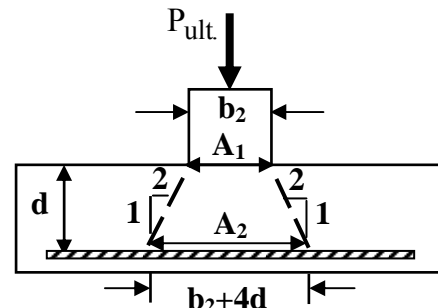
### (8) Check contact pressure between the column and footing:

$$f_c (\text{actual}) = \frac{P_{ult.}}{A_1}$$

$$f_{c(allowable)} = 0.85 \phi f'_c \sqrt{A_2/A_1} \leq 0.85 \phi f'_c ; \text{ where, } \phi = 0.65 \text{ and } \sqrt{A_2/A_1} \leq 2.0$$



Long direction



Short direction

$$A_1 = \text{area of contact (or column)} = b_1 \times b_2. \quad A_2 = \text{projection area} = (b_1 + 4d)(b_2 + 4d)$$

**Note:** If  $f_{c(actual)} > f_{c(allowable)}$ : (1) Increase the section of column, or (2) Design dowels for excess load.

$$A_s \text{ of dowels} = \frac{[f_{c(actual)} - f_{c(allowable)}] A_1}{0.9 f_y}$$

**(9) Design the dowels:**

Minimum number of dowels = **4** for square or rectangular columns.

Minimum number of dowels = **6** for circular columns.

Minimum  $A_s$  of dowels =  $0.005 A_1$  .....ACI 318-14 sec.(15.8.2.1)

where,  $A_1$  = area of column.

**(10) Draw sketches (plan and cross sections) showing all details required for construction.**

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# CHAPTER 6

## STRUCTURAL DESIGN OF FOOTINGS

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### 6.11 DESIGN OF COMBINED FOOTINGS

Combined footings are of three types: (a) Rectangular combined footings, (b) Trapezoidal combined footings, and (c) Strap combined footings.

#### 6.11.1 Design of Rectangular Combined Footings

This type of footings is used when it is possible to make the resultant of loads passes through the centroid of the footing (i.e., the foundation can be extended beyond the property line such that two or more columns can be supported on a single rectangular footing).

##### Steps of Design:

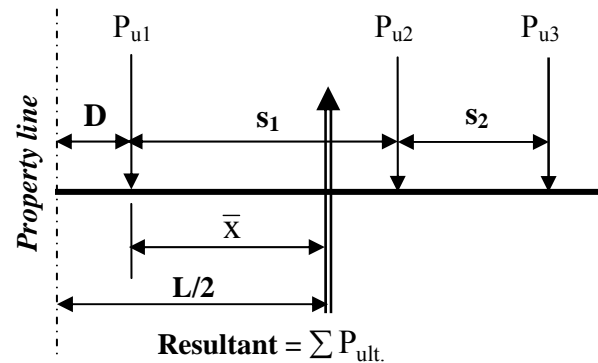
**(1) Convert the loads into ultimate:**

Allowable load:  $\sum P = P_1 + P_2 + P_3$ ;

ultimate load:  $\sum P_{ult.} = P_{u1} + P_{u2} + P_{u3}$ ;

$$\text{ultimate ratio.}(r_u) = \frac{\sum P_{ult.}}{\sum P}$$

Ultimate applied pressure ( $q_u$ ) or  $q_{all} \text{ (factored)}$ :  $q_u = q_a \cdot (r_u)$



**(2) Area and soil pressure distribution per linear meter of the footing:**

The ( $L$ ) dimension is calculated so that the soil pressure is uniform. Therefore, the resultant is at  $L/2$  as shown below:

Take moment at column (1):

$$\sum M_{col..(1)} = 0$$

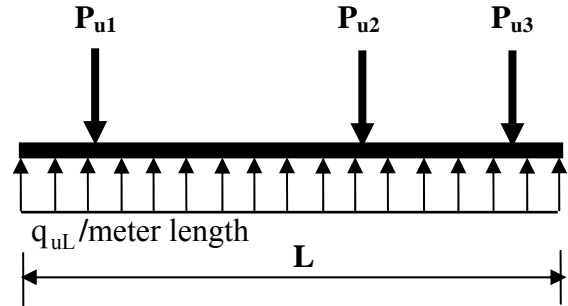
$$P_{u2} \cdot s_1 + P_{u3} \cdot (s_1 + s_2) = \sum P_{ult.} \cdot (\bar{x})$$

Solve for  $\bar{x} = ?$

$$\text{Length of footing: } L = 2(D + \bar{x})$$

$$\text{Ultimate soil pressure per meter } (q_{uL}) = \frac{\sum P_{ult.}}{L}$$

$$\text{Width of footing: } B = \frac{q_{uL}}{q_{all. (factored)}}$$

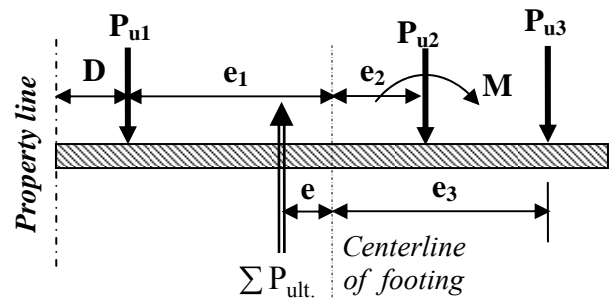


**Note:** If any column is subjected to bending moment, the effect of moment should be taken into account as shown below. Then, try to make the centroid of footing coincide with the line of action of the resultant (i.e., choose  $L$ ) such that  $e = 0$ ).

$$e = \frac{P_{u2} \cdot e_2 + P_{u3} \cdot e_3 + M - P_{u1} \cdot e_1}{\sum P_{ult.}}$$

and

$$L = 2(D + e_1 - e)$$



However, if the resultant of loads does not pass through the centroid of footing; the soil pressure will be non-uniform and determined from:

$$q_{min.}^{max.} = \frac{\sum P}{B \cdot L} \left[ 1 \pm \frac{6 \cdot e}{L} \right] \dots \dots \dots \text{for } e \leq \frac{L}{6}$$

or

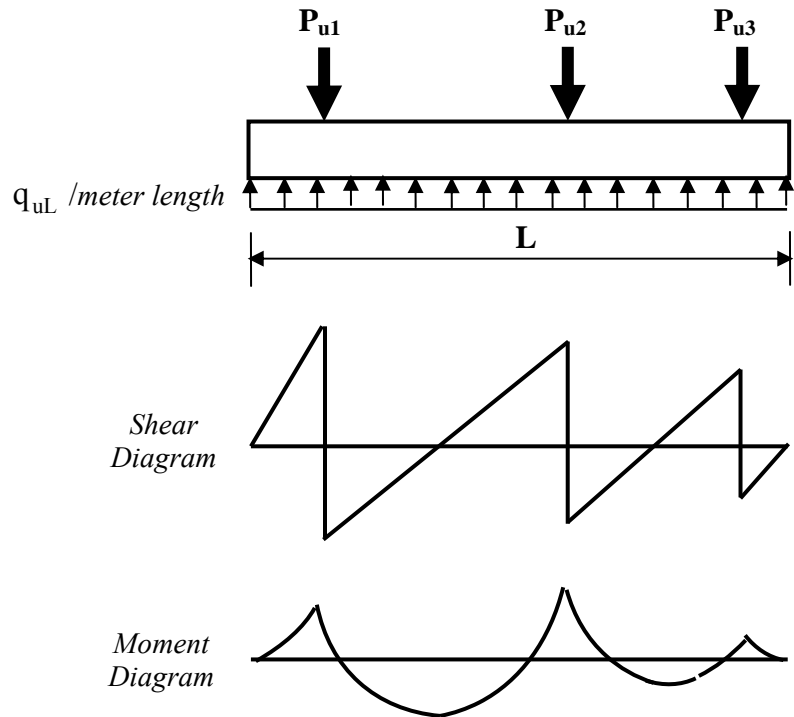
$$q_{max.} = \frac{2 \sum P}{3 \left( \frac{L}{2} - e \right)} \dots \dots \dots \text{for } e > \frac{L}{6}$$

**(3) Check: (a) Bearing capacity [S.F.  $\geq 3.0$ ] and (b) Settlement [ $S \leq S_{allowable}$ ].**

**(4) Draw shear and moment diagrams in L–direction:**

$$V = \text{area of load} = \int_0^x q \cdot dx$$

$$M = \text{area of shear} = \int_0^x V \cdot dx$$



- For negative moment use steel on top.
- For positive moment use steel on bottom.

*Then proceeds with other design steps as before:*

- (5) Determine the thickness of footing.
- (6) Determine steel reinforcement in each direction.
- (7) Check the bond in short direction.
- (8) Check the bearing pressure between the columns and footing.
- (9) Design the dowels.
- (10) Sketch the footing showing all details required for construction.

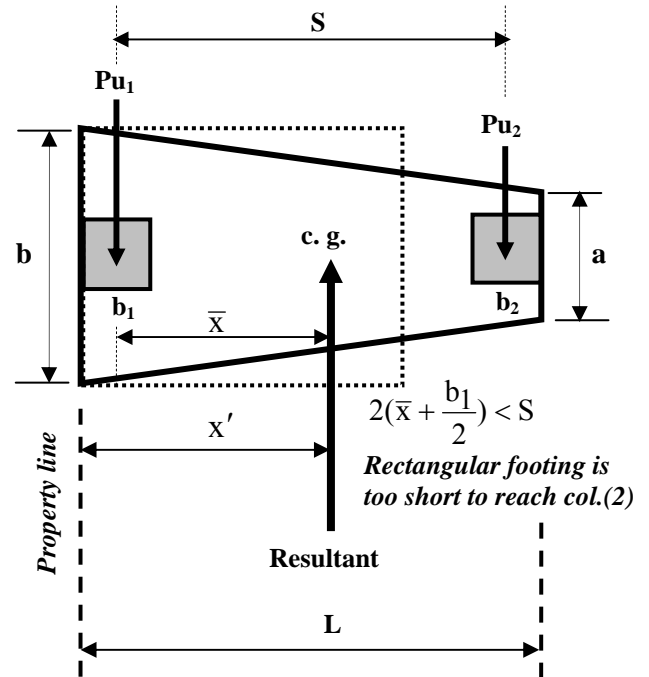
### 6.11.2 Design of Trapezoidal Combined Footings

This type of footing is used in two cases:-

- (1) If the column which has too limited space carries the maximum load. In this case, the resultant of the column loads (including moments) will be closer to the larger column load; and doubling the centroid distance as done for the rectangular footing will not provide sufficient length to reach the interior column as shown in the figure beside.
- (2) If  $\frac{L}{3} < x' < \frac{L}{2}$ ; this limitation is derived from the property of a trapezoid as follows:

$$\text{Area} = A = \left(\frac{a+b}{2}\right).L$$

$$\text{Centroid} = x' = \frac{L}{3} \left( \frac{2a+b}{a+b} \right)$$



It is seen that the solution for ( $a = 0$ ) is a triangle and for ( $a = b$ ) is a rectangle, therefore, a trapezoid solution exists only for  $\frac{L}{3} < x' < \frac{L}{2}$ .

In most cases, trapezoidal footing is used with only two columns, however, it can be used for more than two columns. But, due to variable reinforcing steel required for variable footing width and variable soil pressure, the strap footing is preferred in comparison with trapezoidal one.

#### Steps of Design:

- (1) **Convert the loads into ultimate:**

$$\text{Allowable loads: } \Sigma P = (DL + LL)_{\text{col.1}} + (DL + LL)_{\text{col.2}}$$

$$P_{u1} = 1.2.DL + 1.6.LL$$

$$P_{u2} = 1.2.DL + 1.6.LL$$

$$\text{Ultimate loads: } \Sigma P_{\text{ult.}} = P_{u1} + P_{u2}$$

$$\text{Ultimate ratio.}(r_u) = \frac{\Sigma P_{\text{ult.}}}{\Sigma P}$$

$$\text{Ultimate applied pressure } (q_{\text{ult.}}) \text{ or } q_{\text{all (factored)}}: q_u = q_a \cdot (r_u)$$



## (2) Area and dimensions of the footing:

- **Locate the center of footing:**

Taking moment about column (1); gives  $\bar{x} = ?$ :

$$P_{u2} \cdot S = (P_{u1} + P_{u2}) \cdot \bar{x}; \quad \bar{x} = ?$$

$$x' = \bar{x} + \frac{b_1}{2} \dots\dots\dots (i)$$

But, from moment of areas about the big end:

$$x' = \frac{L}{3} \left( \frac{2a+b}{a+b} \right) \dots\dots\dots (ii)$$

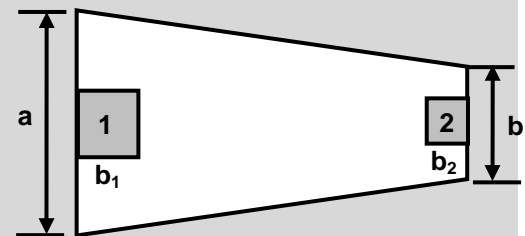
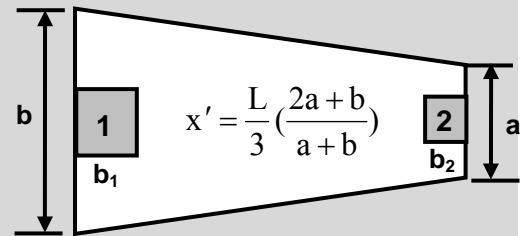
Equating (i) and (ii) gives:

$$\bar{x} + \frac{b_1}{2} = \frac{L}{3} \left( \frac{2a+b}{a+b} \right) \dots\dots\dots (iii)$$

- **Determine the area of footing:**

$$A = \frac{\sum P_{ult.}}{q_{ult.}} = \left( \frac{a+b}{2} \right) L \dots\dots\dots (iv)$$

By solving Eqs. (iii) and (iv), determine **a** and **b**.



**Note:** If large end dimension = **a** and small end = **b**; then

$$x' = \frac{L}{3} \left( \frac{2b+a}{a+b} \right)$$

(3) Check: (a) Bearing capacity [S.F.  $\geq 3.0$ ] and (b) Settlement [ $S \leq S_{allowable}$ ].

## (4) Draw shear and moment diagrams and determine soil pressure at sectional points below the footing:

$$\text{Soil pressure at big end} = q_{ult.} (b) \dots\dots(kN/m)$$

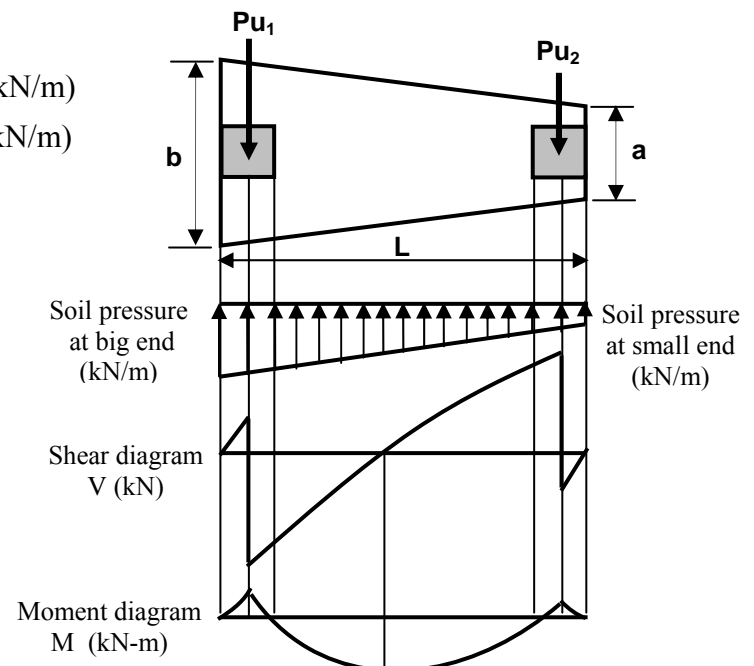
$$\text{Soil pressure at small end} = q_{ult.} (a) \dots\dots(kN/m)$$

**Shear:**  $V = \text{area of load} = \int_0^x q \cdot dx$

**Moment:**  $M = \text{area of shear} = \int_0^x V \cdot dx$

for +ve. moment put steel on bottom.

for -ve. moment put steel on top.



## (5) Determine the thickness of footing.

## (a) Check wide-beam shear at (d) from columns faces:

$$v_{c \text{ all.}} = 0.17 \phi \sqrt{f'_c} \quad \text{where, } \phi = 0.75$$

$$v_{c \text{ act.}} = \frac{V_{(\text{variable})}}{B_{(\text{variable})} d}$$

## • Wide-beam shear from small end:

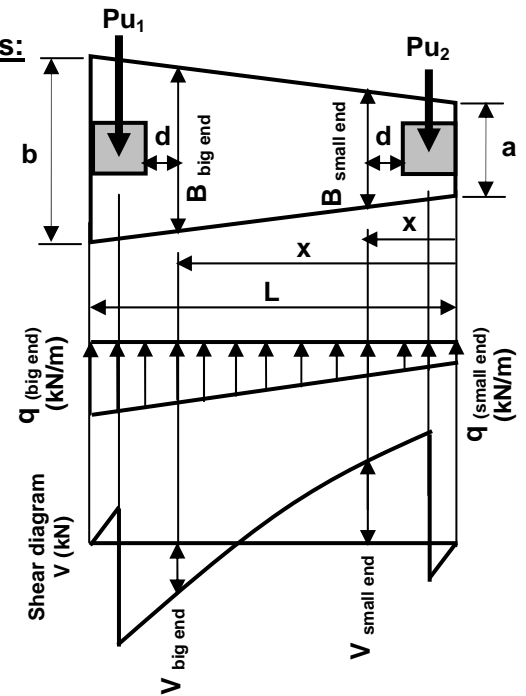
$V = V_{(\text{small end})}$ ; the shear value at (d) from column (2).

$B = B_{(\text{small end})}$ ; the width at (d) from column (2).

## • Wide-beam shear from big end:

$V = V_{(\text{big end})}$ ; the shear value at (d) from column (1).

$B = B_{(\text{big end})}$ ; the width at (d) from column (1).

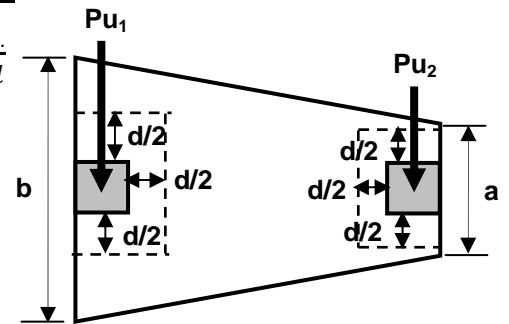


## (b) Check punching shear at (d/2) from columns faces:

$$v_{c \text{ all.}} = 0.33 \phi \sqrt{f'_c} \quad \text{where, } \phi = 0.75 ; v_{c \text{ act.}} = \frac{P_{\text{col.}}}{b_o d}$$

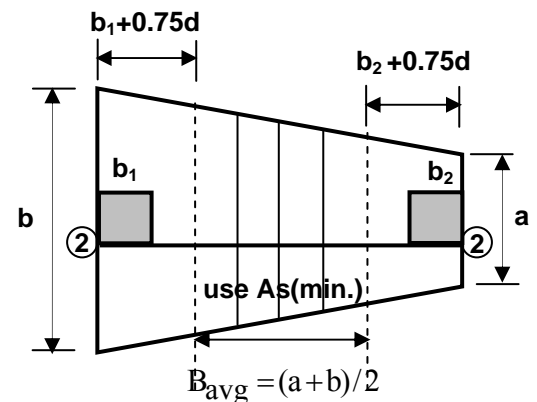
$$\bullet \text{ Punching shear-col.(1): } v_{c \text{ act. col.(1)}} = \frac{P_{u1}}{b_{o \text{ col.(1)}} d}$$

$$\bullet \text{ Punching shear-col.(2): } v_{c \text{ act. col.(2)}} = \frac{P_{u2}}{b_{o \text{ col.(2)}} d}$$



## (6) Determine steel reinforcement in each direction:

	x	$M_{u(1-1)}$	$A_s$	$A_{s \text{ min.}}$	B
L - Direction					variable or take $B_{\text{avg.}}$
B - Direction					



Then proceeds with other design steps as before:

## (7) Check bond in short direction.

## (8) Check bearing pressure between the columns and footing.

## (9) Design the dowels.

## (10) Sketch the footing showing all details required for construction.

### 6.11.3 DESIGN OF STRAP COMBINED FOOTINGS

This type of combined footings is used in the following cases:

- To transmit the moment caused from eccentricity of eccentric loaded column footing to the interior column footing so that a uniform soil pressure will be obtained beneath both footings,
- If the distance between the columns is large so that the case of rectangular combined footing becomes very narrow with high bending moment, and
- If the location of the resultant of loads  $\bar{x} < \frac{L}{3}$ .

Strap footing is found to be more economic than other types of combined footings, however, first try to use rectangular footing, but if not satisfied try to use the trapezoidal one and again if this also not satisfied use strap footing.

#### 6.11.3.1 Limitations of Strap Combined Footings

- $q_1 = q_2$ ,  $B_1 \approx B_2$  in order to prevent or reduce the differential settlement,
- Soil pressure below the strap is zero; (i.e., strap footing should not be in contact with soil),
- $(I_{\text{strap}} / I_{\text{footing}}) \geq 2$ . This rigidity is necessary to prevent the rotation of the exterior footing,
- It is common to neglect strap weight in the design, and
- Check clear span between footing edges to depth ratio; *to see whether it is a deep beam or not (for deep beams, clear span depth ratio  $\leq 4$ )* see ACI Code 318–14 (Art. 10.7).

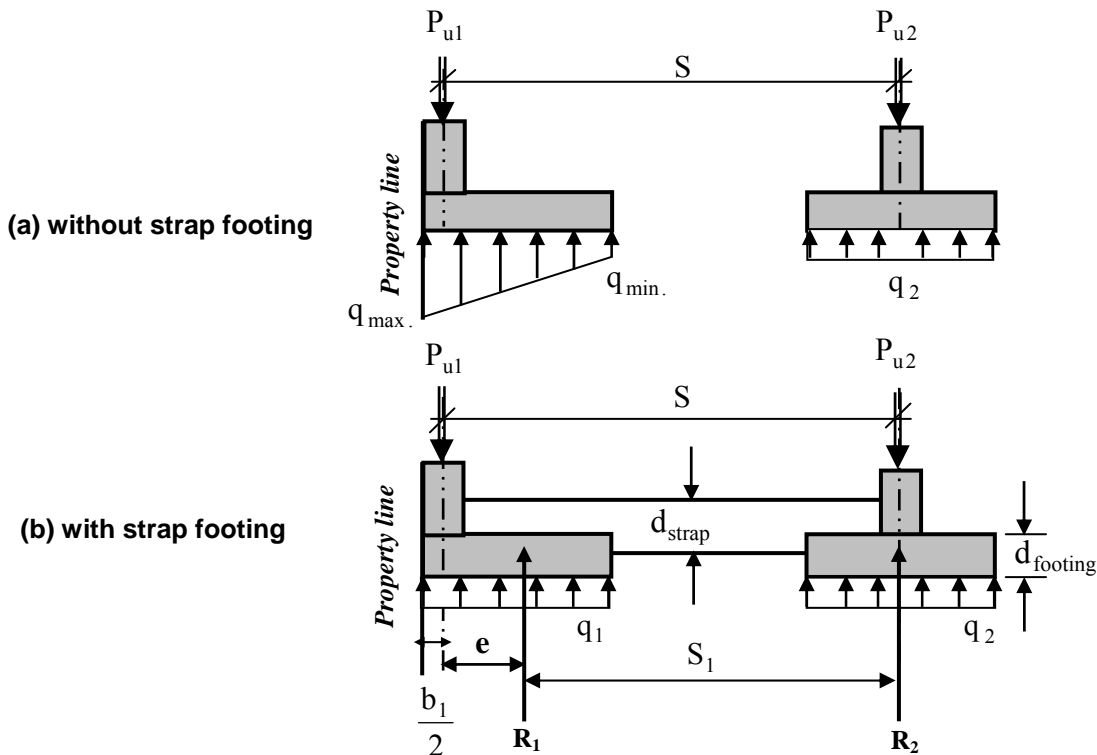


Fig.(6.6): Layout of strap footing.

### 6.11.3.2 Design Steps of Strap Combined Footings

**(1) Convert the loads into ultimate:**

Allowable loads:  $\sum P = (DL + LL)_{col.1} + (DL + LL)_{col.2}$

Ultimate loads:  $P_{u1} = 1.2 DL + 1.6 LL$  ;  $P_{u2} = 1.2 DL + 1.6 LL$

$$\sum P_{ult.} = P_{u1} + P_{u2}$$

$$\text{Ultimate ratio.}(r_u) = \frac{\sum P_{ult.}}{\sum P}$$

Ultimate applied pressure ( $q_{ult.}$ ) or  $q_{all. (factored)}$ :  $q_{ult.} = q_a \cdot (r_u)$

**(2) Find footings dimensions:**

*Referring to Fig.(6.6), assume (e) by trial and error, then find footings reactions as follows:*

Take  $\sum M_{about R_2} = 0$  gives  $R_1 = ?$ :  $P_{u1} \cdot S = R_1 \cdot S_1$ ;  $R_1 = ?$

Take  $\sum M_{about R_1} = 0$  gives  $R_2 = ?$ :  $-P_{u1} \cdot (e) + P_{u2} \cdot S_1 - R_2 \cdot S_1 = 0$ ;  $R_2 = ?$

Check by  $\sum F_v = 0$ :  $R_2 = P_{u1} + P_{u2} - R_1$

Footing dimensions for column (1):  $L_1 = 2(e + \frac{b_1}{2})$ ;  $A_1 = \frac{R_1}{q_{ult.}}$ ;  $B_1 = \frac{A_1}{L_1}$

Footing dimensions for column (2); use square footing:  $A_2 = \frac{R_2}{q_{ult.}}$ ;  $B_2 = \sqrt{A_2}$

Check soil pressure below footings:  $q_1 = \frac{R_1}{B_1 \cdot L_1} \leq q_{ult.}$ ;  $q_2 = \frac{R_2}{B_2 \cdot L_2} \leq q_{ult.}$

**(3) Check: Bearing capacity [S.F.  $\geq 3.0$ ] and Settlement [ $S \leq S_{allowable}$ ].**

**(4) Draw shear and moment diagrams:**

Soil pressure below column footing (1)  $= q_1 \times (B_1) \dots (kN/m)$

Soil pressure below column footing (2)  $= q_2 \times (B_2) \dots (kN/m)$

**Shear:**  $V = \text{area of load} = \int_0^x q \cdot dx$

**Moment:**  $M = \text{area of shear} = \int_0^x V \cdot dx$

*For +ve. moment put steel on bottom; for -ve. moment put steel on top.*

**(5) Determine the thickness of footings.****(a) Check wide-beam shear at (d) from faces of columns:**

$$v_{c \text{ all.}} = 0.17 \phi \sqrt{f'_c} \text{ where, } \phi = 0.75 \text{ .....(ACI 318-14 section 11.3)}$$

$$v_{c \text{ act.}} = \frac{V \text{ (from shear diagram)}}{b d}$$

**(b) Check punching shear at (d/2) from faces of columns:**

$$v_{c \text{ all.}} = 0.33 \phi \sqrt{f'_c} \text{ where, } \phi = 0.75 \text{ .....(ACI 318-14 section 11.33)}$$

$$v_{c \text{ act.}} = \frac{P_{col.}}{b_o d}$$

**(6) Determine the thickness of strap:**

**(a) Check wide-beam shear:**  $v_{c \text{ all.}} = 0.17 \phi \sqrt{f'_c}$  ;  $v_{c \text{ act.}} = \frac{V \text{ (from shear diagram)}}{b_{\text{strap}} d}$

**(b) Check bending moment:**  $M_u = b_{\text{strap}} \cdot d^2 k_m$ ; or  $d = \sqrt{\frac{M_u}{b_{\text{strap}} \cdot k_m}}$

**(c) Check rigidity ratio:**  $(I_{\text{strap}} / I_{\text{footing}}) \geq 2$

**(7) Determine steel reinforcement in footings and strap in each direction:****(a) Steel reinforcement for footing No.(1):**

- Steel at top in long – direction:**

$$A_s = \frac{M_u \text{ ((from } M\text{-diagram)}}}{0.9 f_y 0.9 d}$$

- Steel at bottom in long – direction:**

$$M_{u(1-1)} = \frac{w_L \cdot X_1^2}{2}; \quad A_s = \frac{M_{u(1-1)}}{0.9 \cdot f_y \cdot 0.9 d}$$

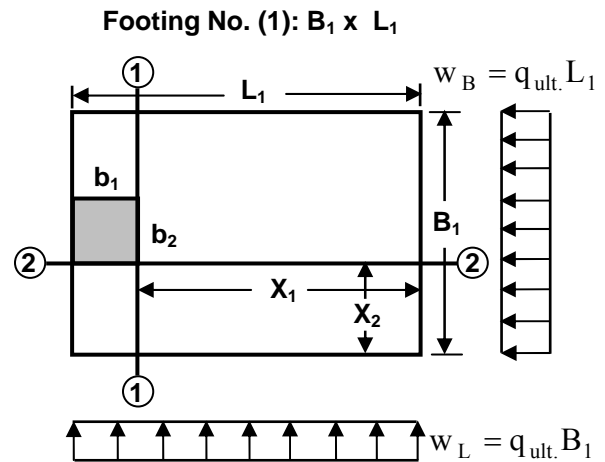
- Steel at bottom in short – direction:**

$$M_{u(2-2)} = \frac{w_B \cdot X_2^2}{2}; \quad A_s = \frac{M_{u(2-2)}}{0.9 \cdot f_y \cdot 0.9 d}$$

$$A_{s \text{ min.}} = \rho_{\text{min.}} \cdot b \cdot d; \text{ where, } \rho_{\text{min.}} \text{ is the larger of: } 1.4/f_y \text{ or } 0.25 \sqrt{f'_c} / f_y$$

$$A_{s \text{ min. (Temp. \& shrinkage)}} = 0.0020 b t$$

Compare  $A_s$  with  $A_{s \text{ min.}}$  and take the larger value for design.



- **Spacing and steel distribution:**

$$\text{Number of bars: } N = \frac{A_{s \text{ total}}}{\text{Area of bar}}$$

$$\text{Spacing (c/c)} = \frac{B - 15 \text{ cm (concrete cover)}}{N-1}$$

**(a) Steel reinforcement for footing No.(2):**

- **Steel at top in long – direction:**

$$A_s = \frac{M_u \text{ (from } M\text{-diagram)}}{0.9 f_y 0.9 d}$$

- **Steel at bottom in long – direction:**

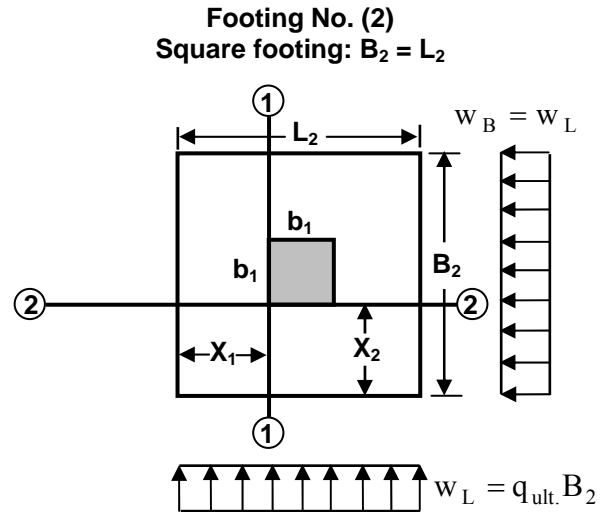
$$M_{u(1-1)} = \frac{w_L \cdot X_1^2}{2}; \quad A_s = \frac{M_{u(1-1)}}{0.9 f_y 0.9 d}$$

- **Steel at bottom in short – direction:**

$$M_{u(2-2)} = \frac{w_B \cdot X_2^2}{2}; \quad A_s = \frac{M_{u(2-2)}}{0.9 f_y 0.9 d}$$

Compare  $A_s$  with  $A_{s \text{ min.}}$  and take the larger value for design.

Or due to footing No.(2) is square, use the same steel at bottom in long – direction.



- **Spacing and steel distribution:**

$$\text{Number of bars: } N = \frac{A_{s \text{ total}}}{\text{Area of bar}}$$

$$\text{Spacing (c/c)} = \frac{B - 15 \text{ cm (concrete cover)}}{N-1}$$

**(c) Steel reinforcement in strap:**

- **Steel at top in long – direction:**  $A_s = \frac{M_u \text{ (from } M\text{-diagram)}}{0.9 f_y 0.9 d}$

- **Steel at bottom in short – direction:** Use  $A_{s \text{ min.}}$

- **Spacing and steel distribution:**

$$\text{Number of bars: } N = \frac{A_{s \text{ total}}}{\text{Area of bar}} \quad \text{at} \quad \text{Spacing (c/c)} = \frac{B - 15 \text{ cm (concrete cover)}}{N-1}$$

## (9) Check bond in short direction for each column footing

**(a) Column footing No.(1):**• **Steel in tension:**

$$l_{d(available)} = X_1 - 7.5 \text{ (concrete cover)}$$

$$l_{d(required)} = \frac{f_y}{1.1 \lambda \sqrt{f'_c}} \frac{\psi_l \psi_e \psi_s}{\left(\frac{c_b}{d_b} + \frac{K_{tr}}{d_b}\right)} d_b$$

provided that  $l_d \geq 300 \text{ mm}$ .

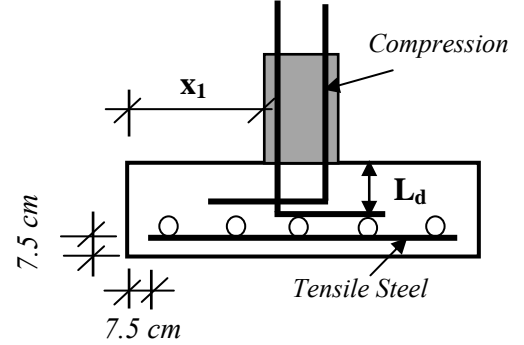
where, all notation above are as defined previously.

• **Steel in compression:**

$$l_{d(available)} = (d) \text{ of footing}$$

$$l_{dc(required)} = \frac{0.24 f_y}{\lambda \sqrt{f'_c}} d_b \geq (0.043 f_y) d_b$$

provided that  $l_d \geq 200 \text{ mm}$ .

**(b) Column footing No.(2):**• **Steel in tension:**

$$l_{d(available)} = X_1 - 7.5 \text{ (concrete cover)}$$

$$l_{d(required)} = \frac{f_y}{1.1 \lambda \sqrt{f'_c}} \frac{\psi_l \psi_e \psi_s}{\left(\frac{c_b}{d_b} + \frac{K_{tr}}{d_b}\right)} d_b$$

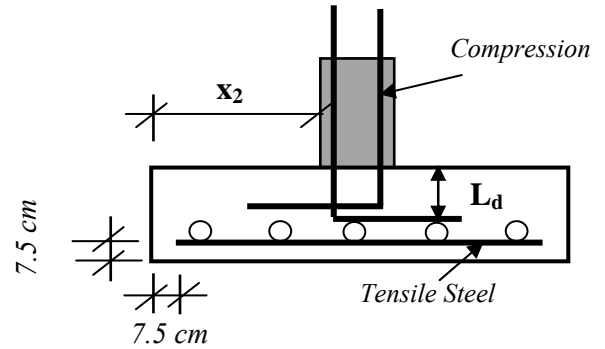
provided that  $l_d \geq 300 \text{ mm}$ .

• **Steel in compression:**

$$l_{d(available)} = (d) \text{ of footing}$$

$$l_{dc(required)} = \frac{0.24 f_y}{\lambda \sqrt{f'_c}} d_b \geq (0.043 f_y) d_b$$

provided that  $l_d \geq 200 \text{ mm}$ .



## (9) Check bearing pressure between the columns and footing:

**(a) Column (1):**  $f_c (actual) = P_{u1}/A_1$

**(b) Column (2):**  $f_c (actual) = P_{u2}/A_2$

$$f_c (allowable) = 0.85 \phi f'_c \sqrt{A_2/A_1} \leq 0.85 \phi f'_c \text{ where, } \phi = 0.65 \text{ and } \sqrt{A_2/A_1} \leq 2.0$$

**(10) Design the dowels:**

Minimum number of dowels = **4** for square or rectangular columns.

Minimum number of dowels = **6** for circular columns.

Minimum  $A_s$  of dowels =  $0.005 A_1$  .....ACI 318-14 sec.(15.8.2.1)

where,  $A_1$  = area of column.

**(1) Draw sketches showing all details required for construction.**



## 6.12 MAT FOUNDATIONS

These types of footings consist of large concrete slabs of (0.75m to 2m) thick and continuous two-way reinforcing at top and bottom that support several lines of columns or walls. Mat foundation may be supported by piles in situations such as high groundwater table to control buoyancy or where the base soil is susceptible to large settlements. In general, the critical point to design a mat foundation is the total settlement but not the bearing capacity failure. Hence, as a rule for rafts, the maximum permissible total settlement is about (2.0 inches) and the maximum permissible differential settlement is about (1.5 inches).

### 6.12.1 USES OF MAT FOUNDATIONS

- (a) When the base soil has a low bearing capacity,
- (b) The column loads are so large such that the use of conventional spread footings covers more than 50% of the total area,
- (c) When the soil strata are erratic or there is a soft layers of soil within the subsoil profile,
- (d) When there is a large differential settlements expected to occur, and
- (e) If there is a basement and groundwater table problems and there is a need to eliminate water infiltration into basement-type installations.

### 6.12.2 TYPES OF MAT FOUNDATIONS

- (1) Flat-plate:** this type is most common, less labor effort, easy to construct, more economical, and has a constant thickness for all the raft; see Fig.(6.7a).
- (2) Plate thickened under the columns:** this is to increase the punching shear capacity, without increasing the thickness for the entire mat; see Fig.(6.7b).
- (3) Waffle-plate:** this type is designed as a grid of beams and slabs in one or two directions to provide rigidity with a minimum concrete thickness; see Fig.(6.7c).
- (4) Wall-plate:** this type consists of slab-wall interaction resulting in a stiffer mat foundation; in this case, check wide-beam shear of walls in each direction; see Fig.(6.7d).
- (5) Plate with pedestals:** this type consists of pedestals as part of mat; to connect steel columns with the mat slab; see Fig.(6.7e).
- (6) Basement walls as part of mat;** see Fig.(6.7f).

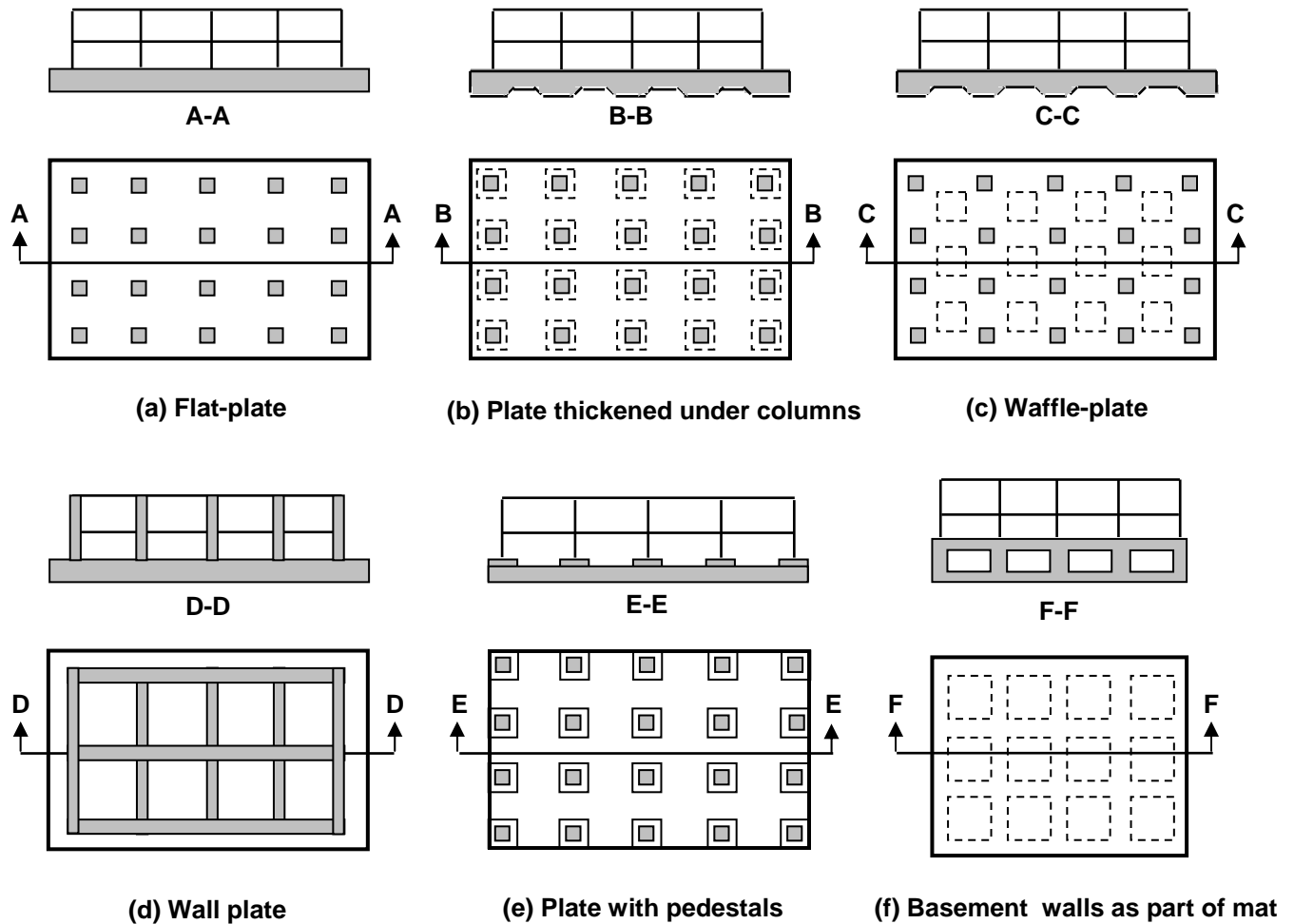


Fig.(6.7): Common types of mat foundations.

**Note:** Depending on local costs, and noting that a mat foundation requires both positive and negative reinforcing steel, it may be more economical to use spread footings even if the entire area is covered. This avoids the use of negative reinforcing steel and can be accomplished as shown in the Fig.(6.8) beside by pouring alternate footings, to avoid formwork, then using fiber spacer boards to separate the footings poured later.

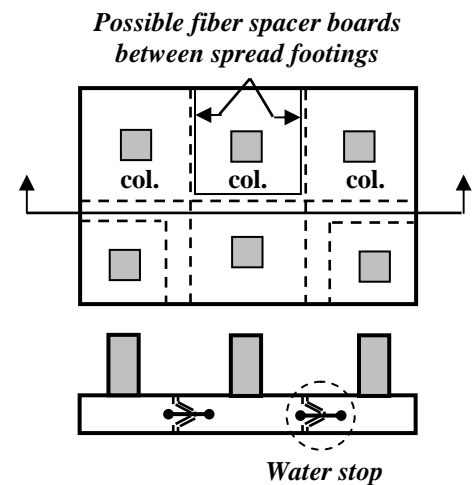


Fig.(6.8): Mat foundations consisting of a series of spread footings.

## 6.13 DESIGN OF MAT FOUNDATIONS

### 6.13.1 The Conventional Rigid Method

This method is used for one of the following cases:

1. If the mat is very thick,
2. The variation in column loads and spacing is  $< 20\%$ ,
3. If the column spacing  $< 1.75/\lambda$ , where  $\lambda = \sqrt[4]{\frac{k_s \cdot B}{4 \cdot E_s \cdot I}}$ ;  $k_s$  = modulus of subgrade reaction,

$E_s$  = modulus of elasticity of soil,  $I = \frac{B \cdot d^3}{12}$ , and  $d$  = effective depth from punching shear.

In this method, the mat is designed as continuous beams or combined footings with multiple column loads. This is done by dividing the mat into strips in each direction loaded by a line of columns and resisted by a uniform or linearly varied soil pressure with the assumption that the mat is rigid.

### 6.13.2 The Approximate Flexible Method

It is preferred when spacing between columns  $> 20\%$ , however, it can be used whatever the spacing between columns. In this method, the mat is designed as flat slab or slabs and beams.

### 6.13.3 Discrete Methods

In these methods, the mat is divided into elements by gridding such as:

1. Finite Difference Method (FDM).
2. Finite Element Method (FEM).
3. Finite Grid Method (FGM).

## 6.14 DESIGN STEPS OF MAT FOUNDATIONS

### BY CONVENTIONAL RIGID METHOD

#### (1) Calculate the total column loads:

$$\sum P = (DL + LL)_{col.1} + (DL + LL)_{col.2} + (DL + LL)_{col.3} + \dots$$

$$P_{u1} = [1.2D.L + 1.6L.L]_{col.1} ; P_{u2} = [1.2D.L + 1.6L.L]_{col.2} ; P_{u3} = [1.2D.L + 1.6L.L]_{col.3}$$

$$\sum P_{ult.} = P_{u1} + P_{u2} + P_{u3} + \dots$$

$$(r_u)_{or.}(L.F.) = \frac{\sum P_{ult.}}{\sum P}$$

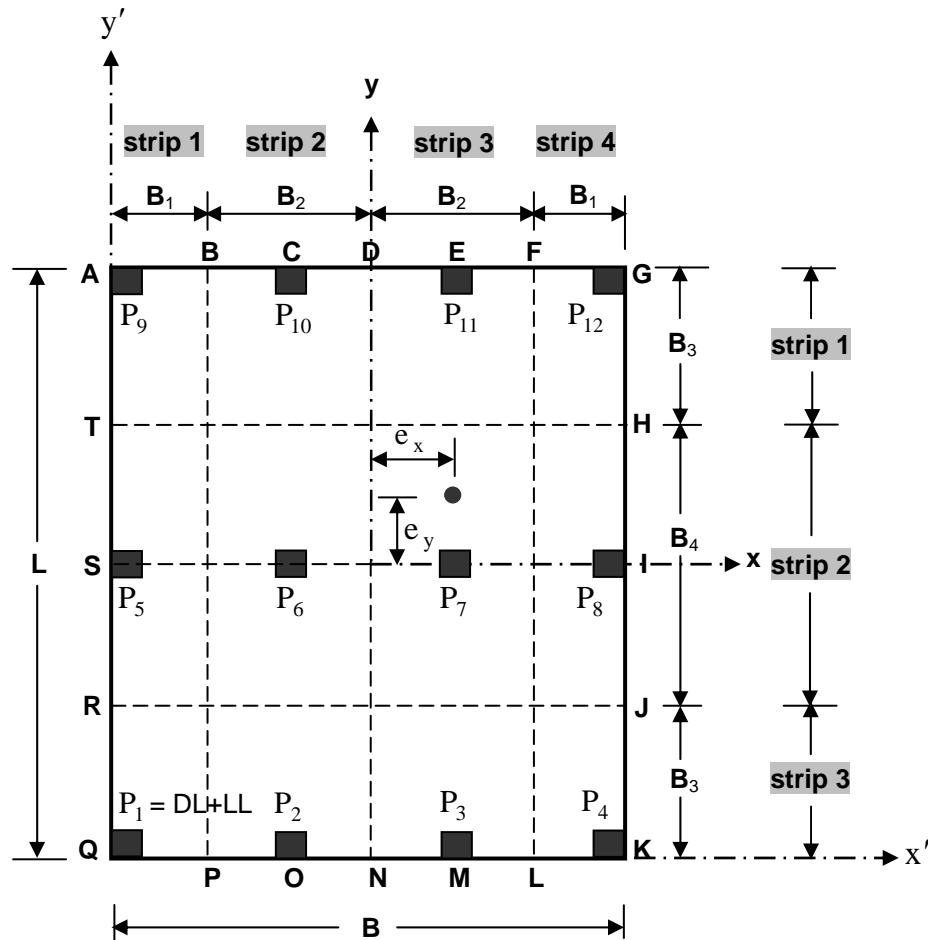


Fig.(6.9): Conventional rigid mat foundation design.

- (2) Determine the load eccentricities,  $e_x$  and  $e_y$  in the x and y directions using  $(x', y')$  coordinates (see Fig.(6.9)):

$$x' = \frac{(P_1 + P_5 + P_9).x'_1 + (P_2 + P_6 + P_{10}).x'_2 + (P_3 + P_7 + P_{11}).x'_3 + \dots}{\sum P}$$

$$e_x = x' - \frac{B}{2}$$

$$y' = \frac{(P_1 + P_2 + P_3 + P_4).y'_1 + (P_5 + P_6 + P_7 + P_8).y'_2 + (P_9 + P_{10} + P_{11} + P_{12}).y'_3 + \dots}{\sum P}$$

$$e_y = y' - \frac{L}{2}$$

- (3) Determine the soil reactions ( $q$ ) below the mat at several points such as A, B, C, D, ...using:

$$q = \frac{\sum P}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x}$$

where,

$$A = B L$$

$$I_x = BL^3/12 = \text{moment of inertia about the x-axis,}$$

$$I_y = LB^3/12 = \text{moment of inertia about the y-axis,}$$

$$M_x = \sum P.e_y = \text{moment of the column loads about the x-axis,}$$

$$M_y = \sum P.e_x = \text{moment of the column loads about the y-axis,}$$

$x$ ,...and.. $y$  = coordinates of the selected points with respect to  $x$  and  $y$  axis.

- (4) Compare the values of the soil pressures determined in step (3) with the allowable net soil pressure to determine whether  $q \leq q_{all(net)}$ .
- (5) Determine the depth ( $d$ ) of the mat by checking the punching shear failure at ( $d/2$ ) from faces of various columns:

$$v_{c all.} = 0.33 \phi \sqrt{f'_c} \text{ where, } \phi = 0.75 \text{ .....(ACI 318-14 section 11.33)}$$

$$v_{c act.} = \frac{P_{col. (L.F.)}}{b_o d}$$

The term  $b_o$  depends on the location of the column with respect to the plan of mat and can be obtained as shown in the Fig.(6.10) below.

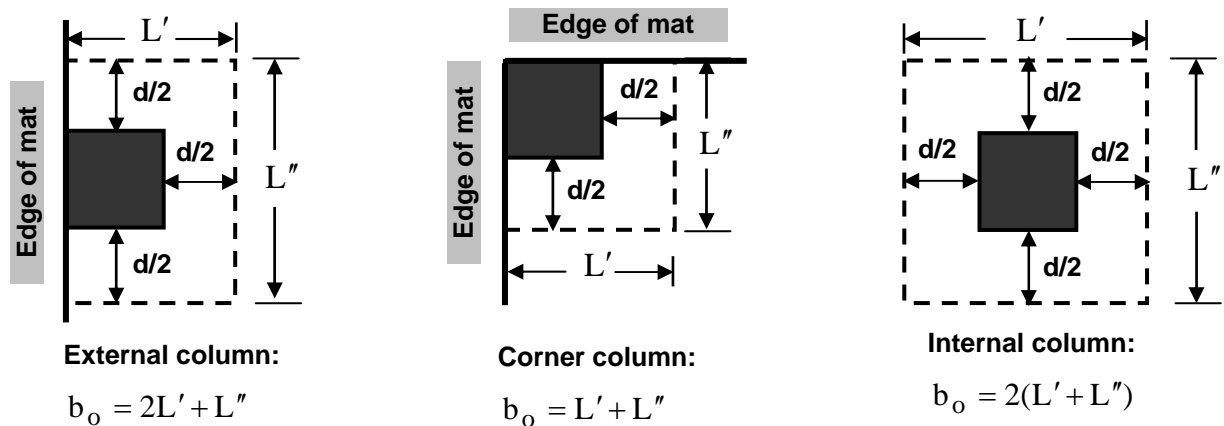


Fig.(6.10): punching area for mat foundation design.

**(6) Divide the mat into several strips in x and y directions.**

*For example*, referring to Fig.(6.9) mentioned previously, take **4 strips in y-direction** with  $B_1, B_2, B_2$ , and  $B_1$  widths, respectively, and **3 strips in x-direction** with  $B_3, B_4$ , and  $B_3$  widths, respectively.

**(7) Draw shear, V, and moment, M, diagrams for each individual strip using the modified loading (in the x and y directions).**

*For example*, the average soil pressure of the bottom (**strip JKQR**) in the x direction is:

$$q_{avg.} = \frac{q_J + q_K + q_Q + q_R}{4}$$

where,  $q_J, q_K, q_Q$ , and  $q_R$  are soil pressures at points J, K, Q, and R as determined from step 3.

$$\text{Total soil reaction} = q_{avg.} (A_{era}) = q_{avg.} (B_3 \cdot B).$$

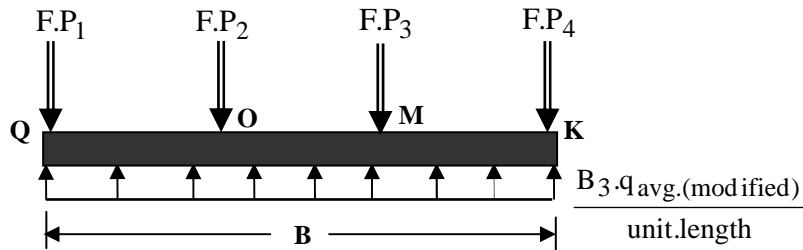
But, the sum of the column loads on the strip  $\sum P_{strip} \neq q_{avg.} B_3 \cdot B$ , because the shear between the adjacent strips has not been taken into account. Therefore, the soil reaction and the column loads need to be modified as follows:

$$\text{Average.load} = \frac{q_{avg.} B_3 \cdot B + \sum P_{strip}}{2}$$

$$\text{Load modification factor: } (F) = \frac{\text{Avg..load}}{\sum P_{strip}}$$

$$q_{avg.}(\text{modified}) = q_{avg.} \left[ \frac{\text{Avg..load}}{(q_{avg.} B_3 \cdot B)} \right]$$

So the modified column loads are:  $F.P_1, ..F.P_2, ..F.P_3$ , and  $F.P_4$ . This modified loading on the strip under consideration is shown in the Fig.(6.11) below.



**Fig.(6.11): Modified loading for strip JKQR.**

**NOTE:** After adjustment, check if the resultant load coincides with the centroid of mat or not. Then after, the shear and moment diagrams for the strip can be drawn. This procedure is repeated for all strips in the x and y directions.

- (8) From the moment diagrams of all strips in one direction (x or y), obtain the maximum positive and negative moments per unit width (i.e.,  $M = M / B_3$ ).
- (9) Determine the areas of steel per unit width for positive and negative reinforcement in x and y directions.

$$A_s = \frac{M_u}{0.9.f_y.0.9d} = \frac{M_u.(L.F.)}{0.9.f_y.0.9d}$$

$$A_{s_{min.}} = \rho_{min.} b. d$$

where,  $\rho_{min.}$  is the larger of:  $1.4/f_y$  or  $0.25\sqrt{f'_c}/f_y$

$$\begin{aligned} A_{s_{min.}}(Temp. \& shrinkage) &= 0.0020 b t \text{ ----- for } f_y < 420 \text{ MPa} \\ &= 0.0018 b t \text{ ----- for } f_y = 420 \text{ MPa} \\ &= \frac{0.0018 \times 420}{f_y} b t \text{ ---- for } f_y > 420 \text{ MPa} \end{aligned}$$

Compare  $A_s$  with  $A_{s_{min.}}$  and take the larger value for design.

**(10) Spacing and steel distribution:**

$$\text{Number of bars: } N = \frac{A_{s_{total}}}{\text{Area of bar}}$$

$$\text{Spacing (c/c)} = \frac{B - 15 \text{ cm (concrete cover)}}{N - 1}$$