CHAPTER 8

DESIGN AND ANALYSIS OF RETAINING WALLS

8.1 INTRODUCTION

Retaining walls are structures used to provide stability for earth or other materials at their natural slopes. In general, they are used to hold back or support soil banks and water or to maintain difference in the elevation of the ground surface on each of wall sides. Also, retaining walls are often used; in the construction of buildings having basements, roads, or bridges when it is necessary to retain embankments or earth in a relatively vertical position. Retaining walls are commonly supported by soil (or rock) underlying the base slab, or supported on piles; as in case of bridge abutments and where water may erode or undercut the base soil as in water front structures.

8.2 TYPES OF RETAINING WALLS

There are many types of retaining walls; they are mainly classified according to their behavior against the soil as shown in Fig.(8.1):-

(a) ***Gravity retaining walls*** are constructed of plain concrete or stone masonry. They depend mostly on their own weight and any soil resting on the wall for stability. This type of construction is not economical for walls higher than 3m.

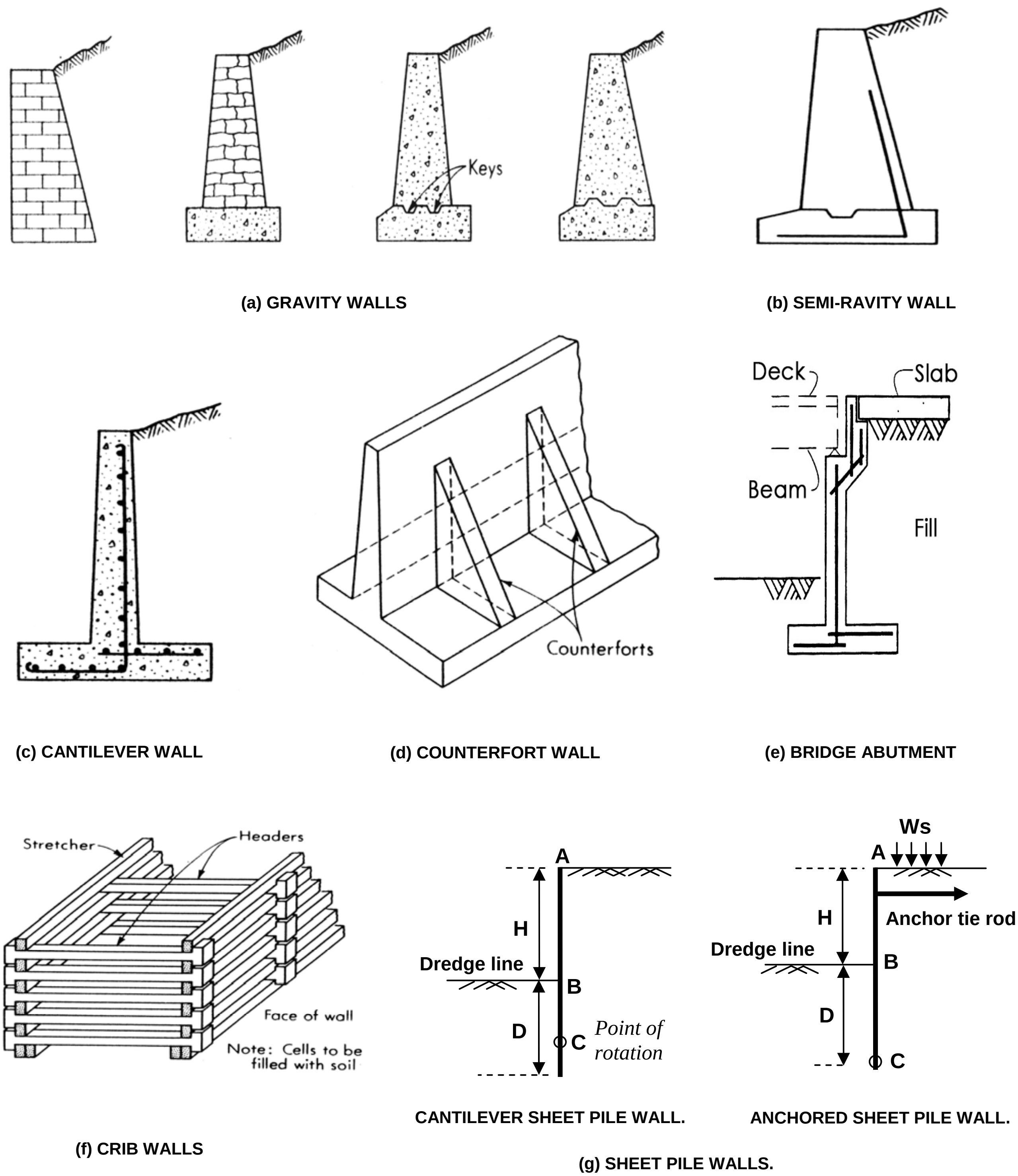
(b) ***Semi-gravity retaining walls*** are modification of gravity wall in which small amounts of reinforcing steel are introduced for minimizing the wall section.

(c) ***Cantilever retaining walls*** are the most common type of retaining walls and are generally used for wall high up to 8m. It derives its name from the fact that its individual parts behave as, and are designed as, cantilever beams. Its stability is a function of strength of its individual parts.

(d) ***Counterfort retaining walls*** are similar to cantilever retaining walls, at regular intervals, however, they have thin vertical concrete slabs behind the wall known as counterforts that tie the wall and base slab together and reduce the shear and bending moment. They are

economical when the wall height exceeds 8m. *Whereas, if bracing is in front of the wall and is in compression instead of tension, the wall is called **Buttress retaining wall**.*

(e) Bridge abutments are special type of retaining walls, not only containing the approach fill, but serving as a support for the bridge superstructure.



(a) GRAVITY WALLS

(b) SEMI-RAVITY WALL

(c) CANTILEVER WALL

(d) COUNTERFORT WALL

(e) BRIDGE ABUTMENT

(f) CRIB WALLS

CANTILEVER SHEET PILE WALL.

ANCHORED SHEET PILE WALL.

(g) SHEET PILE WALLS.

Fig.(8.1): Common types of retaining walls.

(f) Crib walls or coffer dams are cells or units to be filled with soil or built-up members of pieces of precast concrete or metal and are supported by anchor pieces embedded in the soil for stability.

(g) Sheet pile walls are classified as; anchored and cantilevered sheet pile walls; each kind of them may be used in single or double sheet walls. *Of these walls, only the **cantilever retaining walls** and the **bridge abutments** are mostly used at present due to their great economics.*

8.3 DESIGN CONSIDERATIONS

8.3.1 Definitions of Terms

Definitions of retaining wall parts are shown in Fig.(8.2) as:-

(i) **the base slab** constitutes the slab, or footing, on which the wall rests,

(ii) **the stem** is the wall itself, the face of the wall is either the exposed portion (front face) or the portion against which the backfill rests (back face),

(iii) **the toe** is the portion of the base slab which extends beyond the front face of the wall,

(iv) **the heel** is that portion of the base slab which extends away from the back face of the wall. Toe and heel are also used to denote the extreme forward and rear parts of the base slab, respectively,

(v) **a buttress** is a structural member used to tie the stem to the base slab, if the buttress is in tension, the wall is termed a **counterforted wall**, and if it is in compression, the wall is a **buttressed wall**. However, because of front clearances and appearance, the buttressed wall is rarely used. Retaining walls are often built with a batter on the front face sloping toward the backfill.

NOTE: If there is insufficient resisting force for wall stability, a key may be constructed beneath the base slab to project into the subsoil for increasing the passive earth pressure. A key is also often used when the base-slab concrete is poured separately from the stem to affect a more shear-resistant joint between the stem and base. It may also be used to form a vertical joint between the two sections of wall.

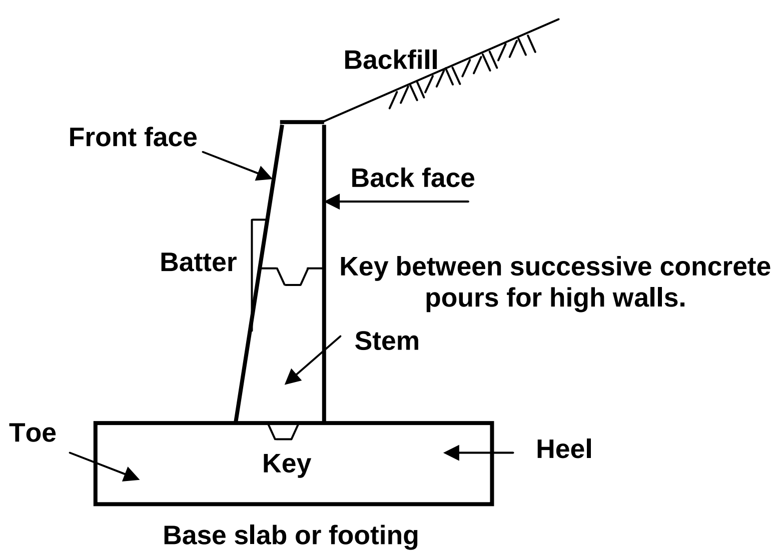


Fig.(8.2): Definitions of retaining wall parts.

8.3.2 Tentative Dimensions of Common Types of Retaining Walls

Retaining wall design proceeds with the selection of tentative dimensions, **see Fig.(8.3)** which are then analyzed for stability and structural requirements and revised as required. Since this is a trial process, several solutions of the problem may be obtained, all of which are satisfactory.

8.3.2.1 Gravity Retaining Walls

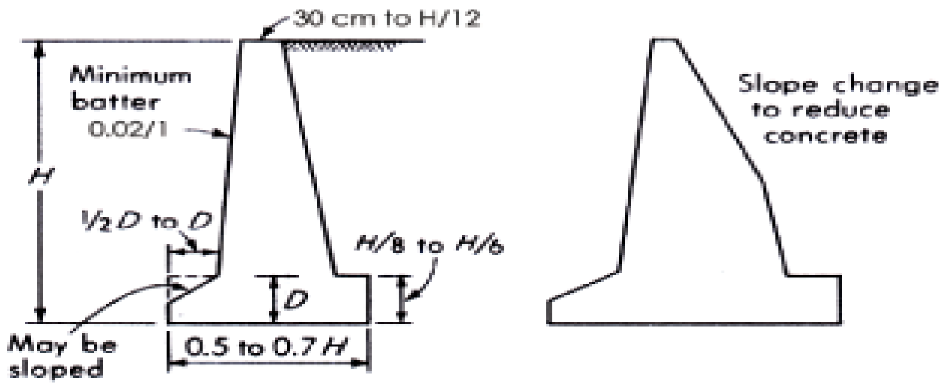
Gravity-wall dimensions may be taken as shown in Fig.(8.3-a). Gravity walls, generally, are trapezoidal in shape, but also may be built with broken backs. The base and other dimensions should be such that the resultant falls within the middle one-third of the base. The top width of the stem should be not less than 30cm. Because of the massive proportions and resulting low concrete stresses, low-strength concrete can generally be used for the wall construction.

8.3.2.2 Cantilever Retaining Walls

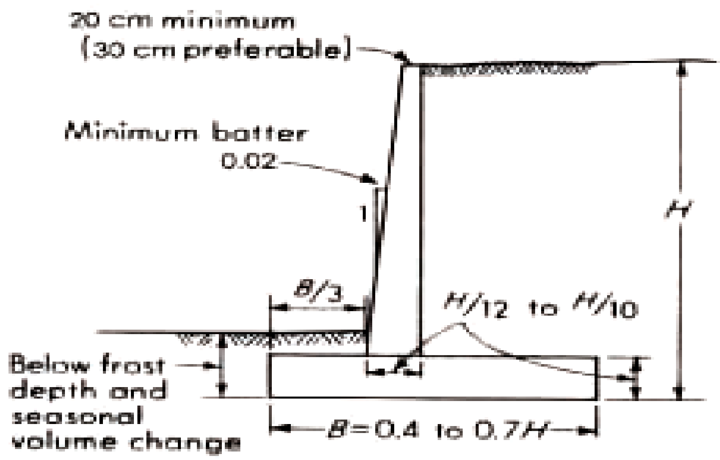
Dimensions of the retaining wall should be adequate for structural stability and satisfy local building-code requirements. The tentative dimensions shown in Fig.(8.3-b) are based in part on the history of satisfactorily constructed walls, and may be used in the absence of other data. However, it may result in an overly conservative design. The top width of the stem should not be less than 30cm. While the base of the stem should be thick enough to satisfy the shear requirements without use of shear reinforcing steel. The base-slab dimensions should be such that the resultant of the vertical loads falls within the middle one-third. If the resultant falls outside the middle one-third, the toe pressures may be excessively large and only a part of the footing will be effective.

8.3.2.3 Counterfort Retaining Walls

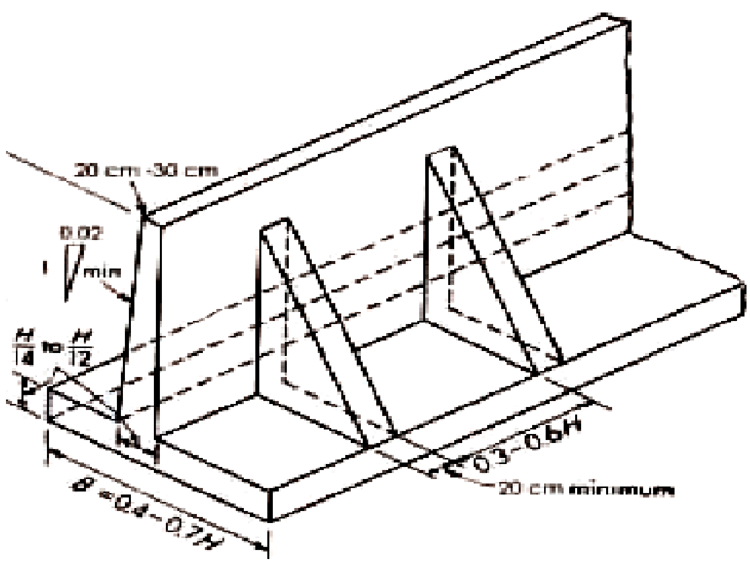
Typical proportions for counterfort retaining walls are as shown in Fig.(8.3c). These dimensions are only a guide, and thinner walls of (10-15) cm thick sections may be used if structural stability is satisfied. The use of a counterfort will be determined by the relative costs of forms; concrete, reinforcing, and labor. The spacing of the counterforts is a trial process to give a minimum cost. The most economical spacing appears to be (1/3-1/2) the height of the wall. A counterfort may be built into the beginning of the wall or by allowing a part of the wall to overhang. The overhanging configuration may prove to be more economical since it saves the concrete and formwork on the two counterforts at the joint. The counterfort wall may be constructed without a toe if additional front clearance is needed and the sliding and overturning stability requirements are met.



(a) GRAVITY WALL



(b) CANTILEVER WALL



(c) COUNTERFORT RETAINING WALL

Fig.(8.3): Tentative dimensions of common types of retaining walls.

8.4 FORCES ACTING ON RETAINING WALLS

The design of a retaining wall must account for all applied loads. The loads that are of primary concern are the lateral earth pressures induced by the retained soil. Under normal conditions, the lateral earth pressure is at rest condition. But, if the wall deflects slightly, stresses were exerted in the soil, these are; a passive earth pressure (P_P) in front of the wall, and an active earth pressure (P_a) behind the wall. For design purposes, the passive earth pressure in front of the wall, is neglected to avoid any problem resulting from removing the soil in front of the wall.

The active and passive pressures are assumed to increase linearly with depth as a function of the weight of soil. The magnitude and direction of these pressures as well as their distribution depend upon many variables; such as height of the wall, the slope of the ground surface (β), type of backfill used, draining of the backfill, level of the water table, added loads applied on the backfill (surcharges either live or dead loads), degree of soil compaction, and movement of the wall caused by the action of the backfill. The forces acting on a retaining wall with level or inclined backfill are shown Fig.(8.4).

The active and passive earth pressures are computed as:

$$P_a = \frac{1}{2}.\gamma H'^2.K_a \quad \textbf{(8.1)}$$

$$P_p = \frac{1}{2}.\gamma H_p^2.K_p \quad \textbf{(8.2)}$$

where, the coefficients of active and passive lateral earth pressures are computed as:

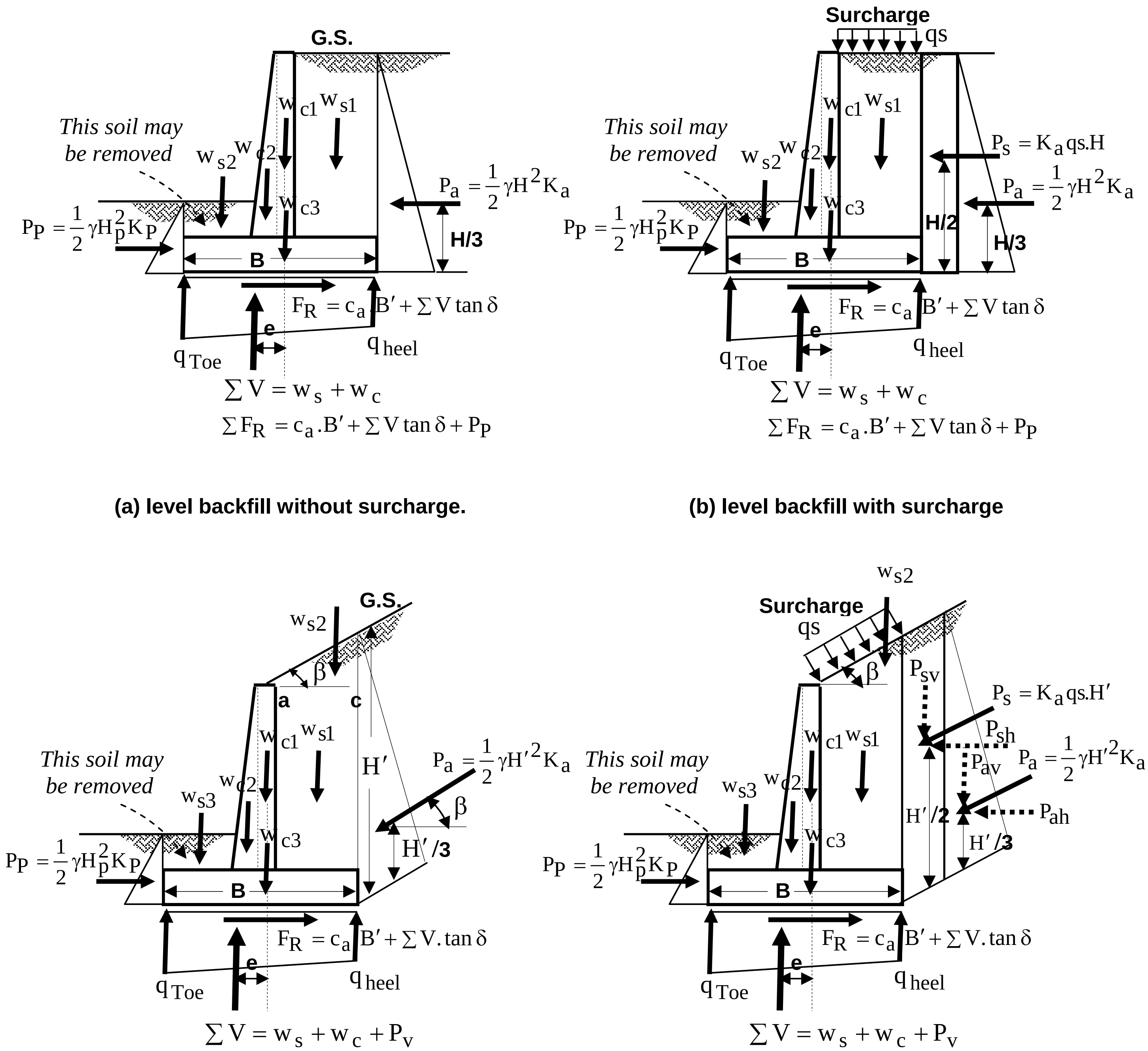
<u>*For a level backfill:*</u>

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} \quad \textbf{or} \quad K_a = \tan^2(45-\phi/2) \quad \textbf{(8.3)}$$

<u>*For an inclined backfill:*</u>

$$K_a = \cos\beta\,\frac{\cos\beta-\sqrt{\cos^2\beta-\cos^2\phi}}{\cos\beta+\sqrt{\cos^2\beta-\cos^2\phi}} \quad \textbf{(8.4)}$$

<u>NOTE:</u> A surcharge load has a same effect as an additional (equivalent) height of earth (H_{su}) above the ground surface obtained as: $H_{su} = W_{su}/\gamma_{backfill}$ where W_{su} is the surcharge load per square unit and $\gamma_{backfill}$ is the unit weight of backfill soil. This additional height due to surcharge, adds a rectangle of pressure behind the wall with a total lateral force assumed acting at its mid-height



(a) level backfill without surcharge. **(b) level backfill with surcharge**

where: $P_{ah} = P_a \cos\beta$, $P_{av} = P_a \sin\beta$, $H' = H + ac.\tan\beta$,

$ws = ws_1 + ws_2 + ws_3$, $wc = wc_1 + wc_2 + wc_3$, $\sum F_R = c_a.B' + \sum V \tan\delta + P_P$

(c) Sloped backfill without surcharge. **(d) Sloped backfill with surcharge**

Fig.(8.4): Forces acting on a retaining wall.

8.5 STABILITY CONSIDERATIONS

At the beginning, tentative dimensions can be used and then analyzed for both external and internal (structural design requirements), for these purposes, computer programs for design and analysis of retaining walls may be helpful.

8.5.1 EXTERNAL STABILITY

This stability includes ***five checks*** as shown below and explained with reference to Fig.(8.5).

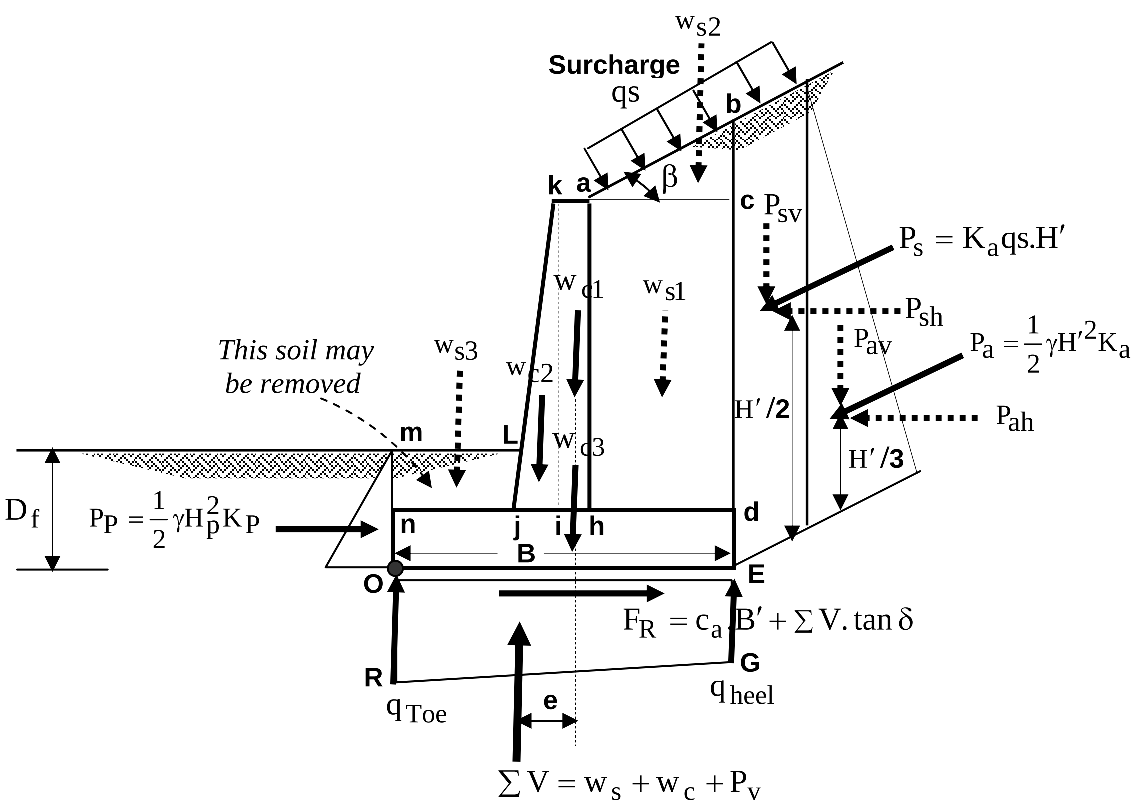
(1) Check for Overturning about Toe (point O),

(2) Check for Sliding along the Base of the Wall,

(3) Check for Bearing Capacity Failure of the Base Soil,

(4) Check for Settlement, and

(5) Check Rotational or Deep Shear Failure.



where, $P_{ah} = P_a \cos\beta$, $P_{av} = P_a \sin\beta$, $H' = H + ac.\tan\beta$,

$ws = ws_1 + ws_2 + ws_3$, $wc = wc_1 + wc_2 + wc_3$ $\sum F_R = c_a B' + \sum V \tan\delta + P_P$

Fig.(8.5): Forces acting on a retaining wall (*Sloped backfill with surcharge*).

(1) Check for Overturning about Toe (point O):

$$SF_{overturning} = \frac{\text{Resisting.Moments}}{\text{Overturning.Moments}} = \frac{\sum M_R}{\sum M_o} \quad \textbf{..(8.5)}$$

≥ 1.5 for cohesionless soils **or** ≥ 2.0 for cohesive soils.

To determine the resisting forces and moments, the following table should be prepared:

Part	Weight (kN/m)	Arm from O (m)	Moment (kN-m/m)
Soil: (1) (2) (3)	ws1 ws2 ws3	xs1 xs2 xs3	
Concrete: (1) (2) (3)	wc1 wc2 wc3	xc1 xc2 xc3	
P_{sv}	$P_s \sin\beta$		
P_{av}	$P_a \sin\beta$		

$\sum V = \boxed{\quad}$ $\sum M_R = \boxed{\quad}$

Overturning moment: $\sum M_o = P_{ah}.(H'/3) + P_{sh}.(H'/2)$

(2) Check for Sliding along the Base of the Wall:

In sliding stability analyses, it is common practice to omit the soil in front of the wall.

$$SF_{Sliding} = \frac{\text{Resisting.Forces}}{\text{Sliding.Force}} = \frac{\sum F_R}{F_S} \quad \textbf{..(8.6)}$$

≥ 1.5 for cohesionless soils **or** ≥ 2.0 for cohesive soils

where, the sliding force $(F_S) = (P_a + P_s)$ **or** $(P_{ah} + P_{sh})$

$P_a = \frac{1}{2}\gamma H^2 K_a$ **……………...*for level ground surface,***

$P_{ah} = \frac{1}{2}\gamma H'^2 K_a \cos\beta$ **……..*for inclined ground surface,***

Resisting force = $\sum F_R = C_a.B' + \sum V.\tan\delta$

$\sum V$ = all the vertical forces, including the vertical component of P_a,

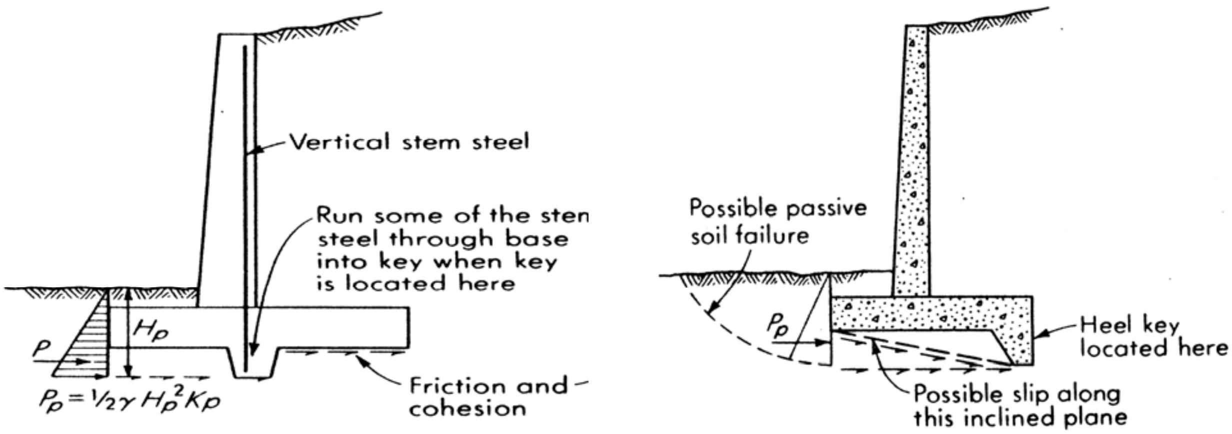
$B' = B - 2e_B$ = the effective length of the base slab,

$e_B = \frac{B}{2} - \bar{x}$,

Location of resultant of $\sum V$ from Toe $(\bar{x}) = \dfrac{Net.Moment}{\sum V} = \dfrac{\sum M_R - \sum M_o}{\sum V}$

$C_a = \frac{2}{3}c..to..\frac{3}{4}c$ and $\delta = \frac{2}{3}\phi..to..\frac{3}{4}\phi$

NOTE: If $SF_{Sliding}$ is **unsafe**: Increase the base dimension **B**, or Use a key beneath the base **near the stem** or **at the heel**, as shown in Fig.(8.21) until $SF_{Sliding} \geq 1.5 - 2.0$



(a) key near the stem. **(b) Key at the heel** (*more effective*)**.**

Fig.(8.21): Effect of shear key on retaining wall stability.

(3) Check for Bearing Capacity Failure of the Base Soil:

$$SF_{Bearing.Capacity} = \frac{Net.ultimate.bearing.capacity}{Max..bearing.pressure} = \frac{q_{ult.(net)}}{q_{actual}} \quad \text{..........................}\mathbf{(8.7)}$$

$\geq 2.5\text{-}3.0$

Calculate the eccentricity by:

$$e_B = \frac{B}{2} - \bar{x} = \frac{B}{2} - \frac{\sum M_R - \sum M_o}{\sum V}$$

Check e_B **with** $B/6$: to see whether the resultant of $\sum V$ (all the vertical forces, including the vertical component of P_a **)** is within the middle third or not, and falls to the right or to the left of the wall centerline.

- If $e \leq B/6$, the maximum bearing pressure is calculated by:

$$q_{actual} = q_{\substack{max.\\min.}} = q_{\substack{Toe\\Heel}} = \frac{\sum V}{B.(1)}(1 \pm \frac{6.e_B}{B})$$

- If $e. > B/6$, the maximum bearing pressure is calculated by:

$$q_{max.} = \frac{2.\sum V}{3.B\left[\frac{L}{2} - e_B\right]} \quad \text{and} \quad q_{min.} = 0$$

NOTE: In this case, it is better to change the dimension (B) until the eccentricity be $e \le B/6$.

The net ultimate bearing capacity of the base soil can be calculated from Hansen's equation, considering the wall as a strip footing with width B' at a depth D_f using c_2 and ϕ_2 shear strength parameters for the base soil.

$$q_{ult.(net)} = cN_c S_c d_c i_c + q(N_q - 1)S_q d_q i_q + 0.5\gamma.B'.N_\gamma S_\gamma d_\gamma i_\gamma \quad \textbf{………..……..(8.8)}$$

where,

c = cohesion of the base soil,
q' = surcharge load or overburden pressure for shallow side,
γ = unit weight of the base soil,
$B' = B - 2e_B$; B' is the retaining wall effective base width,
$N_c, N_q, .N\gamma$ = Hansen's bearing capacity factors obtained from:-

$$N_q = e^{\pi.\tan\phi}\tan^2(45 + \phi/2)\,; \quad N_c = (N_q - 1).\cot\phi\;; \quad N_\gamma = 1.5(N_q - 1).\tan\phi$$

$S_c, S_q, .S\gamma$; $d_c, d_q, .d\gamma$; and $i_c, i_q, .i\gamma$ = Shape, depth, and inclination factors obtained from Table (8.1).

Table (8.1): Shape, depth, and inclination factors for Hansen's equation.

Shape factors	*Depth factors*	*Inclination factors*
$S_c = S_q = S\gamma = 1.0$ since the retaining wall is a continuous footing (L/B >10)	$d_c = 1 + 0.4k*$	$i_c = i_q - \frac{1 - i_q}{N_q - 1}$
	$d_q = 1 + 2.\tan\phi(1 - \sin\phi)^2 k*$	$i_q = \left(1 - \frac{0.5H}{V + A_f C_a \cot\phi}\right)^5$
	$d\gamma = 1.0$ for all ϕ values	$i_\gamma = \left(1 - \frac{0.7H}{V + A_f C_a \cot\phi}\right)^5$

* **NOTE:**

$$k = \frac{D_f}{B} \text{ for } \frac{D_f}{B} \le 1$$

$$k = \tan^{-1}\frac{D_f}{B} \text{ for } \frac{D_f}{B} > 1$$ **(in radians)**, D_f is the depth of footing from the **shallow side.**

(4) Check for Settlement $\left[S_T \le S_{all.}\right]$:

Calculate the total settlement components ***as mentioned in chapter five*** to know whether it will be acceptable or not in comparison of the permissible or tolerable or allowable settlement.

(5) Check Rotational Stability:

Usually, tilting is the result of rotation about toe. This may be attributed to an adequate backfill weight or by the foundation failure in the zone of the toe resulting from a poor layer of soil underlying the footing.

The rotational stability can be investigated using the Swedish circle method as follows:

(1) Draw the wall-soil system and soil layers to convenient and large scale.

(2) Draw a circle with radius sufficient to penetrate into any soft underlying layers.

(3) Compute all the forces acting against the vertical plane through the heel point and moment arm with respect to the trial circle center.

(4) Divide the trial circle into a convenient number of slices and compute the slice weight and the friction and cohesion (tangential) components acting on the base of each slice.

(5) Conduct a moment summation about the circle center to obtain the safety factor as:

$$SF_{Rotational..Stability} = \frac{\sum M_R}{\sum M_o} .. \ge 1.5 \qquad \textbf{(8.9)}$$

- ***For level backfill:*** $SF_{Rotational..Stability} = \dfrac{\left(\sum N.\tan\phi + c.L\right)R}{R(\sum T) + P_a.\bar{y}}$
- ***For inclined backfill:*** $SF_{Rotational..Stability} = \dfrac{\left(\sum N.\tan\phi + c.L\right)R}{R(\sum T) + P_{ah}.\bar{y} + P_{av}.\bar{x}}$

where, $\tan\phi =$ coefficient of friction, c = cohesion of soil, $L = (R.\theta)$;length of trial circle arc.

(6) Make several trials so that the minimum factor of safety is found. If this is too small, a revision may be made to wall dimensions, or the base is placed at a greater depth. The safety factor should not be less than 1.5.

NOTE: when the slip surface passes through several soil layers, c.L will be equal to $c_1.L_1 + c_2.L_2 + c_3.L_3 + \ldots\ldots$

8.5.2 INTERNAL STABILITY

(1) Design of Stem:

Shear and moments in the stem are found using differential equations since the pressure distribution is triangular:-

Load: $q_y = K_a.\gamma.y$ .... *for a level backfill,*

$q_y = K_a \cos\beta.\gamma.y$ .... *for inclined backfill.*

Shear: $V_y = \int_0^h q_y.dh$

$V_y = \frac{1}{2}K_a.\gamma.y^2 + K_a.q_s.y$ ....*for a level backfill with surcharge,*

$V_y = \frac{1}{2}K_a \cos\beta.\gamma.y^2 + K_a \cos\beta.qs.y$ ...... *for inclined backfill with surcharge.*

Moment: $M_y = \int_0^h V_y.dh$

$M_y = \frac{1}{6}K_a.\gamma.y^3 + \frac{1}{2}K_a.q_s.y^2$ ....*for a level backfill with surcharge,*

$M_y = \frac{1}{6}K_a \cos\beta.\gamma.y^3 + \frac{1}{2}K_a \cos\beta.q_s.y^2$. *for inclined backfill with surcharge.*

Divide the stem into (4) sections that is at; y = 0, 0.25H, 0.5H, 0.75H, and H. Then, determine (d) from wide beam shear and moments as shown below and compare the obtained (d) values with those available and use the larger (d) value.

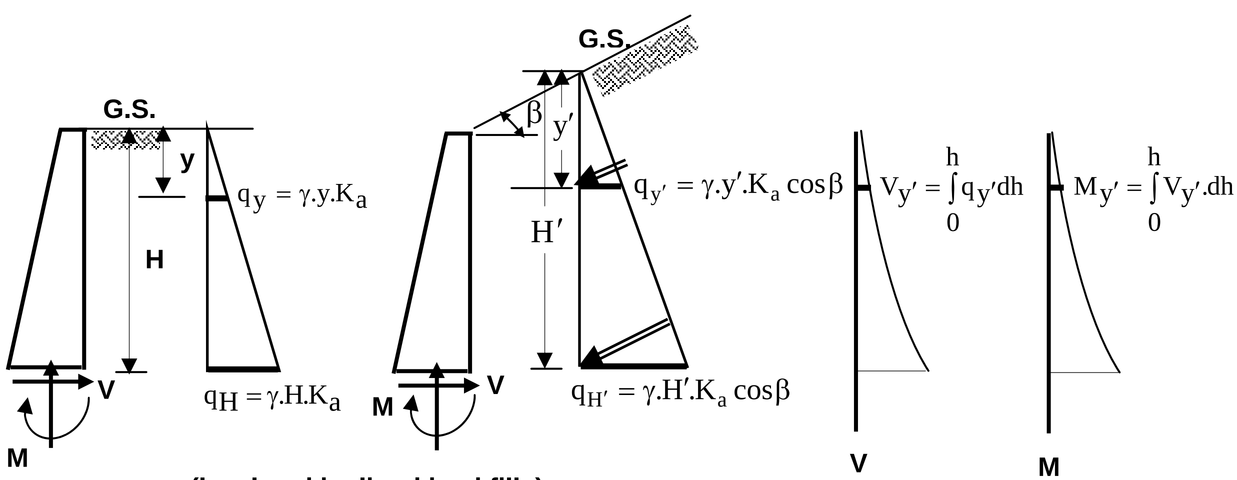


Fig.(8.7): Shear and moment along the stem.

- **(d) from wide beam shear:**

$v_{c\,all.} = 0.17(0.75)\sqrt{f'_c}$ ……..……………..…………….…..…(ACI 318−14 section 11.3)

$v_{c\,act.} = \dfrac{V_y\,(L.F.)}{b\;\;d}$; take $v_{c\,all.} = v_{c\,act.}$ and solve for (d).

- **(d) from moment**:

$$d = \sqrt{\frac{6\;M_y\,(L.F.)}{f_{t\,all.}}}$$ *where,* $f_{t\,all.} = 0.42(0.60)\sqrt{f'_c}$ ……....….…(ACI 318−14 section 22.2)

- **Stem thickness**:

$t_{Bottom} = t_{Top} + S.x$

where, S is the slope of the stem calculated as: $S = (t_{Bottom} - t_{Top})/H$

$d_{available} = t - 7.5$ cm (concrete cover).

- **Stem reinforcement (As):**

$$A_s = \frac{M_u}{0.9\,f_y\;0.9\,d} = \frac{M_y\;(L.F.)}{0.9\,f_y\;0.9\,d}$$

$As_{min.} = \rho_{min.}\,b.\,d$ where, $\rho_{min.}$ is the larger of: $1.4/f_y$ **<u>or</u>** $0.25\sqrt{f'_c}/f_y$

$As_{min.\,(Temp.\,\&\,shrinkage)} = 0.0020$ b t ------------ for $f_y < 420$ MPa

$= 0.0018$ b t ---------- for $f_y = 420$ MPa

$= \dfrac{0.0018 \times 420}{f_y}$ b t ---- for $f_y > 420$ MPa

Compare As with $As_{min.}$ and take the larger value for design as well as extend the steel reinforcement beyond cutoff points to satisfy ACI Code bond requirements.

<u>(a) Development length for stem tensile steel:</u>

$$l_{d(required)} = \frac{f_y}{1.1\,\lambda\sqrt{f'_c}}\;\frac{\psi_t\;\psi_e\;\psi_s}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}\;d_b$$ *provided that* $l_d \geq$ *300 mm.*

<u>(b) Development length for stem compression steel:</u>

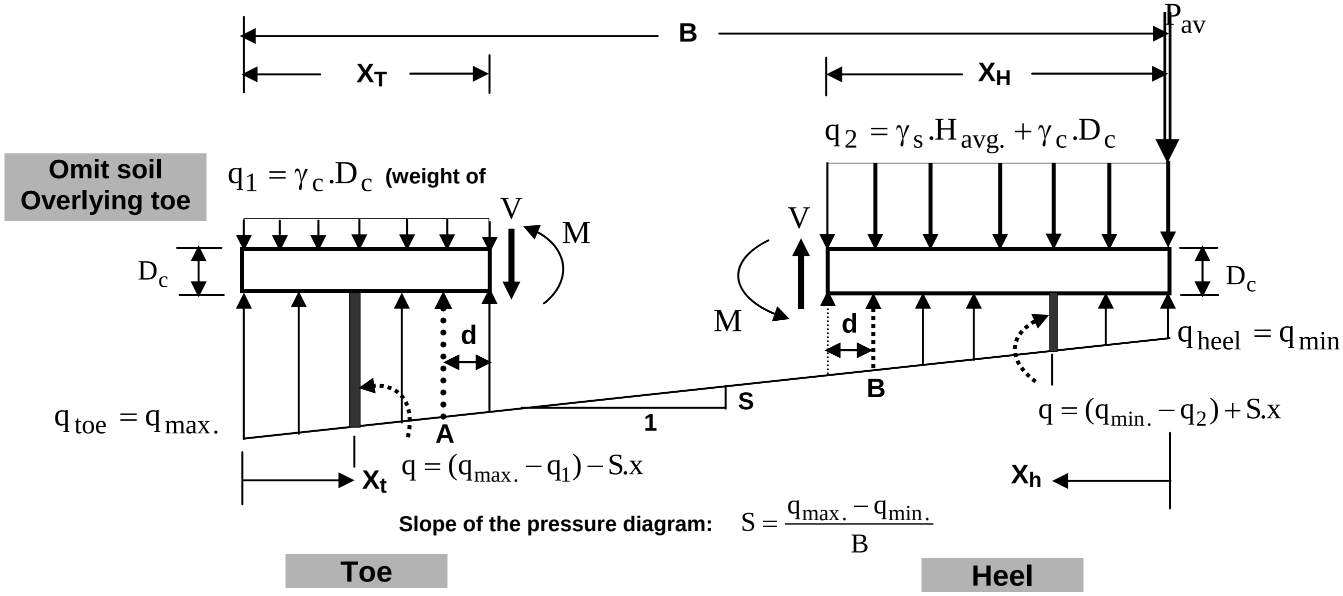
$$l_{dc(required)} = \frac{0.24\;f_y}{\lambda\sqrt{f'_c}}\;d_b \geq (0.043\,f_y)\;d_b$$ *provided that* $l_d \geq$ *200 mm.*

All notations mentioned above are as defined previously in Chapter Six.

Depth y (m)	V_y (kN)	M_y (kN.m/m)	(d) wide beam shear (m)	(d) moment (m)	(d) Available (m)	As (cm^2/m)
0						
0.25H						
0.50H						
0.75H						
H						

(2) Design of the Base Slab:

The pressure distribution on the base is shown below:-



Equations for Toe Design	Equations for Heel Design
$q = (q_{max.} - q_1) - S.x$	$q = (q_{min.} - q_2) + S.x$
V = $\int_0^{XT} q.dx \quad = (q_{max.} - q_1)x - \dfrac{S.x^2}{2}$	**V =** $\int_0^{XH} q.dx \quad = (q_{min.} - q_2)x + \dfrac{S.x^2}{2} - P_{av}$
M = $\int_0^{XT} V.dx = (q_{max.} - q_1)\dfrac{x^2}{2} - \dfrac{S.x^3}{6}$	**M =** $\int_0^{XH} V.dx = (q_{min.} - q_2)\dfrac{x^2}{2} + \dfrac{S.x^3}{6} - P_{av}.x$
The thickness of toe and heel is calculated as:	
Find ***V*** *at (d) from the face of the stem; at point* ***(A)*** where: $(x = x_T - d)$, and $d = t_{base} - 7.5cm - d_b/2$.	*Find* ***V*** *at (d) from the face of the stem; at point* ***(B)*** where: $(x = x_H - d)$, and $d = t_{base} - 7.5cm - d_b/2$.
$\nu c_{all.} = (0.17)(0.75)\sqrt{f'c}$; $\nu c_{act.} = \dfrac{V(x_T - d).L.F.}{b..d}$ Put $\nu_{c\,all.} = \nu_{c\,act.}$ and solve for (d) = ?	$\nu c_{all.} = (0.17)(0.75)\sqrt{f'c}$; $\nu c_{act.} = \dfrac{V(x_H - d).L.F.}{b..d}$ Put $\nu_{c\,all.} = \nu_{c\,act.}$ and solve for (d) = ?
The required reinforcement is calculated as:	
Find ***M*** *at face of stem*; at $x = x_T$: [i.e., $M(x_T)$] $A_s = \dfrac{M_u}{0.9.fy.0.9.d} = \dfrac{M(x_T).L.F.}{0.9.fy.0.9.d}$ *Compare* As *with* $As_{min.}$ *and take the larger value.*	*Find* ***M*** *at face of stem*; at $x = x_H$: [i.e., $M(x_H)$] $A_s = \dfrac{M_u}{0.9.fy.0.9.d} = \dfrac{M(x_H).L.F.}{0.9.fy.0.9.d}$ *Compare* As *with* $As_{min.}$ *and take the larger value.*